

General-relativistic resistive-magnetohydrodynamic simulations of binary neutron stars

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We have studied the dynamics of an equal-mass magnetized neutron-star binary within a resistive magnetohydrodynamic (RMHD) approach in which the highly conducting stellar interior is matched to an electrovacuum exterior. Because our analysis is aimed at assessing the modifications introduced by resistive effects on the dynamics of the binary after the merger and through to collapse, we have carried out a close comparison with an equivalent simulation performed within the traditional ideal magnetohydrodynamic approximation. We have found that there are many similarities between the two evolutions but also one important difference: the survival time of the hypermassive neutron star increases in a RMHD simulation. This difference is due to a less efficient magnetic-braking mechanism in the resistive regime, in which matter can move across magnetic-field lines, thus reducing the outward transport of angular momentum. Both the RMHD and the ideal magnetohydrodynamic simulations carried here have been performed at higher resolutions and with a different grid structure than those in previous work of ours [L. Rezzolla, B. Giacomazzo, L. Baiotti, J. Granot, C. Kouveliotou, and M. A. Aloy, *Astrophys. J. Letters* 732, L6 (2011)], but confirm the formation of a low-density funnel with an ordered magnetic field produced by the black hole–torus system. In both regimes the magnetic field is predominantly toroidal in the highly conducting torus and predominantly poloidal in the nearly evacuated funnel. Reconnection processes or neutrino annihilation occurring in the funnel, none of which we model, could potentially increase the internal energy in the funnel and launch a relativistic outflow, which, however, is not produced in these simulations.

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I. INTRODUCTION

With the rapid progress made in upgrading and testing a series of advanced interferometric gravitational-wave detectors such as LIGO [1], Virgo [2], and KAGRA [3], there are now great expectations that in the next five years we will witness the first direct detection of gravitational waves. Prime sources for such a detection are binary systems of compact objects, namely, binary systems comprising either two black holes, a black hole and a neutron star, or two neutron stars. The latter configuration, in particular, is potentially a very interesting one, as it will represent the most common source, with a realistic expected detection rate of $\sim 40 \text{ yr}^{-1}$ [4]. A detection of gravitational waves from binary neutron stars would yield a wealth of information about the chirp mass, the orientation, and the localization of the binary but also possibly the mass, spin and radius of the individual stars [5, 6]. In turn, this information could set constraints on the equation of state (EOS) of the matter in their interior. Indeed, a number of recent investigations have revealed that it is possible to set serious constraints on the properties of the neutron-star structure and EOS, either when using the inspiral signal only [7, 8], or when exploiting the rich spectral features of the postmerger signal [9–13].

At the same time, the merger of a binary system containing at least one neutron star represents arguably the most attractive scenario to explain the complex phenomenology associated with short gamma-ray bursts (SGRBs), although many alternatives exist [see Ref. [14] for a recent review]. While such a scenario was suggested already 30 years ago [15, 16], numerical simulations (see, e.g., Refs. [17–21]) and new ob-

servations [14, 22] have put this scenario on firmer grounds. In particular, what these simulations have shown is that the merger of a binary system of neutron stars inevitably leads to the formation of a metastable object, which we dub the binary-merger product or BMP. Depending on the total mass and mass ratio of the binary, the BMP can either be a supramassive neutron star (SMNS), that is, a star with mass above the maximum mass for nonrotating configurations M_{TOV} but below the maximum mass for uniformly rotating configurations M_{max} , with $M_{\text{max}} \simeq (1.15 - 1.20) M_{\text{TOV}}$ [23]; a hypermassive neutron star (HMNS), that is, a star more massive than a SMNS; or a black hole¹.

The general-relativistic hydrodynamical modelling of binary neutron stars has seen very considerable progress over the last decade (see, e.g., Refs. [17–19]), and it has now reached a rather mature state. In fact, it is presently possible to calculate inspiral waveforms having a phase accuracy comparable to that of binary black-hole simulations thanks to the use of high-order methods with high-order convergence rates [25–27] or of very high-resolution and long inspirals [28, 29]. At the same time, the space of parameters is also being carefully investigated, both in terms of the variety of the EOSs considered [11–13], and of the treatment of radiative losses via neutrino cooling [30, 31].

When magnetic fields are present, on the other hand, the

¹ In principle, the BMP can also be a stable neutron star, but this would require that the stars have mass $M \lesssim M_{\text{TOV}}/2$. Since the mass distribution in neutron-star binaries is peaked around $1.3 - 1.4 M_{\odot}$ [24], this implies that $M_{\text{TOV}} \gtrsim 2.8 M_{\odot}$. Although this cannot be excluded, there is also no observational evidence that such massive neutron stars exist.

bulk of work carried out so far is considerably more limited. Investigations of the impact that magnetic fields have on the dynamics of the binary have in fact started with the first short-inspiral works in Refs. [32, 33], which were complemented by the longer simulations in Ref. [34]. The latter work, together with Ref. [35], also investigated the possibility that magnetic fields could have an impact on the gravitational-wave signals emitted by these systems during the inspiral. The conclusions reached were that, for realistic strengths $B \lesssim 10^{12}$ G, the presence of magnetic fields could not be revealed by detectors such as advanced LIGO or advanced Virgo. The astrophysical implications of the merger of a magnetized neutron-star binary were explored in Ref. [36], where it was shown that instabilities in the torus orbiting the black hole amplify the magnetic field by 3 orders of magnitude and generate a magnetic-jet structure characterized by an ordered poloidal magnetic field along the black-hole spin axis. The broad consistency with the observations in terms of black-hole spin, torus mass and accretion rate, and magnetic-field topology offered the first evidence that the merger of magnetized neutron stars can provide the basic conditions for the central engine of SGRBs.

Considerable effort has also been dedicated to investigating the properties of the HMNS under more controlled conditions. For instance, using ultrahigh spatial resolutions but axisymmetric initial data, Ref. [37] has provided the first evidence from three-dimensional global simulations that a magnetorotational instability (MRI) [38, 39] is likely to develop during the lifetime of the HMNS (see also Refs. [40, 41] for earlier work in two dimensions). In addition, again using axisymmetric initial data and different magnetic field configurations, it has been shown that a magnetically driven wind can be launched from the outer layers of the HMNS as a result of its differential rotation [42, 43]. These works have also highlighted that for realistic magnetic field topologies the wind is baryon loaded and quasi-isotropic, with bulk velocities of $\sim 0.1 c$ [43]. More recently, instead, the use of subgrid modeling as an effective way to describe the turbulent dynamics that develops in the shear layer between the two neutron stars at merger, has suggested that amplifications of up to 5 orders of magnitude are possible [44], although these amplifications are not produced in direct simulations [35, 36], even at very high resolution [45]. Finally, progress has taken place also on the derivation of improved numerical techniques, such as those in Ref. [46], where the significant advantages of a vector-potential approach and of a Lorentz gauge were discussed.

All of the works mentioned above have been carried out within the ideal-magnetohydrodynamic (IMHD) approximation, in which the electrical conductivity is assumed to be infinite. Under these conditions, the magnetic flux is conserved and the magnetic field is frozen into the fluid, being simply advected with it. This is a very good approximation for the stellar interior before the merger because it neglects any effect of resistivity on the dynamics of the plasma. After the merger, however, there will be spatial regions with very hot plasma where the electrical conductivity is finite and the resistive effects, most notably, magnetic reconnection, will take place.

An obvious improvement over the IMHD description is the

use of the general-relativistic resistive-magnetohydrodynamic (RMHD) equations, which provide a complete magnetohydrodynamic (MHD) description of regions with a high conductivity, such as the stellar interiors, and of regions with small conductivity, such as the electrovacuum exterior. Furthermore, when the conductivity is set to zero, it yields the Maxwell equations in vacuum, thus allowing for the study of the magnetic-field evolution also well outside the stellar magnetosphere [47].

Partly because of the increased complexity of the equations and partly because of the additional difficulties posed by their numerical solution (the equations easily become stiff in regions of high conductivity), RMHD simulations have started only rather recently. Most of the work so far has focussed on problems in flat spacetimes [47–53], but general-relativistic investigations have also been carried out on fixed spacetimes [54]. Indeed, together with the work carried out in Refs. [55, 56], those reported here are, to the best of our knowledge, the only RMHD simulations of the dynamics of binary neutron stars in general relativity. More specifically, we have followed the inspiral, merger, and collapse to a black hole of a neutron-star binary in which the stars have the same gravitational mass of $M = 1.625 M_{\odot}$ and are modelled with a simple ideal-fluid EOS.

