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Effective action, conformal anomaly and the issue of quadratic divergences

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Abstract

For massless ϕ^4 theory, we explicitly compute the lowest order non-local contributions to the one-loop effective action required for the determination of the trace anomaly. Imposing *exact* conformal invariance of the *local* part of the effective action, we argue that the issue of quadratic divergences does not arise in a theory where exact conformal symmetry is only broken by quantum effects. Conformal symmetry can thus replace low energy supersymmetry as a possible guide towards stabilizing the weak scale and solving the hierarchy problem, if (i) there are no intermediate scales between the weak scale and the Planck scale, and (ii) the running couplings exhibit neither Landau poles nor instabilities over this whole range of energies.

1. Introduction

The present work is motivated by the possibility that the standard model (SM) of elementary particle physics could arise from an exactly conformal invariant theory.[1] All observed mass scales of particle physics and their smallness *vis-à-vis* the Planck scale might thus be explained via the quantum mechanical breaking of conformal symmetry induced by the explicit breaking of conformal symmetry that necessarily accompanies any regularization. The explanation of small mass scales via a conformal anomaly would thus be natural in the sense of [3], that is, in terms of a 'nearby' exact conformal symmetry.

Although the attractiveness of such a scenario has been appreciated for a long time since the seminal work of Coleman and Weinberg [4], it appears that attempts at its concrete implementation have so far met only with limited success. In part this may be due to a widely held expectation, according to which the existence of large intermediate scales between the weak scale and the Planck scale is unavoidable if one wants to understand physics beyond the SM, as exemplified by grand unification (GUTs) and the conventional explanation of small neutrino masses via the seesaw mechanism (both of which require new scales above 10^{11} GeV). In this view, low energy physics would be separated from the Planck scale by a cascade of new scales, and there would be no place for conformal invariance, even if only approximate.

In a recent paper [5] we have advocated a different scenario, which is based on the assumption of an exactly conformally invariant tree level Lagrangian, and the absence of new large scales below the Planck scale—a scenario often referred to as the 'grand desert'.[2] Evidence was presented that, with the incorporation of massive neutrinos, all observed features of the SM can be reproduced in this way. Apart from the inclusion of neutrinos, a crucial difference with earlier attempts in this direction (see, e.g., [9]) was the imposition of an extra consistency postulate, namely the requirement that the evolution of all couplings according to the renormalization group equations should be such that the theory remains viable up to a very large scale,

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[1] Classic references on conformal symmetry in (quantum) field theory are [1,2].

[2] See also [6,7] and references therein for a related, but different 'grand desert' scenario where conformal invariance is broken by explicit mass terms for the scalar fields. A similar model with extra scalar fields, but without right-chiral neutrinos is also considered in [8].

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that is, there should exist neither Landau poles nor instabilities below that scale.[3] This obviously requires a special conspiracy of the SM parameters, but as shown in [5] there does exist a 'window' compatible with known phenomenology for which the model can survive up to the Planck scale, and which predicts the Higgs mass(es) to be above 200 GeV.

In this article we wish to address a very specific technical issue, namely the computation of the effective action at one loop beyond the constant field approximation underlying the one-loop effective potential of [4], and the incorporation of *non-local* corrections. For simplicity we will restrict attention to scalar fields, and only consider the massless (and thus classically conformally invariant) ϕ^4 theory. Although there is a huge literature on trace anomalies in gauge theories (see, e.g., [12, 13]) and in curved backgrounds (see, e.g., [13,14]), it appears that the non-local effective action for a self-interacting scalar field has not received so much attention. Because one cannot hope to derive an analytical expression for the complete one-loop effective action, various approximation schemes have been devised in previous work. For instance, in [15] a 'quasi-local' approximation is developed, while [16] sets up a formalism based on 'average actions'; more recently, Ref. [17] derives expressions for the non-local effective action in the small and large field limits, respectively. By contrast, we here focus on the *conformal properties* in that we aim for determining only that part of the one-loop effective action which captures the anomalous behavior under conformal transformations (see also [18,19]). In this way, we can show explicitly that the conformal anomaly [cf. Eq. (28)] is given by a *local* expression which itself arises as the variation of a non-local functional [cf. Eq. (22)]. That the anomaly *must* be local follows from general arguments, and constitutes a crucial consistency check on any approximation scheme for the non-local effective action.

