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ON THE ISS MODEL OF DYNAMICAL SUSY BREAKING

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Abstract. In this letter we would like to apply the superconformal index technique to give one more evidence for the theory proposed by Intriligator, Seiberg and Shenker (ISS) as being described by interacting conformal field theory in its IR fixed point.

The superconformal index technique [1, 2] has already been successfully applied in many cases for checking conjectures of electric-magnetic duality for supersymmetric field theories, finding new examples of dualities and also giving a lot of new identities for mathematical functions standing behind the superconformal indices. The Spiridonov's theory of elliptic hypergeometric integrals is quite a new research area in mathematics [3, 4, 5]. It was realized by Dolan and Osborn in [6] that the superconformal index is given in terms of elliptic hypergeometric integrals and the equalities for the superconformal indices are given by the Weyl group symmetry transformations for elliptic hypergeometric integrals on different root systems [4]. The first example of such integrals is given by the elliptic beta-integral with 7 parameters– an extension of the beta-integral found by Spiridonov in [3].

Later on using the superconformal index technique in a series of papers [7] there were found many new Seiberg dual theories [8] (for example, supersymmetric dualities outside the conformal window for $\mathcal{N}=1$ original Seiberg SQCD theory) coming from the known identities for elliptic hypergeometric integrals and vice versa a lot of new mathematical identities for this class of special functions were conjectured. Also the same technique showing the power of the theory of elliptic hypergeometric integrals can be applied for extended supersymmetric theories in order to check S–duality conjectures [9, 10].

By definition the superconformal index counts the number of gauge invariant operators which satisfy BPS condition and which can not be combined to form long multiplets. The $SU(2,2|1)$ space-time symmetry group of $\mathcal{N}=1$ superconformal algebra consists of $J_i, \overline{J}_i$– the generators of two $SU(2)$s which together give the Lorentz symmetry group of a four-dimensional theory $SO(4)$, translations P_μ, special conformal transformations K_μ, $\mu=1,2,3,4$, the dilatations H and also the $U(1)_R$ generator R. Apart from the bosonic generators there are supercharges $Q_\alpha, \overline{Q}_{\dot\alpha}$ and their superconformal partners $S_\alpha, \overline{S}_{\dot\alpha}$. Then taking the distinguished pair of supercharges [1], for example, $Q=\overline{Q}_1$ and $Q^\dagger=-\overline{S}_1$, one has

$$\{Q,Q^\dagger\}=2\mathcal{H},\quad \mathcal{H}=H-2\overline{J}_3-3R/2, \tag{1}$$

and then the superconformal index is defined by the matrix integral

$$I(p,q,f_k)=\int_{G_c} d\mu(g)\, Tr\Big((-1)^{\mathrm{F}} p^{\mathcal{R}/2+J_3} q^{\mathcal{R}/2-J_3} \times e^{\sum_a g_a G^a} e^{\sum_k f_k F^k} e^{-\beta\mathcal{H}}\Big),\quad \mathcal{R}=H-R/2, \tag{2}$$

where F is the fermion number operator and $d\mu(g)$ is the invariant measure of the gauge group G_c. To calculate the index one should take into account the whole space of states, but, fortunately, only the zero modes of the $\mathcal{H}$ contribute to the trace because the relation (1) is preserved by the operators used in (2). The chemical potentials g_a, f_k are the group parameters of the gauge and flavor symmetries and are given by operators G^a and F^k.

Let us consider $\mathcal{N}=1$ SYM theory based on $SU(2)$ gauge theory with a single chiral field Q in spin $I = 3/2$ representation considered by Intriligator, Seiberg and Shenker (ISS) in [11]. The R–charge for the scalar component is equal $R_Q = 3/5$. The ISS theory is conjectured to be the simplest model of dynamical SUSY breaking (for review of the model and further examples see [12]).

In [11] two possible phases– the confining and the conformal– are proposed for this theory in its IR fixed point. If the former case is realized then deforming a theory by the tree level superpotential $W_{tree} = \lambda Q^4$ which is a relevant operator would dynamically break supersymmetry [11] giving the simplest example of a theory with dynamically broken supersymmetry. In the original paper the authors conjectured the first possibility but later in two papers [13, 14] using different approaches the authors give evidences for the second possibility and still the status of this theory is open. That is why we would like to attack this problem using the superconformal index technique. Although the superconformal index has not yet been shown to be invariant under the RG we rely on the latter fact being true.