Complementing the work reported in Refs. [55, 56], which concentrated on the electromagnetic emission during the inspiral and at the merger, the focus of the simulations reported here is that of assessing the impact that resistive effects have on the dynamics of the binary after the merger and through to collapse to a black hole. To this scope we have carried out a close comparison with an equivalent simulation performed for the same binary within the traditional IMHD approximation. In this way it has been possible to determine both the similarities between the two regimes and the novel features. The most important of such features is the evidence the survival time of the HMNS before collapse to a black hole increases in a RMHD simulation. This difference is associated to a less efficient magnetic braking in the resistive regime, in which matter is no longer perfectly advected with the flow, but can move across magnetic-field lines. In turn, this reduces the transport of angular momentum away from the central regions of the HMNS, increasing its lifetime. Interestingly, a longer-lived magnetized HMNS is of help in those models of SGRBs which invoke the existence of a magnetarlike object produced after the merger [57–62]. Another important result of the simulations reported here, which have been performed at higher resolutions and with a different grid structure than those in the previous work of Ref. [36], is the confirmation that a magnetic-jet structure is formed in the low-density funnel produced by the black hole–torus system. Both in RMHD and in IMHD, the magnetic field is predominantly toroidal in the highly conducting torus and predominantly poloidal in the nearly evacuated funnel. Furthermore, because of the effective decoupling between the matter and the electromagnetic fields achieved in the RMHD simulations, the magnetic-jet structure is coherent on the largest scale of our system. However, as in the IMHD case, also in these RMHD simulations, the magnetic-jet structure does not lead to an ultrarelativistic

tic outflow. Reconnection processes or neutrino annihilation occurring in the funnel could potentially increase the internal energy in the funnel and launch a relativistic outflow; none of these effects is modeled in our simulations.

The plan of the paper is as follows. In Sec. II we briefly review the mathematical setup of our simulations, concentrating mostly on the discussion of the general-relativistic RMHD equations used and on the expression of the generalized Ohm's law we have employed. Section III, on the other hand, is dedicated to illustrate the numerical strategy employed in the solution of the combined set of the Einstein and RMHD equations, including the properties of our computational grid, of our matching to the low-density exterior, and of our initial data. The core of the paper is represented by Sec. IV, where we present our results. After a brief overview, we discuss the magnetic-field topology and magnetic-jet structure produced in the simulations, as well as the comparison with the IMHD case. Such a comparison goes over a number of aspects, from the angular-momentum transfer and lifetime of the HMNS, over to the black hole–torus system, and the electromagnetic luminosities. Finally, Sec. V contains a conclusive summary of our results and the prospects for future research. Although this is not the focus of this work, an illustration of the magnetic-jet structure obtained in the IMHD simulations is presented in Appendix A for completeness.

We use a spacelike signature $(-, +, +, +)$ and a system of units in which $c = G = M_\odot = 1$ unless stated differently.

II. MATHEMATICAL SETUP

A. General-relativistic RMHD equations

Much of the numerical setup used in these simulations has been presented in greater detail in other papers [18, 34, 63–66], and for compactness we will review here only the basic aspects, referring the interested reader to the papers above for additional information. However, given its importance here, we will dedicate some space to a review of our fully general-relativistic RMHD framework, which was first presented in Ref. [66] and represents the extension of the special-relativistic RMHD formalism discussed in Ref. [47]. A similar but independent extension has been presented recently in Ref. [60], which describes the first 3+1 general-relativistic RMHD implementation in fixed spacetimes.

We start by presenting the augmented Maxwell equations

$$\nabla_\nu(F^{\mu\nu} + g^{\mu\nu}\psi) = I^\mu - \kappa n^\mu\psi, \quad (1)$$

$$\nabla_\nu(*F^{\mu\nu} + g^{\mu\nu}\phi) = -\kappa n^\mu\phi, \quad (2)$$

where $g^{\mu\nu}$ is the 4-metric, $F^{\mu\nu}$ is the Faraday tensor, $*F^{\mu\nu}$ is the Maxwell tensor, I^μ is the electric four-current density, and ϕ, ψ are two auxiliary scalar variables added to the Maxwell equations to control the constraints for the magnetic and electric parts, respectively (see below).

After a standard 3+1 splitting of spacetime, the Maxwell and Faraday tensors can be decomposed in terms of the electric (E^i) and magnetic (B^i) fields measured by an observer moving along the normal direction n^ν (i.e., normal or Eulerian observer) as

$$F^{\mu\nu} = n^\mu E^\nu - n^\nu E^\mu + \epsilon^{\mu\nu\alpha\beta} B_\alpha n_\beta, \quad (3)$$

$$*F^{\mu\nu} = n^\mu B^\nu - n^\nu B^\mu - \epsilon^{\mu\nu\alpha\beta} E_\alpha n_\beta, \quad (4)$$

with $\epsilon^{\mu\nu\alpha\beta} := \eta^{\mu\nu\alpha\beta}/\sqrt{-g}$, g the determinant of the 4-metric and $\eta^{\mu\nu\alpha\beta}$ the Levi-Civita symbol. The same can be done for the electric four-current

$$I^\mu := qn^\mu + J^\mu, \quad (5)$$

where q and J^μ are the charge density and the electric current density for an Eulerian observer, respectively. Using these definitions and performing a 3+1 decomposition of Eqs. (1) and (2) with respect to the normal vector n^μ , we arrive at the following evolution equations,

$$(\partial_t - \mathcal{L}_\beta)E^i - \epsilon^{ijk}\nabla_j(\alpha B_k) + \alpha\gamma^{ij}\nabla_j\psi = \alpha K E^i - \alpha J^i, \quad (6)$$

$$(\partial_t - \mathcal{L}_\beta)\psi + \alpha\nabla_i E^i = \alpha q - \alpha\kappa\psi, \quad (7)$$

$$(\partial_t - \mathcal{L}_\beta)B^i + \epsilon^{ijk}\nabla_j(\alpha E_k) + \alpha\gamma^{ij}\nabla_j\phi = \alpha K B^i, \quad (8)$$

$$(\partial_t - \mathcal{L}_\beta)\phi + \alpha\nabla_i B^i = -\alpha\kappa\phi, \quad (9)$$

where γ_{ij} is the spatial 3-metric, $K := K^i_i$ is the trace of the extrinsic curvature K_{ij} , α is the lapse and β is the shift 4-vector. We recall that \mathcal{L}_β denotes the Lie derivative along the shift vector and that ϵ^{ijk} is related to the four-dimensional Levi-Civita tensor via $\epsilon^{\nu\kappa\lambda} = \epsilon^{\mu\nu\kappa\lambda}n_\mu$ or alternatively $\epsilon^{ijk} = \eta^{ijk}/\sqrt{\gamma}$, where γ now is the determinant of the 3-metric. The scalar fields ϕ, ψ measure the deviation from the constrained solution, with ϕ driving the solution of Eq. (9) toward the zero-divergence condition $\nabla_i B^i = 0$, and ψ driving the solution of Eq. (7) toward the condition $\nabla_i E^i = q$. This driving is exponentially fast and over a time scale $1/\kappa$. This approach, named hyperbolic divergence cleaning in the context of IMHD, was proposed in Ref. [67] as a simple way of solving the Maxwell equations and enforcing the conservation of the divergence-free condition for the magnetic field. This method has been extended to the resistive relativistic case in Refs. [47, 49].

An obvious consequence of the Maxwell equations in RMHD is the conservation law associated with the electric charge

$$\nabla_\mu I^\mu = 0, \quad (10)$$

which provides an evolution equation for the charge density

$$(\partial_t - \mathcal{L}_\beta)q + \nabla_i(\alpha J^i) = \alpha K q. \quad (11)$$

Combining the MHD and Maxwell equations we obtain the following set of evolution equations, which we write in a flux-conservative form as

$$\partial_t(\sqrt{\gamma}B^i) + \partial_k(-\beta^k\sqrt{\gamma}B^i + \alpha\epsilon^{ikj}\sqrt{\gamma}E_j) = -\sqrt{\gamma}B^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\gamma^{ij}\partial_j\phi, \quad (12a)$$

$$\partial_t(\sqrt{\gamma}E^i) + \partial_k(-\beta^k\sqrt{\gamma}E^i - \alpha\epsilon^{ikj}\sqrt{\gamma}B_j) = -\sqrt{\gamma}E^k(\partial_k\beta^i) - \alpha\sqrt{\gamma}\gamma^{ij}\partial_j\psi - \alpha\sqrt{\gamma}J^i, \quad (12b)$$

$$\partial_t\phi + \partial_k(-\beta^k\phi + \alpha B^k) = -\phi(\partial_k\beta^k) + B^k(\partial_k\alpha) - \frac{\alpha}{2}(\gamma^{lm}\partial_k\gamma_{lm})B^k - \alpha\kappa\phi, \quad (12c)$$

$$\partial_t\psi + \partial_k(-\beta^k\psi + \alpha E^k) = -\psi(\partial_k\beta^k) + E^k(\partial_k\alpha) - \frac{\alpha}{2}(\gamma^{lm}\partial_k\gamma_{lm})E^k + \alpha q - \alpha\kappa\psi, \quad (12d)$$

$$\partial_t(\sqrt{\gamma}q) + \partial_k[\sqrt{\gamma}(-\beta^kq + \alpha J^k)] = 0, \quad (12e)$$

$$\partial_t(\sqrt{\gamma}D) + \partial_k[\sqrt{\gamma}(-\beta^kD + \alpha v^kD)] = 0, \quad (12f)$$

$$\partial_t(\sqrt{\gamma}\tau) + \partial_k\{\sqrt{\gamma}[-\beta^k\tau + \alpha(S^k - v^kD)]\} = \sqrt{\gamma}(\alpha S^{lm}K_{lm} - S^k\partial_k\alpha), \quad (12g)$$

$$\partial_t(\sqrt{\gamma}S_i) + \partial_k[\sqrt{\gamma}(-\beta^kS_i + \alpha S_i^k)] = \sqrt{\gamma}\left[\frac{\alpha}{2}S^{lm}\partial_i\gamma_{lm} + S_k\partial_i\beta^k - (\tau + D)\partial_i\alpha\right]. \quad (12h)$$

