Temperature and pressure dependence of ion–molecule association and dissociation reactions: the \( \text{N}_2^+ + \text{N}_2 (\pm \text{M}) \leftrightarrow \text{N}_4^+ (\pm \text{M}) \) reaction†‡

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Thermal dissociation and the reverse association reactions of molecular ions can be represented within the same framework as unimolecular reactions of neutral molecules. However, because of different intra- and intermolecular interaction potentials, some differences between ionic and neutral systems should also be expected. Starting from these generalities, the present article analyzes the temperature and pressure dependences of dissociation/association rates of ion–molecule reaction systems. The reaction \( \text{N}_2^+ + \text{N}_2 (\pm \text{M}) \leftrightarrow \text{N}_4^+ (\pm \text{M}) \) is considered in particular and falloff curves for this reaction are constructed over the temperature range 5–600 K.

Falloff corrections are applied to earlier experiments which allows for a representation of the rate coefficients in terms of a limiting low-pressure rate coefficient \( k_{\text{asst}}[\text{N}_2] \text{ cm}^{-6} \text{ molecule}^{-1} \text{ s}^{-1} = 6.8 \times 10^{-29} \text{ (300 K)/T} \), a limiting high-pressure rate coefficient \( k_{\text{hst, as}}[\text{N}_2] \text{ molecule}^{-1} \text{ s}^{-1} = 8.33 \times 10^{-10} \text{ [1 + 0.069(300 K)/T]} \), and a center broadening factor of the falloff curve \( F_c = 0.53 \pm 0.05 \) nearly independent of the temperature. Other examples such as the reactions \( \text{NH}_4^+ + \text{NH}_3 \leftrightarrow \text{N}_2\text{H}_7^+, \text{C}_2\text{H}_7^+ + \text{CH}_3 \leftrightarrow \text{C}_8\text{H}_{10}^+, \text{and C}_2\text{H}_5^+ + \text{C}_2\text{H}_5 \leftrightarrow \text{C}_9\text{H}_{12}^+ \) are illustrated for comparison.

1 Introduction

Radical–radical association and the reverse thermal dissociation reactions of neutral species play an important role, e.g. in combustion and in atmospheric chemical kinetics. The temperature and pressure dependences of their rates are well represented within the framework of unimolecular rate theory and they are documented extensively and in compact form in data evaluations such as refs. 1–4. The corresponding information on ion–molecule reactions, in spite of the importance for various applications, is much less complete and waits for more systematic research. In part this is due to experimental limitations, both in the accessible temperature and pressure ranges. The situation, however, is improving e.g. by the use of the CRESu technique towards lower temperatures (see e.g. refs. 5–7), the SIFT and flowing afterglow techniques towards higher temperatures (see e.g. refs. 8–10), and by the use of the turbulent ion flow tube (TIFT, see e.g. refs. 9–12) or REMPI-LIF studies in static and flow systems (see e.g. refs. 13 and 14) towards higher gas pressures. With these techniques, one can now envisage measurements over broader ranges of conditions such that compact representations of the rate coefficients \( k(T,p) \) as a function of temperature \( T \) and pressure \( p \) will become available as well.

Obviously \( k(T,p) \) should be represented in the same way for ionic as for neutral reactions. Separate formats of data representation, analyses, and terminologies within the fields of ionic and neutral reactions, such as they have often been employed in the past, appear undesirable and should be avoided. Falloff curves characterizing the pressure dependence of \( k(T,p) \), e.g., should be treated by RRKM theory such as demonstrated for proton-bound dimers of amines by Olmstead, Lev-On, Golden and Brauman,25 or by the more advanced versions of unimolecular rate theory available today, see below. Specific rate constants \( k(E,J) \) for the fragmentation of neutrals and ions should be expressed in the same way such as illustrated in unimolecular reaction monographies (see e.g. refs. 16 and 17).

Nevertheless, the interesting question remains in which details of the treatment the neutral and ionic systems differ. The intra- and intermolecular interaction potentials, which are relevant for the reaction, often are of different character which may (or may not) lead to characteristic differences in the rate coefficients \( k(T,p) \). This is one of the aspects of the present article.

Another issue is the applicability of phase space theory (PST) which has been used in various investigations particularly of ionic systems (see e.g. refs. 18–20). It has become clear that specific rate constants \( k(E,J) \) and the corresponding high-pressure rate constants \( k(T,p \rightarrow \infty) \) of ion–molecule dissociation/association reactions often can not be described by PST, see e.g. the recent refs. 12, 21, 22 and earlier work. On the other hand, product translational distributions17 in the fragmentation of molecular ions often are well represented by PST: this apparent contradiction needs to be explained. A further question is the effectivity of intermolecular energy transfer in ion–molecule dissociation/association reactions.11,12 The following analysis of a few model systems tries to shed some light on these issues.