The proposed dual confining phase is described by the single gauge invariant operator coming from the electric theory which is Q^4 whose R–charge is equal to 12/5. The only check which can be applied to show that the original theory has confining phase is the anomaly matching conditions. For the ISS model we have TrR and TrR^3 anomalies which match in two descriptions and are given by the following values 7/5 and $(7/5)^3$ correspondingly.

If the ISS model has the confining phase then it can be thought as Seiberg duality [8] in the IR fixed point and according to the Romelsberger's hypothesis [1] the superconformal indices for the ISS model and its confining phase should coincide. And as a consequence if there is no equality between the superconformal indices then the conformal phase takes place. So assuming existence of the confining phase in the IR fixed point we calculate the superconformal indices for the ISS model and its conjectured confining phase and analyze whether two expressions match or not. The superconformal index for the initial theory is (we use abbreviation I_E for the initial theory and I_M for the conjectured confining phase)

$$I_E = \frac{(p;p)_\infty (q;q)_\infty}{2} \int_{\mathbb{T}} \frac{\Gamma((pq)^{\frac{3}{10}} z^{\pm 1}, (pq)^{\frac{3}{10}} z^{\pm 3}; p, q)}{\Gamma(z^{\pm 2}; p, q)} \frac{dz}{2\pi \mathtt{i} z}, \tag{3}$$

where $\mathbb{T}$ is the unit circle with positive orientation and also we use the following conventions $\Gamma(a,b;p,q) := \Gamma(a;p,q)\Gamma(b;p,q)$, $\Gamma(az^{\pm 1};p,q) := \Gamma(az;p,q)\Gamma(az^{-1};p,q)$, where

$$\Gamma(z;p,q) = \prod_{i,j=0}^{\infty} \frac{1 - z^{-1} p^{i+1} q^{j+1}}{1 - z p^i q^j}, \quad |p|, |q| < 1,$$

is the so-called elliptic gamma function. For a conjectured confining phase the superconformal index is given by one elliptic gamma-function

$$I_M = \Gamma((pq)^{\frac{6}{5}}; p, q). \tag{4}$$

Now we would like to *check the conjectured equality* for the superconformal indices coming from the assumption that there exists a confining phase. First of all we would like to discuss the ellipticity condition [3] for I_E which states that the ratio of the kernel taken from the integral (3) and the same kernel but with the substituted parameter of integration z by pz is the elliptic function of this parameter z. This condition here from physical point of view is interpreted as having anomaly-free gauge theory. It is satisfied trivially since the expression for the superconformal index I_E, using the triplication formula for the elliptic gamma function,

$$\Gamma(z^3;p,q) \;=\; \prod_{i,j,k=0}^{2} \Gamma(zw^i p^{\frac{j}{3}} q^{\frac{k}{3}};p,q), \tag{5}$$

where w is the cubic root of 1 not equal to 1, can be rewritten as the higher order elliptic beta-integral with 28 parameters in addition to p and q [4].

We can calculate the series expansion in chemical potentials for both expressions (3) and (4) and compare the first several terms. Finding the series expansion in the parameter pq for the superconformal index of the initial ISS theory we get

$$I_E \;=\; 1 + \mathsf{o}((pq)^0) \tag{6}$$

and for the assumed confining phase

$$I_M \;=\; -\frac{1}{\sqrt[5]{pq}} + 1 + \mathsf{o}((pq)^0). \tag{7}$$

Comparing now the first terms of series expansions we see that the superconformal indices for two phases do not coincide showing us that our assumption of the existing confining phase with only one gauge invariant operator is not valid and that leads us to evidence that the original ISS theory has interacting conformal field theory in its IR fixed point. The same conclusion can be obtained by taking the limit $p \to 0$ applied to the expressions for the superconformal indices (3) and (4). Applying the formula

$$\lim_{p\to 0} \Gamma(z;p,q) \;=\; \frac{1}{(z;q)_\infty}$$

we see that we obtain different limits for the I_E and I_M.

Now we consider other well understood models with misleading anomaly matchings considered in [15]. The theories under consideration are $\mathcal{N}=1$ SYM with SO gauge group and the matter field Q in the absolutely symmetric tensor representation of second rank of the gauge group. The R–charge of the Q field is $\frac{4}{N+2}$. In [15] it was shown that the anomaly matching conditions are satisfied with the set of gauge invariant operators $\mathcal{O}_n \;=\; \mathtt{Tr}Q^n$ where $n = 2,\ldots,N$ what suggest for the confining phase with the given above operators as fundamental fields to exist. But deforming the theory it was shown that the confining description does not hold further so the theory has conformal phase in its IR fixed point. We would like to show that the same result can be obtained using the superconformal index technique.