The fluid variables D , U , S_i , and S_{ij} are the conserved rest-mass density, the conserved energy density, the conserved momentum, and the fully spatial projection of the energy-momentum tensor, respectively. Their explicit definitions are therefore

$$D := \rho W, \quad (13)$$

$$\tau := U - D = \rho h W^2 - p + \frac{1}{2}(E^2 + B^2) - D, \quad (14)$$

$$S_i := \rho h W^2 v_i + \epsilon_{ijk} E^j B^k, \quad (15)$$

$$S_{ij} := \rho h W^2 v_i v_j + \gamma_{ij} p - E_i E_j - B_i B_j + \frac{1}{2}\gamma_{ij}(E^2 + B^2). \quad (16)$$

Here, $W = \alpha u^0 = u_i/v_i$ is the Lorentz factor, where u^μ are the components of the fluid 4-velocity and v^i are the components of the 3-velocity as measured by the Eulerian observer. Furthermore, $h := (e+p)/\rho = 1+\epsilon+p/\rho$ is the enthalpy, with $e = \rho(1+\epsilon)$ the total energy density, p the pressure, ϵ the specific internal energy, and ρ the rest-mass density [68]. The important difference between the RMHD and IMHD equations is that they involve stiff relaxation terms that pose serious numerical limitations on the time evolution of the equations. For this reason, a distinct class of implicit-explicit evolution methods has been developed, the RKIMEX schemes [69], which we have presented in detail in Ref. [66].

1. Generalized Ohm's law

To close the system of equations presented above, a relation for the electric current density in terms of the other fields is necessary, just like Ohm's law provides a prescription for the spatial conduction current to be proportional to the electric field. A generalized Ohm's law provides the necessary coupling of the electric current density to the electromagnetic and matter fields. Previous work toward relativistic versions of the generalized Ohm's law includes the investigations in Refs. [70–73]. In one of the simpler cases the spatial conduction current can be considered as proportional to the electric field measured by the comoving observer. Therefore the electric four-current density can be written as the superposition of an advective and a conductive current [74], which takes the form of the generalized Ohm's law

$$I^\mu = \tilde{J}_{\text{adv}}^\mu + \tilde{J}_{\text{cond}}^\mu = \tilde{q}u^\mu + \tilde{\sigma}^{\mu\nu}e_\nu. \quad (17)$$

Here, $\tilde{q} := -I^\mu u_\mu = [q + (\tilde{J}_a n^a)]/W$ is the electric charge density measured in the rest frame comoving with the fluid, and should be contrasted with $q := -I^\mu n_\mu$, which is instead the charge density measured by the Eulerian observer. Similarly, e_ν and $\tilde{\sigma}_{\mu\nu}$ are, respectively, the electric field and the electrical conductivity of the medium (which is a rank-2 symmetric tensor) as measured in the same frame. In collisional plasmas the current in the comoving frame can be considered to be carried by the mobile electrons, with charge e , and the conductivity tensor becomes

$$\tilde{\sigma}^{\mu\nu} = \sigma(g^{\mu\nu} + \xi^2 b^\mu b^\nu + \xi e^{\mu\nu\alpha\beta} u_\alpha u_\beta), \quad (18)$$

with b^μ being the magnetic field in the comoving frame, $\xi := e\tau_e/m_e$, $\sigma := n_e e \xi / (1 + \xi^2 + b^2)$, and τ_e the electron collision time scale.

Expressing the four-current density of Eq. (17) in terms of the fields measured by the Eulerian observer we arrive at

$$I^\mu = qn^\mu + qv^\mu + W\sigma[E^\mu + \epsilon^{\mu\nu\alpha}v_\nu B_\alpha - (v_\nu E^\nu)v^\mu] + W\sigma\xi^2(E^\alpha B_\alpha)[B^\mu - \epsilon^{\mu\nu\alpha}v_\nu E_\alpha - (v_\nu B^\nu)v^\mu] \quad (19)$$

In deriving Eq. (17) we have made the implicit assumption that the collision frequency between particles is much larger than the typical oscillation frequency of the plasma, which, we recall, is defined as $\omega_p := (4\pi n_e e^2/m_e)^{1/2}$. This implies that electrons and ions can reach equilibrium on very short time scales and any correction due to the mass difference between electrons and protons can be neglected. As a result, there is no global charge separation and the plasma is neutral. Note also that the first term in Eq. (18) accounts for an isotropic scalar law for the current, while the rest represent anisotropies due to the presence of a magnetic field in the comoving frame.

Ideally, it would be desirable to have a well-defined prescription of the conductivity tensor as a function of the fluid properties, $\sigma_{\mu\nu} = \sigma_{\mu\nu}(\rho, \epsilon, b_\mu)$, which stems from the microphysical properties of the plasma (see Ref. [73] for a recent discussion). In practice, however, we are far from having such a prescription in the extreme physical conditions characterizing merging neutron stars. However, if the collision time scale is much smaller than the electron cyclotron period² or, equivalently, if $\xi \ll |b^\mu b_\mu|^{-1}$, electrons do not have sufficient time to gyrate perpendicular to the magnetic field lines. Under these conditions, the isotropic part of the conductivity is the dominant one (electrons essentially slide along the magnetic-field lines), and expression (18) can be approximated as $\sigma_{\mu\nu} \approx \sigma g_{\mu\nu}$. As a result, the spatial three-current density in the Eulerian frame coming from the generalized Ohm’s law (19) can be simplified so that

$$J^i = qv^i + W\sigma[E^i + \epsilon^{ijk}v_j B_k - (v_k E^k)v^i], \quad (20)$$

where we used the fact that the four-current density can be also written as $I^\mu = qn^\mu + J^\mu$. In our simulations, the conductivity σ is chosen to be either a constant or a function of the rest-mass density and a discussion will be presented in Sec. III C 1. The last term in Eq. (19) represents the Hall effects, which, however, we set to zero for simplicity.

III. NUMERICAL SETUP

A. Field equations

The evolution of the spacetime (i.e., of the 3-metric, extrinsic curvature, and conformal factor) is obtained using the `McLachlan` code, which implements the BSSNOK formulation of the Einstein equations [75–77] employing three-dimensional finite-differencing operators for calculating the fluxes on the right-hand side of the Einstein equations [78]. The time integration is carried out using the third-order accurate strong stability preserving implicit-explicit Runge–Kutta scheme outlined in Ref. [66]. The time step on each grid is limited by the Courant–Friedrichs–Lewy (CFL) condition [68] and hence the Courant coefficient is set to be 0.25 on all refinement levels. A Kreiss–Oliger type dissipation is added to the spacetime evolution equations to ensure that any high-frequency noise produced during the evolution (mostly at the refinement levels boundaries) is damped. The damping parameter in the evolution equation for the shift was carefully chosen to be 0.71 so that the additional term does not become stiff on the coarser grids.

² The electron cyclotron period is defined as $P_{c,e} := 2\pi/\omega_{c,e}$, where $\omega_{c,e} := eB/m_e$ is the cyclotron frequency and represents the frequency at which electrons gyrate perpendicular to the magnetic-field lines.

B. GR-RMHD equations

The general-relativistic RMHD (GR-RMHD) equations are solved with the `WhiskyRMHD` code [66], which uses high-resolution shock-capturing schemes [68, 79]. In particular, the reconstruction of the conserved variables is achieved via the Piecewise Parabolic Method [80], while the fluxes are calculated through the approximate Riemann solver introduced by Harten–Lax–van Leer–Einfeldt [81] and which requires only knowledge about the maximum characteristic speeds of the system, i.e., the speed of light in this case.

The system of RMHD equations is closed by describing the fluid as ideal and with a Γ -law EOS

$$p = \rho\epsilon(\Gamma - 1), \quad (21)$$

where $\Gamma = 2$ and $K = 123.6$. This EOS is clearly an idealization. Much more sophisticated EOSs have been implemented in our numerical infrastructure [13, 82, 83] and have been used by other groups in general-relativistic magnetohydrodynamic (GRMHD) simulations [31]. However, this idealization is probably adequate at this stage as here we are mostly interested in assessing the differences introduced in the dynamics of the BMP by resistive effects. These effects suffer from even larger uncertainties than those associated to the different EOSs.