We choose the reaction system \( \text{N}_2 + \text{N}_2^+ (\pm \text{M}) \leftrightarrow \text{N}_4^+ (\pm \text{M}) \) as a representative example because ample experimental information on this system is available22,23–33 and earlier modellings of the rate constant have been published (see e.g. refs. 13, 32 and 34). We demonstrate how a quick and simple analysis of the experiments can be made which allows for an orientation about the falloff curve of the reaction and the location of the experiments along this curve. Based on the resulting corrections, tentative limiting low- and high-pressure rate coefficients are obtained. The further analysis compares...
2 Experimental results

The pressure dependence of the reaction

\[ \text{N}_2^+ + \text{N}_2 (+ \text{M}) \rightarrow \text{N}_4^+ (+ \text{M}) \]  

(1)

has been investigated only rarely. Mostly the reaction was assumed to be in the termolecular limiting low-pressure range, although some suspicions about deviations from this limit were expressed, e.g. in refs. 31 and 34. Only the REMPI-LIF work by Frost and Sharp\(^1\)) more systematically studied the pressure dependence from 1 to 13 Torr in the bath gas \(M = \text{N}_2\), and from 10 to 490 Torr in the bath gas \(\text{He}\). Fig. 1 shows the results which cover a certain part of the falloff curve with a transition from third to second reaction order. Tentative falloff curves are drawn as full lines such as described below. The temperature dependence has been studied extensively over the broad range 4–530 K, see refs. 23–33. Fig. 2 shows a plot of \(\log k_{\text{ass}}[\text{N}_2] \) vs. \(\log T\) where \(k_{\text{ass}}\) is the pseudo-second order rate coefficient and experiments in the bath gas \(M = \text{N}_2\) are included as full symbols. It has only occasionally been questioned whether the data from Fig. 2 really correspond to the third-order limiting low-pressure rate coefficient of reaction (1) and some deviations from the general trend in ref. 34 were tentatively attributed to falloff effects. However, without data such as shown in Fig. 1 and/or at least a simplified modelling such as demonstrated in the following, corrections for falloff were difficult if not impossible to quantify and falloff effects were difficult to distinguish from experimental artefacts. With the help of our construction of falloff curves, selected experiments at \(T < 100\) K in Fig. 2 have been falloff-corrected which led to the shown open symbols, see below. The presence of artefacts in some experiments became also particularly clear when data in the bath gases \(\text{N}_2\) and \(\text{He}\) over wider temperature ranges were compared, see Fig. 3. The full lines correspond to our preferred rate coefficients for \(\text{N}_2\) (upper curve) and \(\text{He}\) (lower curve). The pronounced differences between various studies obviously wait for an explanation.

The situation initially appeared even less clear for the reaction

\[ \text{NH}_4^+ + \text{NH}_3 (+ \text{M}) \rightarrow \text{N}_2\text{H}_7^+ (+ \text{M}) \]  

(2)

for which representations of \(\log k_{\text{ass}}[\text{M}] \) vs. \(\log T\) in various bath gases showed quite different and – at first sight – unexplainable slopes while plots of \(\log k_{\text{ass}} \) vs. \(\log T\) showed marked deviations from ion–molecule “collision rate coefficients”, see Figs. 4 and 5 from ref. 7. A rationalization of experimental results like those shown in Figs. 1–5 can only come from the complete modelling of the temperature and pressure dependences of \(k_{\text{ass}}(T,[\text{M}])\) or \(k_{\text{ass}}(T,p)\) in the framework of unimolecular rate theory. E.g. Fig. 6 shows the results for reaction (2) with \(M = \text{He}\) from Figs. 4 and 5 in a falloff representation of \(\log k_{\text{ass}} \) vs. \(\log [\text{M}]\) with the parameter \(T\). One realizes that the experimental conditions have corresponded to different locations between the limiting low- and high-pressure ranges. Obviously, one can not provide an analysis of the experimental data before at least a semiquantitative modelling of the falloff curves has been made. Similar to the evaluation of
the N$_2$H$_7^-$-system from ref. 7, we shall illustrate the procedure for the N$_4^-$-system (1) in the following.

3 Representation of rate coefficients $k(T,p)$

Like for neutral reaction systems, we express the pseudo-first order rate coefficients for dissociation or the pseudo-second order rate coefficients for association in doubly-reduced form

$$k = \left[\frac{k_0/k_\infty}{1 + k_0/k_\infty}\right] F(k_0/k_\infty)$$ (3)

where the limiting low-pressure values $k_0$ of the rate coefficients $k$ are proportional to the bath-gas concentration [M] and the limiting high-pressure values $k_\infty$ are independent of [M]. The broadening factors $F(k_0/k_\infty)$ use the [M]-proportional ratio $k_0/k_\infty$ as the reduced pressure scale and they characterize the deviation of the actual falloff curve from the Lindemann–Hinshelwood curve (i.e. the curve with $F = 1$) by the center broadening factor $F_{\text{cent}} = F(k_0/k_\infty) = 1$ and by suitable shape functions.