To calculate the superconformal indices in this case we should distinguish between two cases, namely between even and odd rank of the gauge group. For the case of even rank of the gauge group namely $N = 2K$ we have the following expression for the superconformal index of the initial theory

$$I_E^{SO(2K)} = \frac{(p;p)_\infty (q;q)_\infty}{2^{K-1} K!} \Gamma^{K-1}((pq)^{\frac{2}{2K+2}};p,q) \tag{8}$$
$$\times \int_{\mathbb{T}^K} \prod_{1\leq i<j\leq K} \frac{\Gamma((pq)^{\frac{2}{2K+2}} z_i^{\pm1} z_j^{\pm1};p,q)}{\Gamma(z_i^{\pm1} z_j^{\pm1};p,q)} \prod_{j=1}^K \Gamma((pq)^{\frac{2}{2K+2}} z_j^{\pm2};p,q) \frac{dz_j}{2\pi \mathtt{i} z_j},$$

The possible description of the initial theory in the IR fixed point by the theory without gauge group and having only the gauge invariant operators $\mathcal{O}$ bring us to the possible expression for the index (8) by the superconformal index which can be calculated in this confining phase

$$I_M^{SO(2K)} = \prod_{i=2}^{2K} \Gamma((pq)^{\frac{i}{K+1}};p,q). \tag{9}$$

To see that the two expressions (8) and (9) do not coincide what is the same as there is no confining phase in the origin of moduli space we again take the limit $p \to 0$ in both sides and one can easily see that in the left hand-side the limit is valid while the right hand-side does not have the valid limit.

In the case of odd rank the superconformal index is given

$$I_E^{SO(2K+1)} = \frac{(p;p)_\infty (q;q)_\infty}{2^K K!} \Gamma^K((pq)^{\frac{2}{2K+3}};p,q) \tag{10}$$
$$\times \int_{\mathbb{T}^K} \prod_{1\leq i<j\leq K} \frac{\Gamma((pq)^{\frac{2}{2K+3}} z_i^{\pm1} z_j^{\pm1};p,q)}{\Gamma(z_i^{\pm1} z_j^{\pm1};p,q)} \prod_{j=1}^K \frac{\Gamma((pq)^{\frac{2}{2K+3}} z_j^{\pm1}, (pq)^{\frac{2}{2K+3}} z_j^{\pm2};p,q)}{\Gamma(z_j^{\pm1};p,q)} \frac{dz_j}{2\pi \mathtt{i} z_j},$$

and if there is a possible confining phase for this theory then the superconformal index for this phase is given by the following expression

$$I_M^{SO(2K+1)} = \prod_{i=2}^{2K+1} \Gamma((pq)^{\frac{2i}{2K+3}};p,q) \tag{11}$$

Again taking the limit $p \to 0$ we see that the analytical properties of two sides are different which is interpreted as lack of equality between the two expressions (10) and (11).

One can easily check the ellipticity condition for both cases (8) and (9) (for details see [7]). Also in both cases for even and odd ranks of the gauge group we see mismatch of the superconformal indices of the original theory and the conjectured confining phase. From the physical point of view this discrepancy between the superconformal indices is realized as the fact that in the IR fixed point the theory is given by the interacting conformal field theory rather than by the confining phase.

To conclude in this short letter we describe the application of the superconformal index technique for deriving out theories with misleading anomaly matching [15]. Also we applied this method for the conjectured simplest model of dynamical SUSY breaking proposed long ago in [11] and the situation with which is not understood completely yet. Here we give one more evidence that the proposed theory has interacting conformal field theory description [13, 14] in its IR fixed point rather than the conjectured confining theory with only one gauge invariant operator [11]. We are using completely different approach from the approaches in [13, 14] where the analyze was based on the conjecture that the correct phase in the IR fixed point of some theory is the phase with larger conformal anomaly a and the analyze of the dynamics of the theory by its deforming on $S^1 \times \mathbb{R}^3$ correspondingly.

In [7] it was conjectured that the anomaly matching condition for the dual theories is given by the total ellipticity condition for the elliptic hypergeometric integrals describing the superconformal indices for this duality. Also it was suggested that the ellipticity criterium is a necessary but not a sufficient condition for a true duality. Our examples explicitly show that it is indeed the case.

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