C. Adaptive mesh refinement and symmetries

Our code makes use of the `Cactus` [84] computational framework, which allows us to employ a box-in-box vertex-centered adaptive mesh refinement grid hierarchy that tracks the “center of mass” of the stars as they orbit each other. This was achieved via the `BNSTracker` thorn implemented by W. Kastaun and the `Carpet` driver [85]. The numerical domain consists of six levels of refinement with the resolution doubling between adjacent refinements. In addition, we employ moving refinement boxes in order to track the high-density regions. The outer boundary of the computational domain is located at ≈ 378 km. The finest resolution during the inspiral and merger is $\Delta x \approx 296$ m, but an extra refinement level is activated right after the merger with a resolution of $\Delta x \approx 148$ m. It is important to remark that, although in a small region of spatial extent ~ 13.5 km, this resolution is considerably higher than the one used in Ref. [36] (i.e., ≈ 221 m with a spatial extent of ~ 35.4 km), where the first evidence was given that the merger of a binary system of magnetized neutron stars can lead to the formation of a magnetic-jet structure. Hence, although in a resistive framework, the simulations reported here can be considered as a “higher-resolution” counterpart of those presented in Ref. [36]. We also note that when considering initial data consisting of a binary in quasicircular orbits (see Sec. IV C 2) we do not activate the extra refinement level after the merger, hence keeping a resolution of $\Delta x \approx 296$ m to describe the HMNS. We do this so as to have two different resolutions to describe the HMNS and hence to study the numerical consistency of the solution.

To reduce the computational cost associated with the numerical evolution of an equal-mass binary, we use a reflection-symmetry condition across the $z = 0$ plane and a π -symmetry condition across the $x = 0$ plane. These conditions respect the symmetries of the scenario we are investigating. The outer boundary conditions are set by using simple zeroth-order extrapolation of the hydrodynamic variables. For the electromagnetic and spacetime variables, we employ simple Sommerfeld radiative boundary conditions because of the nature of the fields. Given the long time scale over which our simulations are carried out, reflections due to imperfections of the boundary conditions experience several domain crossings before the end of the simulation. We plan to improve on this in the future by the application of maximally dissipative boundary conditions that minimize the effects of reflections.

1. Exterior matching and atmosphere handling

Our goal with the use of a RMHD framework is that of modelling the exterior of the neutron stars as an electrovacuum, where both the conductivity and the charge density are negligibly small, so that electromagnetic fields should obey in these regions the Maxwell equations in vacuum. On the other hand, we want to model the interior of the stars as highly conducting, so that our equations recover the IMHD limit in such regions. There are several different ways to achieve this; see, for example, Refs. [66, 86–88]. Each of them has, in our view, its advantages and disadvantages. However, because they all try to model the difficult transition region between two regimes that are intrinsically different, they all represent an approximation. This difficulty is not specific to this problem but is a typical feature of physical problems as, for example, in the transition from an optically thick to an optically thin regime in radiative-transfer calculations. Our approach is also an approximation and is similar to the one in Refs. [66, 87] in the sense that the matching from the stellar interior to the stellar exterior is achieved through a carefully chosen conductivity profile. More specifically, the conductivity profile adopted is directly related to the conserved rest-mass density (and hence to the rest-mass density) and given by

$$\sigma := \sigma_0 \max \left[1 - \frac{2}{1 + \exp [2D_{\text{tol}}(D - D_{\text{rel}})/D_{\text{atm}}]}, 0 \right], \quad (22)$$

where $\sigma_0 \gg 1$ corresponds to a uniform scalar conductivity. The parameters D_{tol} , and D_{rel} determine how sharp the transition to the exterior is. For the simulations reported here, we have chosen³, $\sigma_0 = 10^6 = 2.0 \times 10^{11} \text{ sec}^{-1}$, $D_{\text{tol}} = 0.01$ and $D_{\text{rel}} = 100 D_{\text{atm}} = 100 \rho_{\text{atm}}$, where D_{atm} and ρ_{atm} are

the values of the conserved and primitive rest-mass density in the “atmosphere.”

We recall that, as in other Eulerian hydrodynamics and MHD codes, also our `WhiskyRMHD` code makes use of a very low rest-mass density fluid to handle the evolution of the MHD equations in regions which are associated to the exterior of the stars. In such a region, we follow the same prescription initially implemented in Ref. [89] and then adopted in essentially all of the simulations performed with our code, even in its IMHD incarnation [63]. In essence, we treat as atmosphere any region in the computational domain which is below ρ_{atm} , which we take here to be $6.17 \times 10^6 \text{ g cm}^{-3}$, that is approximately 8 orders of magnitude smaller than the maximum rest-mass density.⁴ In such a region, we set the fluid 3-velocity to zero, the rest-mass density to a floor value, and the specific internal energy to the value it assumes for a fluid following a polytropic EOS [68].⁵ In addition, and differently from the IMHD implementation of Ref. [35], the use of the prescription (22) automatically sets the conductivity in the atmosphere to zero, so that the Maxwell equations reduce there to the Maxwell equations in vacuum. Hence, in our prescription the atmosphere is de facto a cold, static, uniform fluid in which electromagnetic waves propagate as if in vacuum.

A few remarks should be made about modelling the neutron stars’ exterior, which in reality is expected to be a highly conducting, low-density, possibly magnetically dominated plasma in very strong magnetic fields.

First, with our prescription for the atmosphere and with the conductivity (22), the latter is zero also in regions that are not at the atmosphere level but close to it, i.e., at $D \leq D_{\text{rel}}$. We do this because the strong winds that are produced at the merger and later on rapidly fill the computational domain. A direct consequence of this is that the atmosphere is present only close to the boundaries and therefore an IMHD prescription would be met essentially everywhere. Yet, we are interested in deviations from perfect-flux freezing and in particular in regions where plasma is tenuous and the magnetic fields essentially decouple from the matter. We can effectively achieve this by setting $\sigma = 0$ in any region that is below D_{rel} . This approach therefore accomplishes our goal of decoupling in the low-density regions the evolution of the electromagnetic fields from the dynamics of the plasma. Of course, an electrovacuum prescription where the rest-mass density is nonzero is, strictly speaking, inconsistent, but we believe that this is a tolerable inconsistency, given that the rest-mass density in these regions takes essentially the smallest values in the whole domain.

Second, albeit somewhat arbitrary, our approach suffers from the same uncertainties of other approaches suggested in the literature to match the two different regimes (see, e.g., Refs. [56, 87, 90]). Ideally, its robustness can be validated by varying the free parameters D_{rel} and ρ_{atm} , although

³ We recall that in units in which the speed of light is not set to 1, $\tau_d := L^2 \sigma_0 / c^2$ represents the Ohmic diffusion time scale, with L the typical length scale of the field. For $L \sim 10^6 \text{ cm}$, $\tau_d \sim 10^2 \text{ sec}$, which is obviously much larger than the timescale over which our simulations are carried out, i.e., $\sim 10^{-2} \text{ sec}$.

⁴ In practice, to avoid being sensitive to the threshold value, we set to atmosphere any cell of which the rest-mass density is below $\rho_{\text{atm}} + \rho_{\text{tol}}$, where $\rho_{\text{tol}} \sim 10^{-2} \rho_{\text{atm}}$ [18].

⁵ A different prescription is used for the specific internal energy in the case of hot, nuclear-physics EOSs (see Ref. [82] for details).

this is something that admittedly we are not able to do here because of the computational costs involved. More importantly, we find that this approach allows us to take an important step beyond the previous IMHD treatment presented in Ref. [91].

Third, while matching to a force-free regime is appealing, it can be rather dangerous in the physical conditions encountered *after* the merger (by contrast, the force-free approximation is probably very good before the merger or in black hole–neutron-star binaries, when only magnetospheric effects are expected to take place). Our calculations reveal in fact that the exterior of the conducting matter (either the HMNS or the torus) is always matter dominated and the plasma beta parameter, i.e., the ratio of the gas-to-magnetic pressure,⁶ $\beta_p := 2p/B^2$, is at least 10^4 and of the order of 10^6 in the polar regions. This is far from the condition of $\beta_p \ll 1$, where the matter inertia can be neglected and the force-free approximation is a good one.

Finally, despite the fact that our implementation offers a control over the amount of resistivity in different parts of the flow, the choice of realistic values for the resistivity is far from trivial (see, e.g., Ref. [92]) and not addressed at all in these simulations.

2. Miscellanea

As already mentioned in Sec. II A, in order to ensure that the magnetic field is essentially divergence free, we have employed the divergence cleaning method of Ref. [67], which, however, requires choosing a suitable value for the constant κ [cf., Eqs. (7) and (9)]. In the simulations presented here, we have set $\kappa = 0.075$. This value does not lead to stiffness problems in the coarser grids and at the same time provides a rapid damping of the constraint violation on a time scale $\tau \sim (B^i B_i)^{1/2} / \nabla_i B^i$.

When after the merger the HMNS collapses to a black hole, steep gradients appear in the rest-mass density and cover just few grid points of the finest grid. If the resolution is sufficiently low, these gradients are simply dissipated numerically and the evolution can proceed without problems. However, for the rather high resolutions used here for the finest grid, i.e., ≈ 148 m, the numerical dissipation is smaller and the gradients are not removed, leading to failures in the conversion of the conserved variables to the primitive ones. To counter this problem we reset to atmosphere the hydrodynamical variables in a mask inside which the lapse function goes below a threshold, e.g., $\alpha < \alpha_{\text{thr}} = 0.1$. This reset is done only for the hydrodynamical variables, while the spacetime and electromagnetic ones are evolved as usual. Furthermore, the reset is applied in practice to a handful of cells, well inside the apparent horizon and thus not influencing the matter dynamics.