It is convenient to factorize the low-pressure rate coefficient $k_{\text{dis},0}$ (for dissociation), which in statistical unimolecular rate theory is given by

$$k_{\text{dis},0} = \beta_c Z [M] \sum_{J=0}^{\infty} (2J + 1) \int_{E_0(J)}^{\infty} f(E,J)dE, \quad (4)$$

with the centrifugal barriers $E_0(J)$ and the equilibrium populations $\beta(E,J)$, into an expression

$$k_{\text{dis},0} = \beta_c Z [M] \sum_{J=0}^{\infty} (2J + 1) \int_{E_0(J)}^{\infty} f(E,J)dE, \quad (5)$$

for the meaning of the various factors in $k_{\text{dis},0}$, see ref. 36 and the calculations described below. Eqsns. (4) and (5) are the results of solving a master equation for intermolecular energy transfer between the bath gas M and the reactant. The properties of this energy transfer, which is rate determining and therefore deserves particular attention, are characterized by the collision efficiency $\beta_c$ and the collision number $Z$ and they can be related to average step sizes $\langle \delta E \rangle$ for energy transfer and overall cross sections. It appears inappropriate to call the calculation of the “trivial”, statistical therodynamical, factor $\beta(Z,J)$ a “phase space theory” such as has become common in the ion–molecule community. This practice turns away emphasis from the properties of $\beta(Z,J)$, i.e. from the rate determining process. Instead, the term “phase space theory” should remain reserved to properties of the dissociative process such as specific rate constants $k(E,J)$, $k_{\text{dis},0}$ or product energy distributions; we suggest that it should even be restricted to reactions on isotropic potential energy surfaces only such as this is also the policy of the present approach.

It is also convenient to express the high-pressure rate coefficients for dissociation ($k_{\text{dis}}$) and for association ($k_{\text{ass}}$) in the form

$$k_{\text{dis},0} = \sum_{J=0}^{\infty} (2J + 1) \int_{E_0(J)}^{\infty} k(E,J)f(E,J)dE, \quad (6)$$

or

$$k_{\text{ass},0} = f_{\text{dis}} \left(\frac{kT}{h}\right) \left(\frac{\hbar^2}{2\pi mk^2}\right)^{3/2} Q_{\text{cent},\text{rigid}}(T), \quad (7)$$

where $f_{\text{rigid}}(T) = 1$ in our nomenclature corresponds to “phase space theory” (PST) and $f_d$ denotes the fraction of encounters leading into the electronic ground state of the adduct (for the meaning of the various factors in $k_{\text{ass},0}$, see ref. 37 and the calculations described below). The centrifugal partition function $Q_{\text{cent}}$ is given by

$$Q_{\text{cent}} = \sum_{J=0}^{\infty} (2J + 1) \exp\left[-\frac{E_0(J)}{kT}\right], \quad (8)$$

where $E_0(J)$ accounts for selected properties of the potential energy surface of the reaction.


\[ k_{\text{diss}} \text{ and } k_{\text{ass}} \text{ are linked by the equilibrium constant } K_c \text{ through} \]

\[
\frac{k_{\text{ass}}}{k_{\text{diss}}} = K_c = \left( \frac{[A]}{[B][C]} \right)_{eq} = Q_{\text{ vib.rot.el.}}(A) \exp(E_0/kT) Q_{\text{ vib.rot.el.}}(B) \left( 2\pi mkT \right)^{3/2} \tag{9}
\]

and

\[ f_{sa} = Q_d(A)[Q_d(B)Q_d(C)]. \]

The shape of the broadening factors \( F(k_0/k_c) \) will be illustrated below. It should be repeated that, at various levels of simplification, the expressions commonly used in the ion community and notably elaborated by Bates, Herbst, Bowers and others (see e.g. refs. 38–43) are equivalent to eqns. (3)–(9) from standard statistical unimolecular rate theory of neutral reactions, and a common language appears more than timely today.

4 Simplified falloff curves and falloff corrections

The detailed theoretical determination of the factors entering eqns. (3)–(9) in general meets with a lot of uncertainties and unknown molecular properties. For this reason an iterative combination of experimental and theoretical information appears necessary. We demonstrate this approach in the following. We start with a very simple estimate of falloff corrections to be applied to the experiments presented in Figs. 1 and 2.