Our main purpose in taking up again these (rather old) questions here is to reconsider their implications for physics beyond the standard model, in particular the question of naturalness and stability of the weak scale. The appearance of quadratic divergences in theories with scalars is usually invoked as the main (theoretical) argument for low energy supersymmetry. By contrast, we here argue—following earlier arguments by Bardeen [20]—that (quantum mechanically broken) conformal symmetry may provide an equally good mechanism for the stabilization of the weak scale, if we can impose *exact* conformal invariance on the counterterm dependent terms of the effective action (for a discussion of scale stability with softly broken conformal symmetry see also [21], which likewise casts some doubt on the usual arguments for low energy supersymmetry). The main role of supersymmetry could then be in ensuring finiteness of quantum gravity.

The plan of this Letter is as follows. In Section 2 we review the construction of the one-loop effective action for pure ϕ^4 theory; the conformal properties of the resulting expression are discussed in Section 3. In Section 4 we consider the implications of these results for the issue of quadratic divergences in scalar field theory. Some useful formulae are collected in Appendix A.

[3] Landau poles exist *for all* extensions of the SM, so that none of these extensions is likely to exist as a rigorous quantum field theory (see, e.g., [10,11]). At issue is only the *scale* at which these poles occur.

2. Massless ϕ^4 theory revisited

The simplest (non-trivial) example of a classically conformally invariant theory in four space–time dimensions is massless ϕ^4 theory, whose action reads[4]

$$S_{\rm cl}^E = \int d^4x \sqrt{g}\left(-\frac{1}{2}\phi\Box_g\phi + \frac{1}{12}R\phi^2 + \frac{\lambda}{4}\phi^4\right), \tag{1}$$

where

$$\Box_g := \frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}g^{\mu\nu}\partial_\nu\right) \tag{2}$$

is the generally covariant Laplace–Beltrami operator. It is a standard result [2] that the addition of the term involving the Ricci scalar R in (1) makes the action also invariant under local re-scalings (Weyl transformations). Our main concern in this Letter is the flat space theory, but we nevertheless have included the Weyl invariant gravitational couplings here because they will contribute to the conformal (trace) anomaly even in the flat space limit. Accordingly, we set $g_{\mu\nu}(x) = \eta_{\mu\nu}$, except in those places relevant to the computation of the trace anomaly.

As is well known [22–24], the effective action is defined as the Legendre transform

$$\Gamma[\phi_c] = W[J] - \int d^4x\, J(x)\phi_c(x), \tag{3}$$

with the generating functional $W[J]$ of one-particle irreducible (1PI) Green's functions, where the classical field $\phi_c(x)$ is defined by

$$\phi_c(x) = \frac{\delta W[J]}{\delta J(x)}. \tag{4}$$

The effective action can be expanded in powers of $\hbar$ ("loop expansion")

$$\Gamma[\phi_c] = \Gamma^{(0)}[\phi_c] + \hbar\Gamma^{(1)}[\phi_c] + \cdots, \tag{5}$$

where

$$\Gamma^{(0)}[\phi_c] = S_{\rm cl}^E(\phi_c). \tag{6}$$

It is also well known that the one-loop contribution to the effective action is given by [23]

$$\Gamma^{(1)}[\phi_c] = \frac{1}{2}\,{\rm Tr}\ln\frac{\delta^2 S_{\rm cl}}{\delta\phi(x)\delta\phi(y)}. \tag{7}$$

For the classical action (1) one thus straightforwardly derives the (still formal) result

$$\Gamma^{(1)}[\phi_c] = \frac{1}{2}\,{\rm Tr}\ln\left(-\Box + 3\lambda\phi_c^2(x)\right), \tag{8}$$

[4] For convenience, we will work with a Euclidean metric throughout this Letter.

which, in turn, can be rewritten as (dropping an infinite constant)

$$\Gamma^{(1)}[\phi_c] = -\frac{1}{2}\operatorname{Tr}\ln\left(1 - \frac{3\lambda\phi_c^2(x)}{-\Box + 3\lambda\phi_c^2(x)}\right). \tag{9}$$

This expression can be (formally) expanded as

$$\Gamma^{(1)}[\phi_c] = \frac{3\lambda}{2}\int d^4x\,\sqrt{g(x)}D(x,x;\phi_c)\phi_c^2(x) + \frac{9\lambda^2}{4}\int d^4x\,\sqrt{g(x)}\int d^4y\,\sqrt{g(y)}D(x,y;\phi_c) \times \phi_c^2(y)D(y,x;\phi_c)\phi_c^2(x) + \cdots, \tag{10}$$

where the field dependent propagator $D(x,y;\phi_c)$ is defined as

$$\left(-\Box_g + \frac{1}{6}R + 3\lambda\phi_c^2(x)\right)D_g(x,y;\phi_c) = g^{-3/8}(x)\delta^{(4)}(x,y)g^{-1/8}(y). \tag{11}$$