D. Initial data

The initial data consist of a magnetized binary neutron-star system of total Arnowitt–Deser–Misner (ADM) mass $M_{\text{ADM}} = 3.25 M_\odot$ and an initial orbital separation of 45 km. Each star has a baryon mass equal to $1.625 M_\odot$ and an equatorial radius of $R_{\text{eq}} = 13.68$ km, so that the initial separation corresponds to approximately $3.3 R_{\text{eq}}$. The initial orbital velocity is $\Omega_0 = 1.85 \text{ rad ms}^{-1}$, and the maximum rest-mass density is $5.91 \times 10^{14} \text{ gr cm}^{-3}$. Lacking self-consistent initial data for magnetized binaries, our initial data are generated by the LORENE library as an unmagnetized irrotational binary in equilibrium on a quasicircular orbit [93, 94]. The magnetic field is then superimposed on the unmagnetized constraint-satisfying solution. Following Refs. [34, 35], the initial magnetic field is fully contained inside the stars and purely poloidal. This is achieved after prescribing the toroidal vector potential A_ϕ to have the form

$$A_\phi = A_b [\max(P - P_{\text{cut}}, 0)]^2, \quad (23)$$

where $P_{\text{cut}} = 0.04 P_{\text{max}}$ determines the point at which the magnetic field goes to zero (typically before it reaches the surface). The resulting (poloidal) magnetic field is just the curl of the vector potential and leads to a maximum magnetic-field strength of $1.97 \times 10^{12} \text{ G}$ at the pressure maximum. Because the initial data constructed in this way are not a solution of the full Einstein–Euler–Maxwell system, they will introduce an increased violation of the constraint equations, which amount to $\sim 2.5 \times 10^{-6} / M_{\text{ADM}}^2$ and $\sim 1.7 \times 10^{-6} / M_{\text{ADM}}^2$ in the L_2 -norm, of the Hamiltonian and momentum constraints, respectively. These values should be compared with those before the introduction of the magnetic field and which are about 1 order of magnitude smaller, i.e., $5 \times 10^{-7} / M_{\text{ADM}}^2$ and $1.5 \times 10^{-7} / M_{\text{ADM}}^2$.

The perturbations introduced by the addition of the magnetic fields are small enough so as not to have a significant effect on the dynamics of the binary. Indeed, our experience is that the L_2 -norm of the constraints relaxes to values comparable to those of simulations of unmagnetized binaries after about one crossing time or, equivalently, one orbit. Obvious ways to improve this approach, and which could become important for an accurate modelling of the gravitational-wave emission during the inspiral, exist. Among them are the use of consistent initial data for magnetized binaries or the simulation of binaries with much larger separations, so that the system has several orbits to reach a more consistent MHD equilibrium.

In addition, because the main focus of this work is the assessment of resistive effects on the postmerger dynamics and hence a comparison between IMHD and RMHD simulations, our interest in an accurate treatment of the initial data is rather limited here. As a result, and once again to reduce the computational costs, we accelerate the inspiral as first suggested in Ref. [95]. More specifically, for most of our simulations we modify the initial linear momenta by adding an initial inward radial velocity which is $\sim 20\%$ of the orbital velocity. This reduces the number of orbits at this separation from ~ 3.5 to

⁶ We note that our definition of β_p is the one normally used in plasma physics, but the inverse of the one employed in other numerical-relativity calculations, e.g., Ref. [21].

only ~ 1.5 . Of course, the constraint violations introduced in this way are even larger than those discussed above with the introduction of magnetic fields⁷, but we have also verified that this does not introduce qualitative differences by comparing the results of these grazing collisions with those obtained from the corresponding binaries in quasicircular orbits.

IV. RESULTS

The dynamics of the same neutron-star binary considered here has been previously investigated in hydrodynamic simulations [18], in IMHD simulations [35], and, more recently, also in RMHD simulations [56]. In all cases, it was shown that after the merger this specific model forms a rapidly rotating HMNS with a high degree of differential rotation. The transient object collapses later to form a rotating black hole of mass $\sim 2.9 - 3.0 M_{\odot}$. The magnetic-field strengths chosen here and in previous works [35, 56] are not sufficiently high to affect the bulk dynamics of the binary during the inspiral [34], which can be considered to be equivalent to the purely hydrodynamical case for all practical purposes.

In the following subsections, we focus on the results obtained from the application of our RMHD implementation. We start by providing a general description of the basic features of the RMHD dynamics (Sec. IV A) and then move to make a detailed comparison with an IMHD simulation of the same neutron-star binary (Sec. IV C). We note that our focus here is different from the one in Ref. [56], which matched the resistive description to a force-free one to study the interaction of the two stellar magnetospheres before the merger. Here, on the other hand, we are mostly interested in the postmerger object and on the effects that resistivity has on the dynamics of the HMNS and subsequent black hole–torus system. Because of this, and because the scenario we are investigating is polluted by the large baryonic winds produced after merger, our resistive matching is made to an electrovacuum exterior. In this sense, the work carried out here and in Ref. [56] provide a complementary description of the dynamics of magnetized binary neutron stars in full general relativity.

A. Rapid overview

The dynamics of the binary when evolved within the RMHD framework is summarized in Figs. 1 and 3. The first one, in particular, reports two-dimensional cuts in the (x, y) and (x, z) planes of the rest-mass density ρ (left panel), of the specific internal energy ϵ (middle panel), and of the modulus of the magnetic field $|B| := (B^i B_i)^{1/2}$ (right panel). Marked

with white lines are the projection of the magnetic-field lines on the different planes, while marked with green solid lines are the isocontours of the rest-mass density. The snapshots refer to times $t = 2.43$ ms (top row), $t = 3.91$ ms (middle row), and $t = 11.60$ ms (bottom row). The positions of the two stars are marked with \times and $+$ symbols, and the stars' trajectories are marked with red dashed lines.

Given the small initial orbital separation of 45 km and the reduced linear moment, the two stars merge very rapidly. More specifically, at approximately $t \simeq 0.5$ ms, the two stellar surfaces start entering in contact, although the actual merger takes place at $t \simeq 3.91$ ms.⁸ As the merger takes place, the two stellar cores become significantly distorted by the large tidal fields and produce spiral arms. At the leading edges of these spiral arms, the specific internal energy increases through shock heating (cf., the middle column of Fig 1).

The time $t = 2.42$ ms in the top row of Fig. 1 corresponds to one orbit of the binary, which is sufficient for the magnetic field to diffuse over the thin transition layer close to the surface of the stars in an attempt to settle to a new equilibrium configuration (cf., the discussion in Sec. IV.C.1 of Ref. [66]). Once the magnetic field has diffused out of the star it continues to propagate also in regions that are dynamically treated as atmosphere and where the electrical conductivity is set to zero as if the medium was an electrovacuum. In this way, we achieve a rather smooth transition between the highly conducting stellar interior and the electrovacuum exterior. This is shown in Fig. 2, where we show the electrical conductivity at three reference times on the (x, z) plane. Note that, the region in black corresponds to our electrovacuum but does not coincide with the atmosphere. Indeed, the region in black is filled with tenuous plasma as can be seen in the left columns of Fig. 1. Note also that at this time the magnetic-field topology is still predominantly poloidal in the exterior of the star. However, a toroidal component is also being generated as the highly conducting material in the stellar interior shears the poloidal magnetic field in the lower-density, high-conductivity spiral arms.

As mentioned above, at $t = 3.91$ ms the merger takes place, at least as measured from the position of the first peak in the gravitational-wave amplitude. When this happens, a vortex sheet is created between the two stars, which could lead to the onset of a Kelvin–Helmholtz instability and the generation of a large-scale and ultrastrong magnetic field [97] (see the middle row of Fig. 1). It is presently a matter of debate whether such large-scale magnetic fields can be produced with amplifications of several orders of magnitude. Present direct simulations are not able to reach the resolutions necessary to resolve the turbulent motion produced by the instability [34, 35]. The results obtained so far with direct, very high-resolution simulations, either local [98] or global [45], indicate that the amplification of the magnetic field is of a factor 20 at most, most likely because resistive instabilities dis-

⁷ For completeness, we can compare the violations in the L_2 -norm of the Hamiltonian and momentum constraints for binaries with reduced initial momenta with the corresponding violations for binaries on quasicircular orbits. The latter amount to $\sim 2 \times 10^{-7}/M_{\text{ADM}}^2$ for the Hamiltonian constraint and to $\sim 4 \times 10^{-8}/M_{\text{ADM}}^2$ for the average of the momentum constraints.

⁸ As is customary, we define the time of merger as the time of the first peak of the gravitational-wave amplitude [13, 83, 96].