To a first approximation we assume that the experimental data given in Fig. 2 have been obtained near to the low-pressure third-order limit of reaction (1). We then also assume that the high-pressure second-order limiting value of \( k_{\text{ass}} \) is of the order of the Langevin ion-induced isotropic dipole capture rate constant

\[
k_{\text{ass.\infty}} \approx k_L = 2\pi \sqrt{a/\mu} \tag{10}
\]

With the polarizability \( \alpha = 1.77 \times 10^{-24} \text{ cm}^3 \) of \( \text{N}_2 \) this leads to \( k_{\text{ass.\infty}} \approx 8.33 \times 10^{-10} \text{ cm}^3 \text{ molecule}^{-1} \text{ s}^{-1} \). To a first approximation, the centers of the falloff curves are then located at those values of \([\text{N}_2]\) were \( k_{\text{ass.\infty}} \approx k_{\text{ass.0}} \). With \( k_{\text{ass.0}[\text{N}_2]} \) from Fig. 2 and \( k_{\text{ass.\infty}} \) from eqn. (10) one obtains \([\text{N}_2]_{\text{cent}} \approx 4 \times 10^{13}, 8 \times 10^{13}, 8 \times 10^{13}, 5 \times 10^{15}, 1.4 \times 10^{15} \) and \( 4 \times 10^{15} \text{ molecule}^{-3} \) at \( T = 4, 20, 70, 200, 300 \) and 500 K, respectively. Falloff corrections, within this first approximation, then are estimated through the approximate relationship

\[
F(x) \approx f_{\text{cent}}^{1/[1+(\log x/\gamma]^2]} \tag{11}
\]

with \( x = k_{\text{ass.0}}/k_{\text{ass.\infty}} \text{ and } \gamma \approx 0.75--1.27 \log f_{\text{cent}} \). While the rigorous calculation of \( f_{\text{cent}} \) is fairly involved, see below, a quick estimate follows from the modelling of ref. 44 where \( f_{\text{cent}} \), for “normal cases”, are tabulated as a function of the number of transitional modes of the adduct. For the \( \text{N}^+_2 \text{-system with four transitional modes, one has } f_{\text{cent}} \approx 0.5 \text{ independent of the temperature. Provided that the values of } [\text{N}_2] \text{ applied in the experiments are given, falloff corrections with eqns. (3) and (11) then immediately can be estimated.} \]

Applying the described procedure to the near-third order experiments of Fig. 2, one obtains a second approximation for the experimental \( k_{\text{ass.0}} \). Repeating the procedure practically does not further change the values. We have selected representative experimental points (full symbols) from Fig. 2 and indicated the corresponding falloff-corrected results by open symbols in Fig. 2. In several publications unfortunately the experimentally used \([M]\) were not given. However, there were enough data which could be falloff-corrected. The shift of \([\text{N}_2]_{\text{cent}} \) towards smaller values with decreasing temperature makes low-temperature experiments particularly susceptible to these corrections. The free-jet experiments from ref. 32 between 4 and 20 K, which have employed bath-gas concentrations of \( \text{N}_2 \) between \( 5 \times 10^{14} \text{ and } 1.5 \times 10^{15} \text{ molecule cm}^{-3} \), and the CRESU experiments from ref. 30, therefore, have been of particular interest. As shown in Fig. 2, considerable falloff corrections to these experiments are suggested by the described simplified procedure. Nevertheless, these corrections are only tentative at this stage and require a validation by a more thorough theoretical modelling such as presented in the following.

The apparent agreement between “phase space calculations” of \( k_{\text{ass.0}} \) and the measured \( k_{\text{ass}} \), which was found in ref. 32, on the basis of the present analysis has to be considered as accidental and misleading.

5 Low-pressure rate coefficients for \( \text{N}_2^+ + \text{N}_2 \rightarrow \text{N}_4^+ \rightarrow \text{M} \)

In this section we compare the derived low-pressure rate coefficients \( k_{\text{ass.0}} \) of reaction (1) from Figs. 1 and 2 with calculations based on eqns. (5) and (9). These calculations are not as straight-forward as one would like. Information on the essential collisional energy transfer parameters \( Z \) and \( \beta_k \) are scarce. It appears reasonable to identify \( Z \) with the \( \text{N}_4^+ \text{-M} \) capture rate coefficients.\(^{45-49}\) These are the Langevin rate coefficient \( Z = k_L = 5.56 \times 10^{-10} \text{ cm}^3 \text{ molecule}^{-1} \text{ s}^{-1} \) for \( \text{N}_4^+ \text{-He} \) collisions and the ion-induced dipole + quadrupole capture rate coefficient \( 44 \) for \( \text{N}_4^+ \text{-N}_2 \) collisions \( Z = 7.20 \times 10^{-10} \text{ cm}^3 \text{ molecule}^{-1} \text{ s}^{-1} \). \(^{41,42}\) \( \text{N}_2 \) is approached (\( r \approx 3.8 \text{ Å} \), leads to a MEP potential of the linear \( \text{N}_4^+ \text{ species of Morse type} \)

\[
V(r) = D_0[1 - \exp(-2\beta_k(R - R_0))]^2 \tag{12}
\]