The gravitational couplings have been re-instated here so we can discuss the conformal properties of these objects. Namely, under local conformal transformations we have

$$g_{\mu\nu}(x) \to e^{-2\omega(x)}g_{\mu\nu}(x), \qquad \phi_c(x) \to e^{\omega(x)}\phi_c(x). \tag{12}$$

The conformally covariant Laplacian transforms as

$$\left(-\Box_g + \frac{1}{6}R\right)\phi_c(x) \to e^{3\omega(x)}\left(-\Box_g + \frac{1}{6}R\right)\phi_c(x). \tag{13}$$

It is important here that the operator in parentheses acts on a quantity of conformal weight one; it would not be conformally covariant when acting on a field of different conformal weight, such as $\phi_c^2(x)$. From these formulae we deduce the transformation properties of the propagator (11) under conformal rescalings, viz.

$$D(x,y;\phi_c) \to e^{\omega(x)}D(x,y;\phi_c)e^{\omega(y)}. \tag{14}$$

Therefore (10) is not only generally covariant, but in addition also formally invariant under conformal transformations—only formally, because the first two terms in the expansion are divergent and require renormalization which introduces, as we will see, a conformal anomaly.

Let us start with a constant field $\phi_c(x) = \phi_0$. The propagator reads then

$$D(x-y;M) \equiv D(x,y;\phi_c)|_{\phi_c(x)=\phi_0} = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2+M^2}, \tag{15}$$

where $M^2 = 3\lambda\phi_0^2$. Summing up the series we obtain the standard result

$$\Gamma^{(1)}[\phi_0] = \frac{1}{2}\int d^4x \int \frac{d^4k}{(2\pi)^4}\ln\left(1 + \frac{M^2}{k^2}\right). \tag{16}$$

Before we proceed to the general case we emphasize the following point. To renormalize the theory, we should impose *exact conformal invariance on the counterterm dependent terms of the effective action*. This requirement in particular *leaves no room for a finite mass term* in the local part of (10). For the further computation it is therefore more convenient to work with dimensional regularization (see, e.g., [23,24]) because the latter automatically satisfies the requirement of not introducing any local terms that break conformal invariance in the effective action at any given order. In particular, all divergent counterterms (poles in ϵ) are conformally invariant. The regularization is performed in the usual way by continuing all loop integrals to $D = 4 - 2\epsilon$ dimensions via the replacement

$$\int \frac{d^4k}{(2\pi)^4} \to (Cv^2)^{\epsilon}\int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}, \tag{17}$$

where $C = e^{\gamma}/(4\pi)$. Like any other regulator, this procedure breaks classical conformal invariance, here via the dimensionful parameter v. Using the formulae from Appendix A we get, in the constant field case,

$$\Gamma^{(1)}[\phi_0] = \frac{9\lambda^2}{64\pi^2}\int d^4x\,\phi_0^4\left[-\frac{1}{\epsilon} + \left(\ln\frac{3\lambda\phi_0^2}{v^2} - \frac{3}{2}\right)\right] + \mathcal{O}(\epsilon), \tag{18}$$

which, after renormalization, is just the familiar Coleman–Weinberg effective potential [4].

When the field $\phi_c(x)$ depends on x we obviously cannot sum the series in (10). Since we are mainly interested in the conformal anomaly we analyze only the first two terms of the expansion (those displayed explicitly in (10)) because all other terms are convergent and therefore conformally invariant. Let us write these two terms expanding the propagators around a constant field ϕ_0^2

$$\begin{aligned}\Gamma^{(1)}(\phi_c) &= \frac{3\lambda}{2}\int d^4x\,\phi_c^2(x)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+3\lambda\phi_0^2} \\ &\quad - \frac{9\lambda^2}{32\pi^2}\int d^4x\,d^4y\,\left(\phi_c^2(x)-\phi_0^2\right)K\left(x-y;3\lambda\phi_0^2\right)\phi_c^2(y) \\ &\quad + \frac{9\lambda^2}{64\pi^2}\int d^4x\,d^4y\,\phi_c^2(x)K\left(x-y;3\lambda\phi_0^2\right)\phi_c^2(y) + \cdots,\end{aligned} \tag{19}$$

where the dots stand for higher powers of $(\phi_c^2(x) - \phi_0^2)$ and higher number of propagators (so that these expressions are finite), and

$$K\left(x-y;3\lambda\phi_0^2\right) = 16\pi^2\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4} \times \frac{e^{-ip(x-y)}}{(k^2+3\lambda\phi_0^2)((k+p)^2+3\lambda\phi_0^2)}. \tag{20}$$