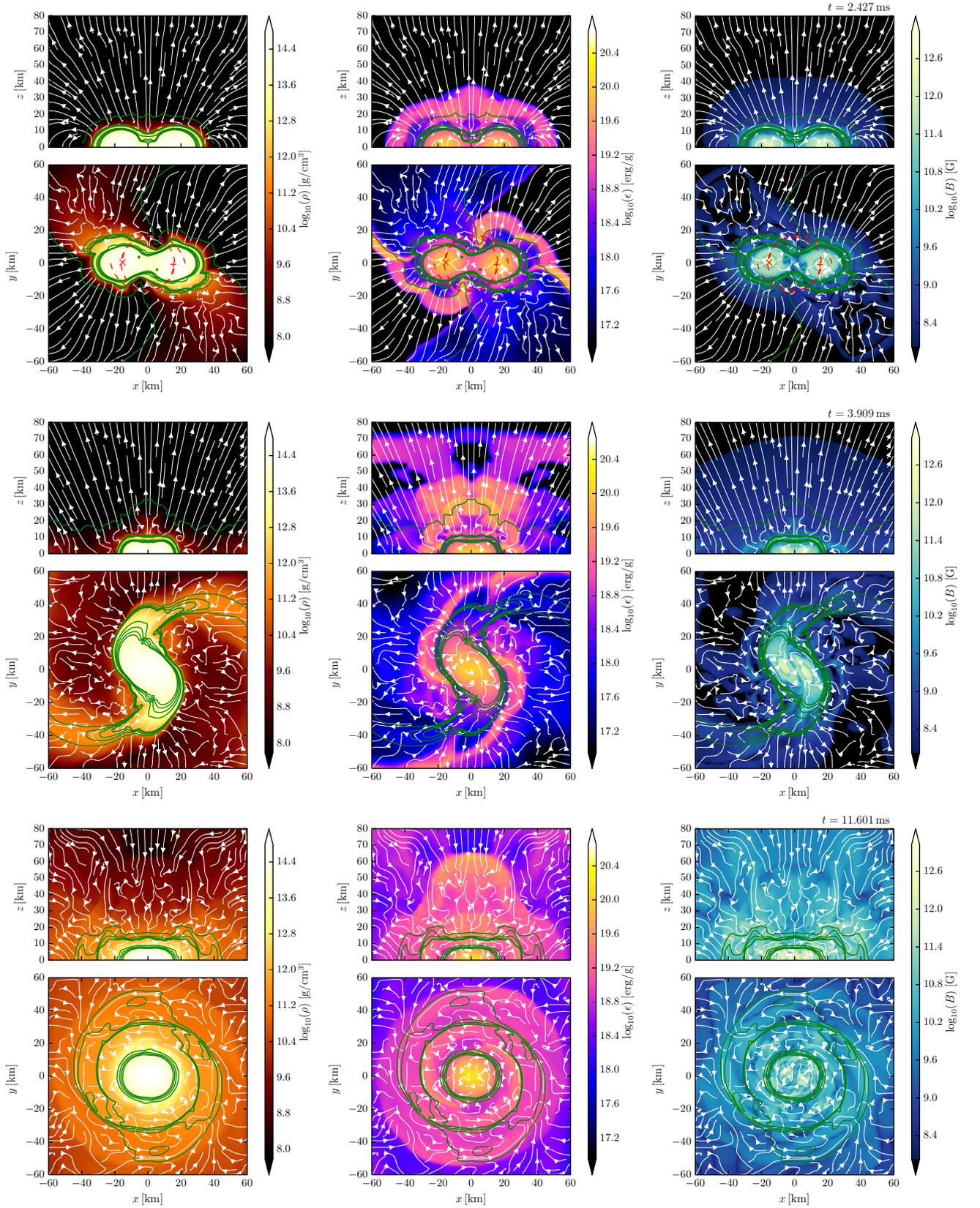


FIG. 1. Snapshots of two-dimensional cuts in the (x, y) and (x, z) planes of the rest-mass density ρ (left panel), of the specific internal energy ϵ (middle panel), and of the modulus of the magnetic field $|B| := (B_i B_i)^{1/2}$ (right panel). From the top, the snapshots correspond to times $t = 2.43$ ms (top row), $t = 3.91$ ms (middle row), and $t = 11.60$ ms (bottom row). Shown with white lines are the projection of the magnetic-field lines on the different planes, while marked with green solid lines are the isocontours of the rest-mass density at $\rho = \{6.2 \times 10^7, 1.2 \times 10^{11}, 2.5 \times 10^{11}, 3.7 \times 10^{11}, 4.9 \times 10^{11}, 6.2 \times 10^{11}, 1.3 \times 10^{13}, 2.5 \times 10^{13}, 3.7 \times 10^{13}, 4.9 \times 10^{13}, 6.2 \times 10^{13}\}$ gr cm $^{-3}$. The positions of the two stars are marked with \times and $+$ symbols, and the stars' trajectories are marked with red dashed lines. The different panels represent the evolution of the HMNS and highlight that no ordered magnetic-field topology emerges; this will change when the HMNS collapses to a black hole (see also Figs. 3–6).

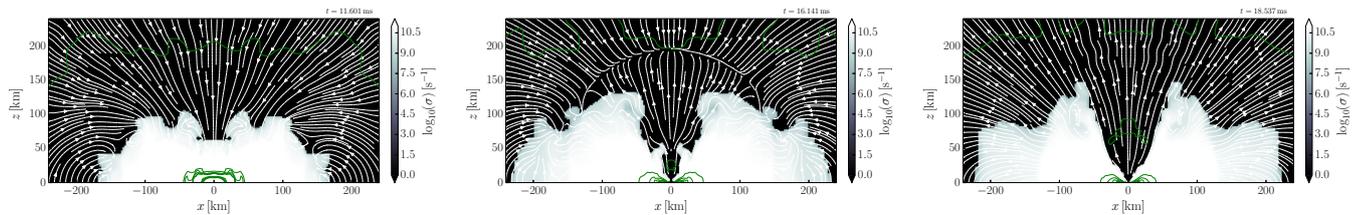


FIG. 2. Two-dimensional cuts on the (x, z) plane of the electrical conductivity distribution at different times $t = \{11.601, 16.141, 18.537\}$ ms (left to right). Note that, the conductivity is very large in the HMNS (left panel) and torus (middle and right panel), where the IMHD is recovered. On the other hand, the conductivity is outside the HMNS/torus, where the electrovacuum limit is reached. Note also that the region in black does not coincide with the atmosphere, but is filled with tenuous gas as can be seen in the left columns of Figs. 1 and 3.

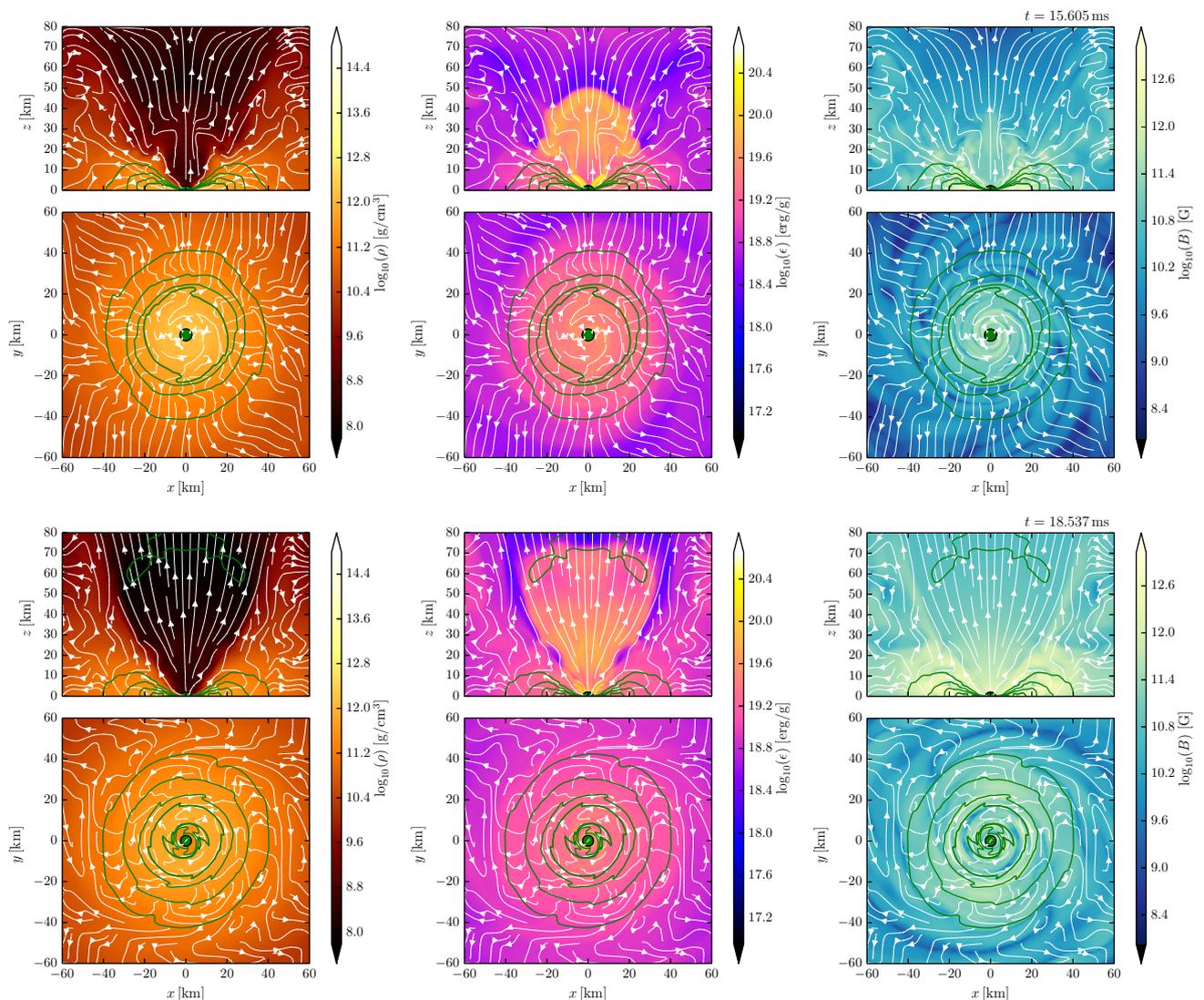


FIG. 3. The same as in Fig. 1 but for times $t = 15.61$ ms (top row) and $t = 18.54$ ms (bottom row), when a black hole has already been formed. Note that, in contrast to the dynamics of the HMNS shown in Fig. 1, the collapse of the HMNS leads to the generation of large-scale coherent magnetic fields and the emergence of a magnetic-jet structure around the black-hole rotation axis. The magnetic field is mostly poloidal in the funnel and mostly toroidal in the torus (see also Figs. 5 and 6).