with the parameters \( D_0/hc = 9330 \text{ cm}^{-1} \), \( R_0 = 2.00 \text{ Å} \) and \( \beta_k = 3.31 \text{ Å}^{-1} \). \( R \) here denotes the \( \text{N}-\text{N} \) distance of the two central \( \text{N} \) atoms. Outside the valence range, the graphical representation of the \( \text{N}_4^+ \text{ potential from ref. 49, for } (r - r_0) < 3.8 \text{ Å} \), leads to a MEP potential of the linear \( \text{N}_4^+ \text{ species of Morse type} \)

\[
V(r, \beta) = \frac{q^2}{2\pi} + \left[ \frac{qQ}{\pi} + \frac{q^2}{\pi} \Delta \right] \frac{3\cos^2(\beta - 1)}{2} \tag{13}
\]

is approached (\( r \approx \text{center-of-mass distance between } \text{N}_4^+ \text{ and } \text{N}_2 \approx 1.77 \times 10^{-24} \text{ cm}^3, \Delta \xi = 0.7 \times 10^{-24} \text{ cm}^3, Q = 1.5 \times 10^{-26} \text{ esu cm}^{-3} \text{ from ref. 50}. \)) Unfortunately, the \( ab \text{ initio} \) calculations of ref. 49 have not been extended up to the range of \( R \)-values where the transition between the valence and electrostatic potential occurs. We have, therefore, calculated \( E_{\text{el}}(J) \) for a potential which switches abruptly at the crossing of the MEP potentials of eqns. (12) and (13) (the difference between bond lengths \( R \) and c.o.m. distances \( r \) to have been taken into account). Fig. 7 shows the MEP potentials for eqns. (12) and (13) together with the positions and heights of the
centrifugal barriers $E_{0}(J)$. Following the way from $E_{0}(J)$ to $F_{rot}$ such as described in ref. 36, one obtains $F_{rot}$ such as given in the Appendix. Obviously, with the chosen potential model the long-range potential matters most over a wide temperature range and the valence potential becomes important only at very high $T$. The same applies to centrifugal partition functions $Q_{cen}$, see below.

A further point of uncertainty is the anharmonicity correction factor $F_{anh}$. Using fundamental instead of harmonic frequencies such as done here, a substantial part of the anharmonicity correction is already included but there is additional anharmonicity. Our experience from “normal” triatomic molecules indicates that an additional factor of about $h\Delta E$ is needed. However, this factor is somewhat uncertain and requires more work in the future.

The evaluation of eqns. (5) and (9) is straightforward. It leads to results which remain most uncertain in the factor $\beta_{c}^{2}F_{anh}F_{rot}$. A fine-tuning by comparison with the most reliable experiment, at least at one temperature, can not be avoided. If a calculation of $k_{ass,0}$ would agree with the experiments without fine-tuning, today this still has to be considered as accidental. In our present work we rely most on the results at 296 K from ref. 13 with $M$ = He and $N_{2}$ (after minor changes due to the falloff extrapolation, see below) and we fine-tune the $\Delta E$-values. For $M$ = He, agreement with $k_{ass,0}(296 K)$ is found if $\beta_{c}^{2} \approx 0.39$ and $-\Delta E/hc = 240$ cm$^{-1}$ is chosen (instead of the initial guess of 180 cm$^{-1}$, see above). The markedly larger value of $k_{ass,0}(296 K)$ for $M = N_{2}$ would require to increase $-\Delta E$ by 250 to values above 1000 cm$^{-1}$ where $\beta_{c}$ approaches unity and where $\beta_{c}$ becomes insensitive to the value of $\Delta E$. However, it should be emphasized that the fitted $\beta_{c}$ and $\Delta E$-values also include errors from other factors such as $\beta_{c}F_{anh}F_{rot}$ such that no safe conclusions on $\Delta E$ can be drawn for $M = N_{2}$. Nevertheless, the unusually large difference between the experimental values of $k_{ass,0}$ in He and $N_{2}$, see Fig. 3, points towards unusually efficient energy transfer in $N_{4}^{+}$-$N_{2}$ collisions, perhaps due to complex-forming collisions between $N_{4}^{+}$ and $N_{2}$. While an apparent ratio $\beta_{c}(He)/\beta_{c}(N_{2}) \approx 0.49$ is derived from the experiments of ref. 13, the apparent ratio $\beta_{c}(He)/\beta_{c}(N_{2}) \approx 0.14$ from ref. 31 in our analysis would lead to an apparent value of $\beta_{c}(N_{2}) \approx 2.8$ which would be difficult to understand in terms of uncertainties of other factors. We, therefore, consider the value of the ratio $\beta_{c}(He)/\beta_{c}(N_{2})$ from ref. 31 as unrealistic and probably due to an unexplained experimental artefact.