Calculating the terms displayed in (19) (the relevant integrals can be found in Appendix A) we arrive at the following expression

$$\Gamma^{(1)}(\phi_c) = \frac{9\lambda^2}{64\pi^2}\int d^4x\,\phi_c^2(x)\left[-\frac{1}{\epsilon} + \int_0^1 d\alpha\,\ln\left(\frac{-\alpha(1-\alpha)\Box + 3\lambda\phi_0^2}{v^2}\right)\right]\phi_c^2(x). \tag{21}$$

The above result (21) still contains the spurious parameter ϕ_0^2. Because the final result must be independent of it, have definite conformal properties and at the same time coincide with the Coleman–Weinberg potential in the limit of constant $\phi_c(x)$, we conclude that, after including higher order terms in (19) and (10), the relevant part of the renormalized non-local effective action should be obtained by the replacement $\phi_0^2 \to \phi_c^2(x)$. This gives

$$\Gamma_R^{(1)}[\phi_c] = \frac{9\lambda^2}{64\pi^2}\int d^4x\,\phi_c^2(x)\left[\int_0^1 d\alpha \times \ln\left(\frac{-\alpha(1-\alpha)\Box + 3\lambda\phi_c^2(x)}{v^2}\right)\right]\phi_c^2(x). \tag{22}$$

To be sure, one cannot hope to obtain a closed form analytical expression for the complete one-loop effective action. Although the above expression is certainly modified by further non-local functionals at higher orders in the expansion, we claim that (22) fully captures the *anomalous* conformal properties of the one-loop effective action, in the sense that the (unknown) functional modifications arising from convergent contributions at higher orders will not affect the anomaly. Observe that the result (22) cannot be expanded in λ because the zeroth order term would be $\propto \ln(\Box)$ which is ill-defined when acting on constant fields. Expanding in $\Box$, on the other hand, the resulting series would involve inverse powers of λ, and thus diverge at small λ.

While dimensional regularization automatically implements our postulate of an exact conformal invariance of counterterm dependent terms of the effective action, this is not so for other regulators for which there appear non-conformal local terms at intermediate steps of the calculation. For instance, with a standard UV cutoff Λ, the divergent part of the effective action (19) would read

$$\Gamma^{(1)}[\phi_c] = \frac{3\lambda\Lambda^2}{32\pi^2}\int d^4x\,\phi_c^2(x) - \frac{9\lambda^2}{64\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) \times \int d^4x\,\phi_c^4(x) + \cdots. \tag{23}$$

It thus breaks conformal invariance, and furthermore depends on the unphysical parameter M^2. Our basic requirement then amounts to an *exact* cancellation of the first term by an appropriate counterterm, not leaving any finite mass term either. Similarly, the M^2-dependence of the second term is gotten rid of by renormalization.

3. The conformal anomaly

Exact conformal invariance is reflected in the conservation of the conformal currents $J_\mu = \xi^\nu T_{\mu\nu}$ [1,2], with the conformal Killing vectors $\xi_D^\mu = x^\mu$ for dilatations, and $\xi_{K(\nu)}^\mu = 2x^2\delta_\nu^\mu - x^\mu x_\nu$ for conformal boosts, where $T_{\mu\nu}$ is the conserved energy momentum tensor; we have

$$\partial^\mu J_\mu = \partial^\mu\left(\xi^\nu T_{\mu\nu}\right) = \frac{1}{4}\left(\partial_\nu\xi^\nu\right)T^\mu{}_\mu. \tag{24}$$

Current conservation thus implies $T^\mu{}_\mu = 0$. If, on the other hand, this symmetry is broken by quantum effects there will appear an anomaly (trace anomaly) on the r.h.s. such that $T^\mu{}_\mu \neq 0$. As is well known (see, e.g., [23,24]), anomalies may occur when a symmetry of the classical Lagrangian cannot be maintained at the quantum level. An anomaly is unavoidable when a symmetry breaking term in the regulated effective action cannot be removed by a *local* counterterm before the regulator is removed. This applies in particular to conformal symmetry which is necessarily broken by *any* regularization, see [12,13].