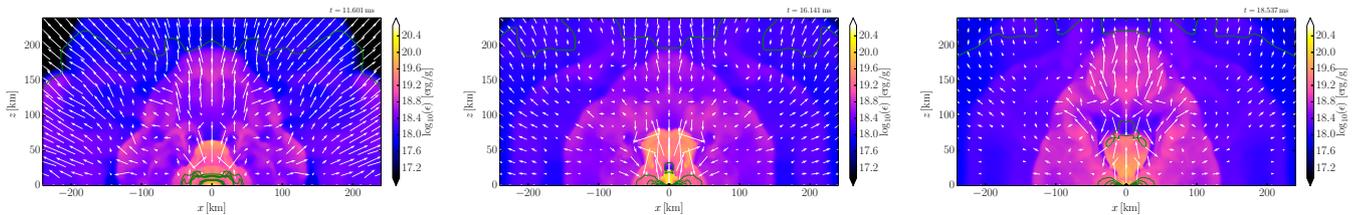


FIG. 4. Two-dimensional cuts on the (x, z) plane of the specific internal energy and of the velocity field in this plane. Note that the times selected coincide with those presented in Fig. 2, and are useful to gain information on the properties of the flow during the HMNS stage and after the collapse to a black hole. The flow is essentially given by the magnetically driven wind during the HMNS stage (left panel). However, once a funnel is produced, the flow is ingoing at heights $z \lesssim 100$ km, but becomes outgoing for $z \gtrsim 100$ km (middle and right panels).

rupt the Kelvin–Helmholtz unstable vortex [98] (but see also Ref. [44] for recent simulations with subgrid modeling which could lead to much larger amplifications).

In the simulations reported here, we find that the magnetic-field magnitude increases slightly less than an order of magnitude during this stage (see Fig. 12 and the discussion in Sec. IV C). This moderate growth of the magnetic field is possibly due to insufficient resolution and our inability to capture the dynamics of the relevant scales. On the other hand, it could also indicate that a Kelvin–Helmholtz instability simply does not develop. A possible reason is that the time scale of the instability might be longer than the dynamical time scale and the shearing motion could very rapidly get destroyed as the two stellar cores collide on a time scale of a fraction of a millisecond. We should also note that resistive effects might become important at this stage as the reconnection of magnetic-field lines might lead to the acceleration of matter due to Ohmic heating. We believe that higher-order schemes, as those presented in Refs. [25, 26, 48], could help resolve this issue.

The bar-deformed HMNS produced after the merger is differentially rotating, and magnetic braking transfers angular momentum from the inner core to the outer parts of the star. The spiral arms widen and merge together generating more shock heating and dissipation. A magnetically driven wind as a result of differential rotation is launched from the outer layers of the HMNS [42, 43]. The wind could play an important role in the modelling of short gamma-ray bursts (SGRBs), which show an extended x-ray emission [61, 62]. The wind is not constant in time, but rather characterized by a bursty activity in which high internal energy plasma blobs (i.e., local concentrations of specific internal energy a few kilometers in size) are launched from near the black-hole horizon and propagate along the z -direction (bottom row of Fig. 1). Interestingly, the bursts observed in the specific internal energy of the hot rotating halo that forms around the central object are anticorrelated with the bursts observed in the modulus of the magnetic field; a more detailed discussion on these bursts follows in Sec. IV C 4.

Unfortunately, the resolution used here is insufficient to be able to track the development of an MRI, which is, however, expected to develop [37, 45] and could significantly amplify the magnetic field. Blind to this effect, our simulation shows that at $t = 11.60$ ms (see the bottom row of Fig. 3) magnetic braking has managed to store enough rotational energy in the

winding of the magnetic-field lines so that the inner core of the star is now less differentially rotating. The direct consequence of this is that the HMNS collapses to a black hole of mass $M = 2.88 M_{\odot}$ and dimensionless spin $a = J/M^2 = 0.87$, as measured from the apparent horizon [99, 100]. We postpone the discussion on how angular momentum is transported outward and how this affects the lifetime of the HMNS to Sec. IV C 2.

At time $t = 15.61$ ms, the black hole is surrounded by a thick accretion torus that is responsible for confining and collimating the magnetic field along the z -axis (cf., the top row of Fig. 3). The properties of the black hole-torus system are shown in Table I at approximately 4.74 ms after the collapse and show that the mass of the torus is $0.095 M_{\odot}$. The magnetic-field topology and matter dynamics soon after the collapse are highly turbulent, but the high degree of symmetry introduced by the black hole, which is gravitationally dominant over the torus, rapidly establishes some order in this system. After about one orbital period, in fact, the torus becomes essentially axisymmetric and the matter in the polar region is rapidly accreted onto the black hole, giving rise to a funnel where the rest-mass density reaches values close to that of the atmosphere (cf., the bottom row of Fig. 3).

After black-hole formation, the plasma dynamics in the funnel is far from being stationary and continues with repeated bursts having a period of about 2.4–3.7 ms. While a more detailed discussion of these bursts is postponed to Sec. IV C 4, it is useful to remark here that the ejected material does not have sufficient energy to reach large distances away from the black hole. This is probably due to the fact that, although the magnetic field is comparatively strong, the material in the funnel is still matter dominated, with $\beta_p \sim 10^4 - 10^6$. Such large values are not particularly surprising since the initial magnetic field in the stars is rather small, i.e., $\sim 10^{12}$ G, and is not amplified significantly. At the same time, the torus angular velocity profiles have become nearly Keplerian, and the MRI could develop (cf., Fig. 14). However, as for the HMNS, also the spatial resolution of the grid covering the torus is too small to capture the fastest growing modes of the instability in the torus (see discussion in Sec. IV C 5).

For completeness, we report in Fig. 4 three different two-dimensional cuts on the (x, z) plane of the specific internal energy and of the velocity field in this plane. The times selected are the same as those presented in Fig. 2, and are use-

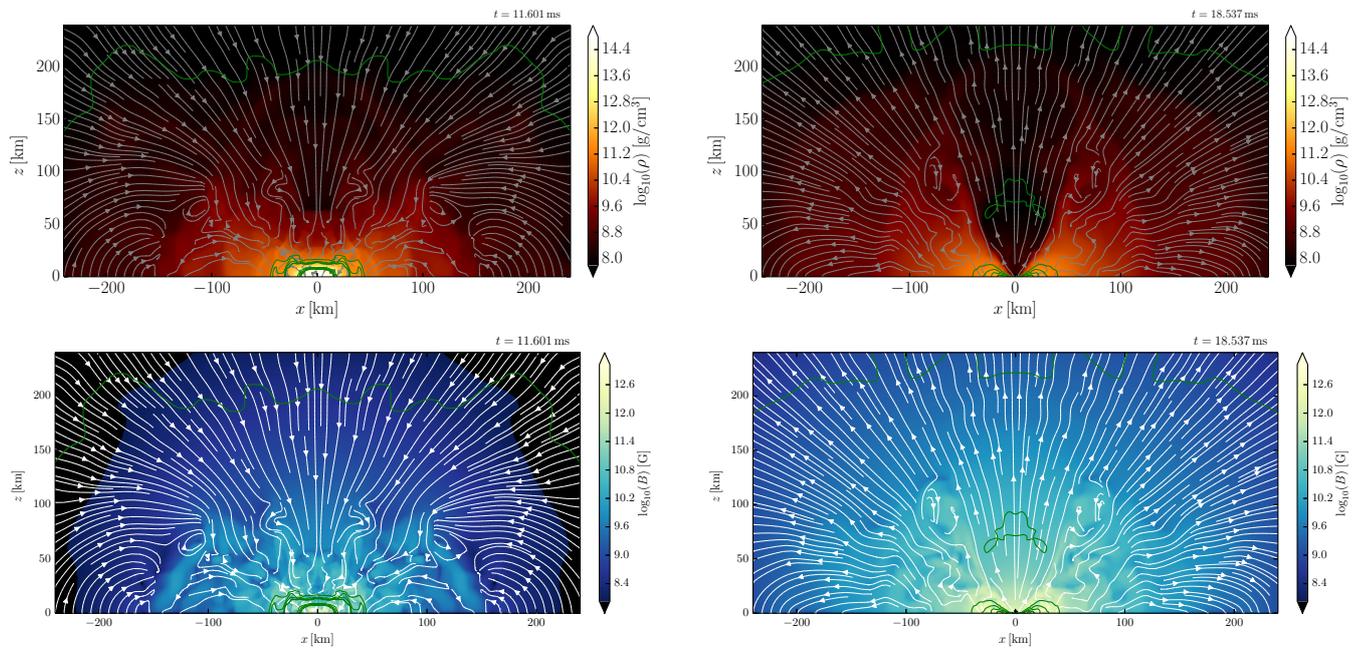


FIG. 5. Large-scale two-dimensional snapshots on the (x, z) planes of the rest-mass density (top row) and of the magnetic field (bottom row). The two columns refer to $t = 11.60$ ms (left column), when the HMNS has not yet collapsed and to $t = 18.54$ ms (right column), when a black hole has already been formed. Note again the formation of a magnetic-jet structure around the black-hole rotation axis, which extends on scales that are much larger than those of the accreting torus (i.e., $\sim \pm 60$ km) and of the black hole (i.e., $\sim \pm 5$ km) (see also Figs. 3, 6, 16, and 17).