Since $\Delta E$ in the analysis of experiments for neutral systems often has been found to be temperature independent (see e.g. ref. 54), we here assume the same and test the predicted temperature dependence of $k_{ass,0}$. Our results over the range 5–600 K can be represented by

$$k_{ass,0}(He) \text{ cm}^{6} \text{ mole}^{-2} \text{ s}^{-1} \approx 2.42 \times 10^{-29} \text{ (300 K/T)^{2.60} - 1.79 \times 10^{-30}(300 K/T)^{3.15}}$$

(14)

$$k_{ass,0}(N_{2}) \text{ cm}^{6} \text{ mole}^{-2} \text{ s}^{-1} \approx 6.80 \times 10^{-29} \text{ (300 K/T)^{2.23} - 5.60 \times 10^{-31}(300 K/T)^{3.12}}$$

(15)

where the second terms only matter at low temperatures. The two-term form of Eqns. (14) and (15) was chosen in order to reproduce small deviations of the calculated $k_{ass,0}(M)$ from simple $T^{-n}$ expressions. Eqns. (14) and (15) are included in Figs. 2 and 3. The agreement with the falloff-corrected experimental values looks very good. However, one should keep in mind that a number of uncertain assumptions have been made such as the values of the anharmonicity correction, $\Delta E$, the potential leading to $F_{rot}$ and the collision frequency $Z$ being taken as the ion–dipole + quadrupole capture rate constant. In addition, falloff corrections so far came from the simplified method of section 4. The latter procedure requires a more detailed control such as given in the following, but it does not lead to noticeable changes of the derived values of $k_{ass,0}$.

### 6 High-pressure rate coefficients for $N_{2}^{+} + N_{2} \rightarrow N_{4}^{+}$

We now consider $k_{ass,\infty}$ for reaction (1). So far only the measurements from ref. 13 have accessed a certain part of the falloff curve, see Fig. 1. However, no reliable extrapolations to the high-pressure limit could be made on the basis of these experiments alone. Instead, $k_{ass,\infty}$ in ref. 13 was fixed by an estimate from AQO theory. Abandoning AQO theory, we now use the more correct SACM/CT methods such as described in the Appendix, assuming that the ion-induced dipole + quadrupole potential governs $Q_{cen}$. For higher temperatures ($T > 600$ K), minor modifications from the valence potential arise which, however, can not be quantified as long as the transition between valence and electrostatic potential in Fig. 7 is not better characterized (see also the analogous investigation of the contribution from valence forces in the $N_{2}$H$_{7}^{+}$ system described in ref. 7). For this reason, we represent $k_{ass,\infty}$ by the ion-induced dipole + quadrupole potential only. For this potential, the SACM/CT calculations from ref. 48 (for adiabatic dynamics such as realized here) give $k_{ass,\infty}/cm^{3}$ molecule$^{-1} \text{ s}^{-1} = 8.33 \times 10^{-10}$

$$[1 + 0.069(300 K/T)^{0.33}]$$

(16)

where the first term corresponds to the ion-induced dipole (Langevin) and the second to the ion-quadrupole capture rate constant. Using a value of $k_{ass,\infty}(296 K) = 8.3 \times 10^{-10}$ cm$^{3}$ molecule$^{-1} \text{ s}^{-1}$ instead of $9.0 \times 10^{-10}$ cm$^{3}$ molecule$^{-1} \text{ s}^{-1}$ from eqn. (16), the experimental falloff curve in ref. 13 was also well reproduced employing a broadening factor $F_{cen}$ at 0.5. However, the question remained whether this estimated value and the employed shape of the modelled falloff curve (similar to the method from section 4), were adequate. This question will be further examined in the following section.

Before moving on to the full falloff curves we come back to the question of the validity of phase pace theory in calculating $k_{ass,\infty}$. In Fig. 8 we compare $k_{ass,\infty}$ from the SACM/CT calculations for the ion-induced dipole + quadrupole potential with the corresponding results for an ion-induced dipole + locked quadrupole potential which is isotropic and hence corresponds to PST. The two rate coefficients are different such that PST does not apply. The difference, i.e. the rigidity factor $f_{rigid}$ in eqn. (7), with decreasing temperature approaches the value 0.534 (for the even case $Q > 0$, see ref. 48). With
increasing temperature, the increasing influence of the isotropic ion-induced dipole short-range potential leads to an increase of $f_{\text{rigid}}$. This may be an artefact of our model since finally the anisotropic valence potential will take over. In any case, however, $k_{\text{ass,}x}$ is far from the value given by PST. Keeping the rigidity factor for the long-range ion-quadrupole and the short-range valence potential the same, i.e. equal to 0.534, the short range-valence/long range-electrostatic switching potential probably leads to an optimum prediction of $k_{\text{ass,}x}$. This is probably a better estimate of $k_{\text{ass,}x}$ than given by the ion-induced dipole + quadrupole potential alone. However, in the absence of a better $ab\ initio$ potential in the switching range it appears premature to change $k_{\text{ass,}x}$. Obviously this uncertainty is of no practical relevance for the range of conditions accessible so far, but it will matter when more high temperature/high pressure data become available.