The trace of the energy–momentum tensor can be calculated following [2], where it was shown that this trace vanishes for a conformally invariant classical action if one makes use of the so-called improved energy–momentum tensor. The latter can be directly obtained by varying the classical action (1) w.r.t. the metric, which gives

$$T_{\mu\nu}^{(0)} \equiv \left.\frac{\delta S}{\delta g^{\mu\nu}}\right|_{g_{\mu\nu}=\eta_{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi + \frac{1}{4}\lambda\phi^4\right) + \frac{1}{6}(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu)\phi^2. \tag{25}$$

By contrast, the contribution to $T_{\mu\nu}^{(1)}$ calculated from (22) has non-vanishing trace and constitutes the conformal anomaly. The easiest way to calculate it is to use the formula

$$T^\mu{}_\mu(x) = \left.\frac{\delta\Gamma[\phi_c,\omega]}{\delta\omega(x)}\right|_{\omega(x)=0}, \tag{26}$$

where the functional derivative is calculated with respect to conformal transformations (12) and we put $g_{\mu\nu}(x) = \eta_{\mu\nu}$ at the end. In order to derive this from (22) we must thus properly covariantize all expressions by re-inserting the metric $g_{\mu\nu}(x)$ in the appropriate places.

To proceed further we rewrite (22) as

$$\Gamma_R^{(1)}[\phi_c] = \frac{9\lambda^2}{64\pi^2}\int d^4x\,\sqrt{g}\phi_c^2(x) \times \left\{\ln\left(\frac{3\lambda\phi_c^2(x)}{v^2}\right) + \int_0^1 d\alpha\,\ln\left[1 + \frac{\alpha(1-\alpha)}{3\lambda} \times \frac{1}{\phi_c(x)}\left(-\Box_g + \frac{1}{6}R\right)\frac{1}{\phi_c(x)}\right]\right\}\phi_c^2(x). \tag{27}$$

Namely, using (12) and (13), it follows that the second term inside parentheses is *invariant* under conformal transformations: expanding the logarithm[5] we obtain an infinite series of terms, each of which is scale-invariant due to the inverse powers of $\phi_c(x)$. Let us mention here that the first order term in this expansion produces a *finite* correction (*alias* wave function renormalization) to the kinetic term in (1). The first term under the

[5] As we said above, this should not be thought of as a proper perturbative expansion, although (27) may nevertheless serve as a possible definition of the logarithmic differential operator. The point here is simply to verify the proper behavior of this operator w.r.t. the Weyl scalings (12).

integral in (27) breaks conformal invariance and gives the conformal anomaly

$$T^{\mu}{}_{\mu}(x) = \frac{9\lambda^2}{32\pi^2}\phi_c^4(x) \tag{28}$$

(on a curved space–time manifold this result is supplemented by the well-known terms quadratic in the Riemann tensor). We have thus confirmed (at this order) the general result that the anomaly itself is a local expression, but is obtained as the functional variation of a non-local expression (see, e.g., [25] for a discussion of this point).

The above result can be easily generalized to the case of $O(N)$ symmetry, that is, to N real scalar fields $\{\phi_c^i(x) \mid i = 1, \dots, N\}$ transforming in the fundamental representation of $O(N)$. The effective action is then equal to

$$\Gamma^{(1)}[\phi_c] = -\frac{N-1}{2}\,\mathrm{Tr}\ln\left(1 - \frac{\lambda\phi_c^2(x)}{-\Box + \lambda\phi_c^2(x)}\right) - \frac{1}{2}\,\mathrm{Tr}\ln\left(1 - \frac{3\lambda\phi_c^2(x)}{-\Box + 3\lambda\phi_c^2(x)}\right), \tag{29}$$

where $\phi_c^2(x) \equiv \sum_i \phi_c^i(x)\phi_c^i(x)$, and the corresponding conformal anomaly is

$$T^{\mu}{}_{\mu}(x) = \frac{(N+8)\lambda^2}{32\pi^2}\big(\phi_c^2(x)\big)^2. \tag{30}$$

These results can be rewritten in the form, familiar from general discussions of the trace anomaly (see, e.g., [13,14]), viz.

$$T^{\mu}{}_{\mu}(x) = \beta(\lambda)O_4(x), \tag{31}$$

where $O_4 \propto \phi_c^4$ is a dimension four operator, and the prefactor is the β-function for the $O(N)$ model (which is known up to five loops [26])

$$\beta(\lambda) = \mu\frac{\partial\lambda}{\partial\mu} = \frac{N+8}{8\pi^2}\lambda^2 + O\big(\lambda^3\big). \tag{32}$$

Hence, at least to this order,

$$T^{\mu}{}_{\mu}(x) = \frac{1}{4}\beta(\lambda)\big(\phi_c^2(x)\big)^2. \tag{33}$$

It is this relation, the *anomalous Ward identity*, which encapsulates the content of the symmetry, and how it is broken by quantum effects. A 'good' regularization of the theory should therefore preserve the structure of the anomalous Ward identity (31) as far as possible [20]. From this point of view the advantages of dimensional regularization are evident: it preserves the structure of (31) throughout the calculation by maintaining *exact* conformal symmetry at the level of the counterterms and the local part of the effective action, and in such a way that the parameter M nowhere appears on the r.h.s. of (31). Had we used a UV cutoff Λ instead, there would have appeared spurious mass terms (depending on M and Λ) on the r.h.s. of (31) at intermediate steps of the calculation, as is evident from (23). It is precisely the requirement of conformal invariance of the counterterm dependent terms of the effective action which ensures the absence of such spurious terms (as well as true mass terms) at any step of the calculation.

4. The issue of quadratic divergences

Let us now return to the issue of quadratic divergences in scalar field theories. The calculation of the foregoing section shows that the absence of (quadratically) divergent mass term corrections can be consistently imposed order by order by insisting on the *exact conformal invariance of the counterterm dependent terms of the effective action*; the effective CW potential then is the restriction of the renormalized effective action to constant field configurations. This statement is valid both in dimensional regularization (which does not distinguish between quadratic and logarithmic divergences) as well as in other schemes such as regularization in terms of an explicit UV cutoff Λ. Therefore, the issue is not whether the divergences which appear at intermediate steps of the calculation are quadratic or not, but only whether or not a symmetry can be imposed by means of local counterterms, and how the remaining anomalous terms can be uniquely characterized and computed. All this is, of course, in complete accord with standard renormalization theory [23,24].

To see that the way by which (classical) conformal invariance disposes of quadratic divergences is really no different from the way in which supersymmetry takes care of the problem, it is useful to recall that there is no problem whatsoever (other than convenience) in regularizing a supersymmetric theory by means of a *non*-supersymmetric regulator [27]—such as, for instance, ordinary dimensional regularization, or the use of *different* cutoffs for bosonic and fermionic loops. In both cases supersymmetry is violated but can be re-instated order by means of appropriate counterterms[6] (which themselves then also violate supersymmetry). In other words, the celebrated cancellation of quadratic divergences in supersymmetric theories is thus by no means automatic, but the result of an order by order imposition of supersymmetry at the level of the counterterm dependent terms of the effective action in perturbation theory. Furthermore, as emphasized in [20], quadratic divergences have no import on the general structure of the anomalous Ward identity (31) because the one-loop β-function on the r.h.s. of (31) 'does not know' about them, and therefore their appearance should be rather viewed as an artifact of the particular method employed to regulate the theory.

While the perturbative renormalization procedure thus does not care as to whether the divergences, which appear at intermediate steps of the calculation, are logarithmic or quadratic, the picture is, however, different in a Wilsonian perspective. There, one views the SM as being embedded as an effective low energy theory in some more unified theory whose modes above the weak scale have been 'integrated out'. Because one would then expect the mass corrections to be of the order of the unification scale (which would act as an effective UV cutoff), the existence of nearly massless modes (in comparison with the unification scale) indeed becomes a problem in the absence of an inde-

[6] This is true because there do exist perturbative regulators preserving supersymmetry, such as higher derivative regulators [28], dimensional regularization by dimensional reduction [29] (whose status at higher loops remains uncertain, however), or superspace methods (see, e.g., [30]).

pendent reason for their existence (as we pointed out already, conformal invariance cannot be invoked in the presence of a large mass scale). This conclusion seems inevitable if the large scale theory is also described by quantum field theory—as is the case for most 'beyond the SM' scenarios, such as GUTs and the MSSM. For this reason, and as already stated in the introduction, *the scenario proposed in* [5] *can only work if there are no intermediate mass scales of any kind between the weak scale and the Planck scale*. In addition we must require that the running couplings stay positive and bounded over a large interval of energies from $\Lambda_{\rm QCD}$ up to the Planck scale, since otherwise the theory would break down in between. The QCD scale is a natural IR cutoff because below that scale the conformal symmetry is known to be broken by nonperturbative QCD effects (quark and gluon condensates) which introduce their own mass scale. The upper limit derives from our assumption that physics at and beyond the Planck scale is no longer described in terms of standard quantum field theory, see the comments below. For this reason neither possible IR fixed points nor UV fixed points are relevant to the present considerations.