Similar remarks apply to the $\text{N}_2\text{H}^+$-system for which the ion-induced dipole + permanent dipole potential was used in the calculations of $k_{\text{ass,}x}$ from Figs. 5 and 6. Again, due to the anisotropy of the long-range ion-permanent dipole potential $k_{\text{ass,}x}$ is markedly below the values for the isotropic ion-locked dipole potential used in PST. Also in this case an anisotropic short-range valence potential will cause a further drop of $k_{\text{ass,}x}$ with increasing $T$ below the values shown in Figs. 5 and 6, see the estimates in ref. 7.

Phase space theory is even more inadequate in cases with weaker isotropic long range potentials and being governed by a markedly anisotropic short range valence potential. We use the $\text{C}_6\text{H}_7^+ + \text{CH}_3 \leftrightarrow \text{C}_6\text{H}_{10}^+$ systems and $\text{C}_7\text{H}_7^+ + \text{CH}_4 \leftrightarrow \text{C}_8\text{H}_{12}^+$ as examples. In these cases thermal dissociation rate coefficients $k_{\text{diss,}x}$ in the high-pressure limit have been measured\textsuperscript{11,12} and related to experimental specific rate constants $k(E,J)$. SACM/CT calculations have been made\textsuperscript{21} employing simple switching models between anisotropic short-range valence and isotropic long range-ion-induced dipole components, one parameter of which was calibrated through the experimental $k(E,J)$. One observes that both, the specific and the thermally averaged rate coefficients are far below the PST results, being further away at higher energies and higher temperatures. Figs. 9 and 10 demonstrate the behaviour by a plot of $k_{\text{ass,}x}$ (converted with $K_i$ from $k_{\text{diss,}x}$) as a function of temperature. The MEP of the potential causes $k_{\text{ass,}x,\text{PST}}$ to slightly increase with temperature as compared to the Langevin rate constant. This increase is also an indication of the increasing importance of the valence potential. The rigidity factors $f_{\text{rigid}}$ are small, of the order of $10^{-2}$ to $10^{-3}$ near 600 K, indicating the dominance of the valence potential under these conditions. Only at very low temperatures the electrostatic range of the potential with $k_{\text{ass,}x} \Rightarrow k_1$ is predicted to become relevant. In this case the values of the effective Massey parameters = 0.5 and 0.15, for the $\text{C}_8\text{H}_{10}^+$ and $\text{C}_9\text{H}_{12}^+$ systems respectively, indicate markedly nonadiabatic dynamics in the transitional modes such that here only SACM/CT and even not conventional variational transition state theory can be used to determine rate coefficients.

The identification of nonadiabatic dynamics may also provide the answer to the apparent paradox mentioned above: rate constants for fragmentation often are far from PST results while product energy distributions often are close to PST predictions. It may well be that the final energy distributions of the dissociation products are being established by nonadiabatic dynamics as well as Coriolis coupling long after the rate-determining dynamical bottlenecks of the reaction have been passed.

7 Reduced falloff curves

Having discussed the detailed estimation of limiting low- and high-pressure rate coefficients $k_{\text{ass,}0}$ and $k_{\text{ass,}x}$, we now proceed to the determination of the falloff curves. Calculating full falloff curves is the most difficult part of the treatment and there is no complete treatment available as yet. However, once $k_{\text{ass,}0}$ and $k_{\text{ass,}x}$ are characterized sufficiently well, doubly reduced falloff curves connecting these rate coefficients are less sensitive to details of the problem. The falloff curves for the
N\textsubscript{2}H\textsuperscript{+} system shown in Fig. 5 have been determined in this way.\textsuperscript{7} It was shown that PST, accounting for E- and J-dependences of specific rate constants, gives already satisfactory results for the reduced falloff curves, being in contrast to rigid activated complex RRKM theory. The E- and J-dependences of the rigidity factors \(f_{\text{rigid}}(E, J)\) did not much change the reduced falloff curves from PST. In the following, for the \(N_2^\text{+} + N_2 \rightarrow N_4^\text{+}\) reaction, we used either the SACM/CT results from ref. 56 or simple PST and we followed the method described in ref. 7.