Let us explain why, in our opinion, the Wilsonian arguments outlined above may not be applicable to the Planck scale M_P, the only large scale in nature of whose existence we can be sure. Namely the usual arguments leading to divergent loop integrals cut off at M_P are based on the tacit assumption that ordinary quantum field theory works smoothly all the way up to M_P, then to be abruptly replaced by a Planck scale theory of quantum gravity. However, we would expect that such a theory which (by some as yet unknown mechanism) is supposed to give rise to an effectively conformal theory below the Planck scale, itself can *not* be a space–time based quantum field theory. Rather, space–time and its concomitant symmetries would then be emergent properties in a theory of quantum gravity,[7] as would be the case for quantum field theory. Even if field theory methods were to apply right up to M_P, such arguments do not take into account the anticipated UV finiteness of a proper (unified) theory of quantum gravity, and the fact that the resulting UV cancellations at the Planck scale may survive to low energy scales with conformal symmetry and in the absence of intermediate scales.

We note that several of the points raised here were also made in a recent preprint [7]. The main difference is that the νMSM model proposed there breaks conformal invariance already at the classical level, because the extra scalar field there is supposed to play the role of the inflaton. This requires not only a special fine-tuning of the parameters, but in particular, an explicit mass term at variance with conformal invariance; the latter is needed because the CW mechanism is not compatible with the values of the scalar self-couplings required to reproduce inflation. By contrast, we here make no attempt to use scalar fields for such purposes; rather, it is supposed that the mechanism leading to inflation—or what effectively *looks like it* from our low energy vantage point—is intrinsically quantum gravitational in nature. Interestingly, the scenarios of [5] and [7] differ in their predictions for the Higgs mass spectrum, and may thus be subject to experimental discrimination (and falsification).

To conclude: it is often said that the worst case for high energy physics would be if LHC discovered only the Higgs particle(s), but nothing else. We think otherwise: if there are no intermediate scales there is nothing to obstruct our view of the Planck scale. The challenge would then be to explain the observed structure of low energy physics directly and in a minimalistic way from a Planck scale theory of quantum gravity and quantum space–time, rather than evade the problem by introducing myriads of new particles and couplings, whose direct verification may well remain out of experimental reach. Besides, when trying to solve the hierarchy problem one is a priori in a much better position if the only terms in the effective action which break conformal invariance are logarithms.

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Appendix A

For the convenience of the reader, we here collect some integrals used in the main body of this Letter. For the dimensionally regulated integrals we have

$$\int \frac{d^d k}{(2\pi)^d} \ln\left(1+\frac{M^2}{k^2}\right) = \frac{4\Gamma(2-d/2)M^d}{(4\pi)^{d/2}d(2-d)},$$
$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2+M^2} = \frac{2\Gamma(2-d/2)M^{d-2}}{(4\pi)^{d/2}(2-d)},$$
$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2+M^2)((k+p)^2+m^2)}$$
$$= \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 \frac{d\alpha}{(\alpha(1-\alpha)p^2+M^2)^{2-d/2}}. \qquad \text{(A.1)}$$

For $d=4-2\epsilon$ and $C=e^\gamma/(4\pi)$ it gives up to $O(\epsilon)$

$$16\pi^2 (Cv^2)^\epsilon \int \frac{d^d k}{(2\pi)^d} \ln\left(1+\frac{M^2}{k^2}\right)$$
$$= -\frac{M^4}{2\epsilon} + \frac{M^4}{2}\left(\ln\frac{M^2}{v^2} - \frac{3}{2}\right),$$
$$16\pi^2 \int \frac{d^d k}{(2\pi)^d} \frac{(Cv^2)^\epsilon}{k^2+M^2} = -\frac{M^2}{\epsilon} + M^2\left(\ln\frac{M^2}{v^2} - \frac{1}{2}\right),$$
$$\int \frac{d^d k}{(2\pi)^d} \frac{16\pi^2 (Cv^2)^\epsilon}{(k^2+M^2)((k+p)^2+M^2)}$$
$$= \frac{1}{\epsilon} - \int_0^1 d\alpha \ln\left(\frac{\alpha(1-\alpha)p^2+M^2}{v^2}\right).$$

[7] Indeed, recent investigations on infinite dimensional hidden symmetries in supergravity suggest precisely such a scenario, see, e.g. [31].

References

[1] G. Mack, A. Salam, Ann. Phys. 53 (1969) 174.
[2] C. Callan, S. Coleman, R. Jackiw, Ann. Phys. 59 (1970) 42.
[3] G. 't Hooft (Ed.), in: Recent Developments in Gauge Theory, Plenum Press, New York, 1980.
[4] S. Coleman, E. Weinberg, Phys. Rev. D 7 (1973) 1888.
[5] K. Meissner, H. Nicolai, Phys. Lett. B 648 (2007) 312, hep-th/0612165.
[6] M. Shaposhnikov, I. Tkachev, Phys. Lett. B 639 (2006) 414, hep-ph/0604236.
[7] M. Shaposhnikov, arXiv: 0708.3550 [hep-th].
[8] J.R. Espinosa, M. Quiros, Phys. Rev. D 76 (2007) 076004, hep-ph/0701145.
[9] M. Sher, Phys. Rep. 179 (1989) 273.
[10] J. Glimm, A. Jaffe, Quantum Physics: A Functional Integral Point of View, Springer-Verlag, 1987.
[11] P. Ramond, Journeys Beyond the Standard Model, Perseus Book Group, 1999.
[12] J.C. Collins, A. Duncan, S.T. Joglekar, Phys. Rev. D 16 (1977) 438.
[13] K. Fujikawa, H. Suzuki, Path Integrals and Quantum Anomalies, Clarendon Press, Oxford, 2004.
[14] M.J. Duff, Class. Quantum Grav. 11 (1994) 1387, hep-th/9308075.
[15] B.L. Hu, D.J. O'Connor, Phys. Rev. D 40 (1984) 743.
[16] C. Wetterich, Nucl. Phys. B 352 (1991) 529.
[17] A.O. Barvinsky, V.F. Mukhanov, Phys. Rev. D 66 (2002) 065007, hep-th/0203132.
[18] I.L. Buchbinder, S.D. Odintsov, I. Shapiro, Effective Action in Quantum Gravity, IoP Publishing, 1992.
[19] M. Asorey, G. de Berredo-Peixoto, I.L. Shapiro, Phys. Rev. D 74 (2006) 124011, hep-th/0609138.
[20] W.A. Bardeen, On Naturalness in the Standard Model, preprint FERMILAB-CONF-95-391-T;
W.A. Bardeen, Mechanisms of Electroweak Symmetry Breaking and the Role of a Heavy Top Quark, preprint FERMILAB-CONF-95-377-T.
[21] P.H. Frampton, C. Vafa, hep-th/9903226.
[22] B. Zumino, in: S. Deser (Ed.), in: Brandeis Lectures, vol. 2, The MIT Press, Cambridge, MA, 1970.
[23] C. Itzykson, J.B. Zuber, Quantum Field Theory, McGraw–Hill, 1985.
[24] S. Weinberg, The Quantum Theory of Fields, vols. 1–3, Cambridge Univ. Press, 2000.
[25] R. Manvelyan, R. Mkrtchyan, H.J.W. Müller-Kirsten, hep-th/0103082.
[26] H. Kleinert, J. Neu, V. Schulte-Frohlinde, K.G. Chetyrkin, S.A. Larin, Phys. Lett. B 272 (1991) 39, hep-th/9503230;
H. Kleinert, J. Neu, V. Schulte-Frohlinde, K.G. Chetyrkin, S.A. Larin, Phys. Lett. B 319 (1993) 545, Erratum.
[27] D. Maison, Phys. Lett. B 150 (1985) 139.
[28] J. Iliopoulos, B. Zumino, Nucl. Phys. B 76 (1974) 310.
[29] W. Siegel, Phys. Lett. B 84 (1979) 193.
[30] S. Ferrara (Ed.), Supersymmetry, vol. 1, North-Holland, World Scientific, 1987.
[31] T. Damour, H. Nicolai, arXiv: 0705.2643 [hep-th].