At first, in Fig. 11 we show the calculated broadening factors \(F(k_0/k_\text{ass})\) of the \(N_2^\text{+} + N_2 \rightarrow N_4^\text{+}\) reaction as a function of the temperature (including weak collision effects). Second, in Figs. 12 and 13 we provide complete sets of falloff curves for the \(N_4^\text{+}\) system which are constructed by means of the calculated broadening factors, eqns. (3), (14) and (16). These curves are completely analogous to Fig. 6 for the \(N_2\text{H}^\text{+}\) system. (The curves do not look completely smooth which is not an error of the calculation but a real effect. The shallow dips of the curves are located close to the centers of the falloff curves where \(k_\text{ass}/k_0\) has broad minima.) One realizes pronounced shifts of the falloff curves with changing temperatures. At lower temperatures, much lower pressures have to be employed to approach the lower pressure limit of \(k_{\text{ass}}\) than at higher temperatures. The results well agree with the conclusions from section 4 such that the falloff corrections shown in Fig. 2 essentially remain unchanged. Many investigations, contrary to the assumptions of the authors, have not been performed in the low-pressure termolecular limit and falloff corrections would have been required before true experimental values of \(k_{\text{ass},0}\) could be given. It should be emphasized that this conclusion is quite independent of the remaining uncertainties of the treatment. The construction of falloff curves from ref. 13, which was based on a simpler approach than used here, turned out to nearly coincide with the present more detailed calculations (only a minor difference in the extrapolated \(k_{\text{ass},0}\) was observed).

Having done the full falloff calculations, see Figs. 11–13, one may also test the approximate analytical representations of \(F(k_0/k_\text{ass})\) from eqn. (11). A comparison of eqn. (11) with Fig. 11 gives reasonable agreement once \(F(\text{cent})\) is known. Improvements are obtained by replacing eqn. (11) by

\[
F(x) \approx F(\text{cent})\left[1+\alpha \frac{\log x-0.2}{N\Delta N}\right]^{-1}
\]

with \(F(\text{cent}) \approx 0.53, N \approx 1.3, \Delta N \approx 0.1, +\Delta N \text{ for } \log x < -0.2, -\Delta N \text{ for } \log x > -0.2. \) However, this more complicated expression instead of eqn. (11) appears only necessary when more precise data over wider pressure ranges would be available.

8 Conclusions
We have illustrated the possibilities and uncertainties of today’s modelling of the temperature and pressure dependence of rate coefficients \(k(T,p)\) for ion–molecule dissociation/association reactions. Our analysis clearly demonstrates the common and different properties of ionic and neutral reaction systems. The reaction \(N_2^\text{+} + N_2 (\pm M) \rightarrow N_4^\text{+} (\pm M)\) has served as a model system for which the determination of \(k_{\text{ass},0}, k_{\text{ass,\text{bare}}}, k_{\text{ass,g}}\) and reduced falloff curves were illustrated. A representation with \(k_{\text{ass,0}}\) from eqns. (14) and (15), \(k_{\text{ass,\text{bare}}}\) from eqn. (16), and broadening factors of the falloff curve from eqns. (11) or (17), provides a compact format for representing the rate coefficients.

This is the same format as routinely employed for neutral radical–radical reactions and it appears advisable that ion–molecule reactions are represented in the same way as neutral radical–radical reactions.

Appendix
Ion-quadrupole capture rate coefficients
Following ref. 48, the effective mass \(M\) of the collision system \(M = (aB/\hbar^2)(Qq/2kT)^{1/2}\) is calculated and found to be much larger than unity such that expressions for adiabatic collisions can be used. These are formulated with the parameters \(\theta = z(2kT)^{1/2}Q^{1/2}, z = 1.352 + \ln \theta\) and \(\ln k = 0.2344 + 0.0861(z^{1/2}/16)^{1/2}\). \(k_{\text{cap}}\) then follows via \(k_{\text{cap}}/k_L = \kappa/(2\theta^2)^{1/2}\).
and $k_l = 2\pi^2 (q^2/\mu)^{1/2}$. We use $\mu = 1.77 \times 10^{-24}$ cm$^3$ and $Q = -1.5 \times 10^{-18}$ esu cm$^3$ for N$_2$.

Molecular parameters$^9$

Frequencies of N$_4^+$: $v/cm^{-1} = 2275, 2239, 390, 134 (2)$ and 91 (2). Rotational constant of N$_4^+$: $B/cm^{-1} = 0.112$. Bond energy of N$_4^+$: $E_{\text{b}}/hc = 8908$ cm$^{-1}$, $D_{\text{b}}/hc = 9330$ cm$^{-1}$.

Rotational factors $F_{\text{rot}}(T)$

$F_{\text{rot}}$ in the absence of centrifugal barriers: $F_{\text{rot}} = F_{\text{rot max}} = 467, 117, 46.7, 23.4, 7.72$ and 3.74 for $T = 5, 20, 50, 100, 300$ and 600 K, respectively. $F_{\text{rot}}$ for the switching potential from eqns. (12) and (13): $F_{\text{rot}} = 52.9, 21.9, 12.1, 7.72$ and 3.74 for the long-range potential of eqn. (13): $F_{\text{rot}} = 53.8, 22.4, 12.4, 7.88, 3.81$ and 2.4. $F_{\text{rot}}$ for the short-range potential of eqn. (12): $F_{\text{rot}} = 4.0, 3.5, 3.1, 2.8, 2.2$ and 1.8. All $F_{\text{rot}}$ are given for the same series of T as $F_{\text{rot max}}$.

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