ful to gain information on the properties of the flow during the HMNS stage and after the collapse to a black hole. Note that during the HMNS stage the flow consists mostly of the intense magnetically driven outgoing wind discussed above (left panel). However, once a funnel is produced, the flow is ingoing at heights $z \lesssim 100$ km but becomes outgoing for $z \gtrsim 100$ km (middle and right panels). The location of this stagnation point varies with time and follows the expansion of hot fluid which is produced by the accretion process and that can be followed via the increases in the specific internal energy. In all cases considered, the vertical flow very close to the black hole is *ingoing*.

B. Magnetic-field topology and magnetic-jet structure

As mentioned above, the high degree of symmetry near the rapidly rotating black hole induces a quick rearrangement of the matter and of the magnetic fields. As a result, the magnetic-field topology at time $t = 18.54$ ms changes in the funnel and develops a dominant poloidal component, giving rise to a well-defined magnetic-jet structure, which is almost axisymmetric. This result is similar to what was already found in the simulations of Ref. [36], with the important difference that this configuration has been reached with a higher spatial resolution and a consistent treatment of the resistivity.

The rest-mass density of the plasma in the funnel is close to that of the atmosphere, but also slightly larger. Hence, given our choice of the conductivity profile in Eq. (22), the conduc-

tivity is essentially zero everywhere in the funnel (see the middle and right panels of Fig. 2). This has two important consequences. First, the dynamics of the electromagnetic fields in this region is not that prescribed by the IMHD equations but rather that of electromagnetic waves in vacuum. At the same time, because of its (comparatively) small rest-mass density and pressure, the matter in the funnel tends to move along the field lines. We should clarify that this behavior is not achieved because the test-particle limit of the RMHD equations is reached (the rest-mass density and pressure are in fact nonzero) but rather because the matter in the funnel can only move in the vertical direction, either accreting onto the black hole or moving outward (the matter in the funnel has low or zero specific angular momentum).

Modelling the dynamics of the matter in the funnel is among the most challenging aspects of these calculations. Although the matter there has the largest magnetic-pressure support (i.e., $\beta_p \gtrsim 10^4$), this is still about 4 orders of magnitude away from being magnetically dominated. The reason for this behavior is most likely due to the comparatively weak magnetic fields that we are able to build in the funnel at these resolutions and the short evolution times. Second, the zero-conductivity plasma in the funnel is just adjacent to the high-conductivity plasma of the torus; this large jump in the conductivity helps in preserving the magnetic-jet structure and in providing a natural agent for the collimation of the flow at low latitudes.

It is useful to remark that in close analogy with what found in Ref. [36] the magnetic-jet structure produced here is *not* a

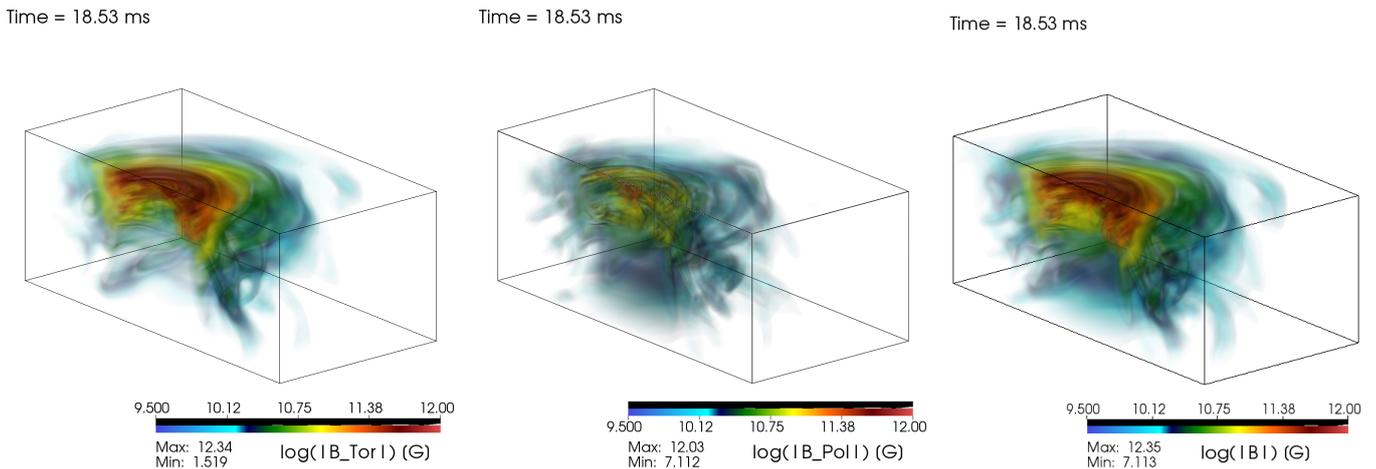


FIG. 6. Three-dimensional snapshots of the norm of the toroidal magnetic field $|B_{\text{tor}}|$ (left panel), of the poloidal one $|B_{\text{pol}}|$ (middle panel), and of the total magnetic field $|B|$ (right panel), all at time $t = 18.3$ ms. Additionally, we plot the modulus of the magnetic field at the same time to illustrate where the toroidal and poloidal contributions become dominant. Note that the magnetic field at the edges of the funnel starts developing a toroidal component that exhibits signs of twisting, while the magnetic field in the evacuated region is predominantly poloidal. The figures show a cut through the torus, with the z -axis facing down, in order to be able to see the evacuated region formed along the z -axis and the funnel-wall structure that develops at the interface with the interstellar medium. The domain plotted corresponds to a rectangular grid with dimensions $[0 \text{ km}, 115.8 \text{ km}] \times [-115.8 \text{ km}, 115.8 \text{ km}] \times [0 \text{ km}, 92.16 \text{ km}]$

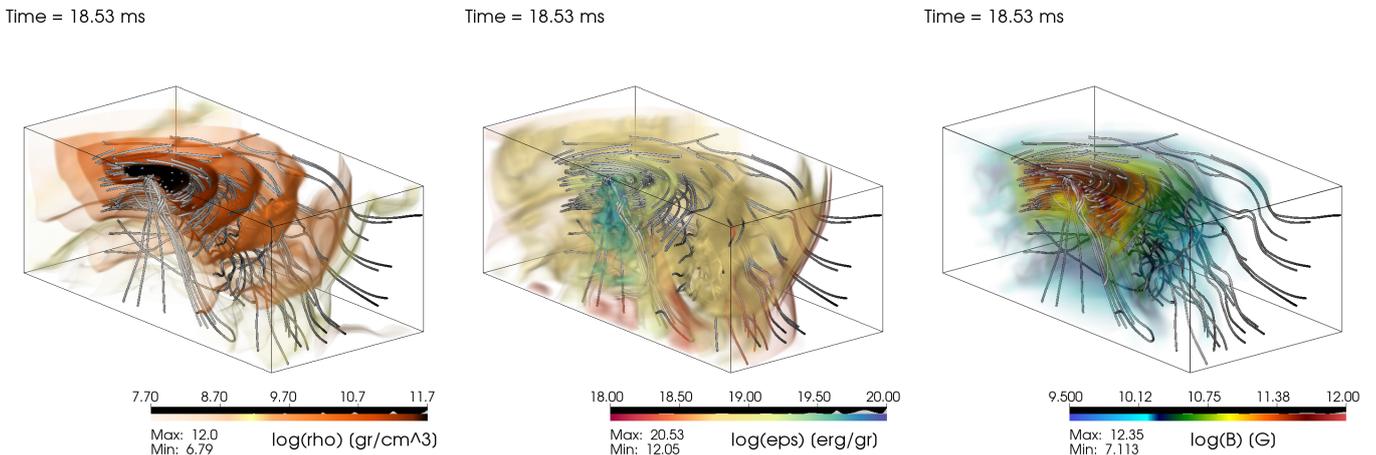


FIG. 7. Three-dimensional snapshots of the rest-mass density ρ (top left panel), specific internal energy ϵ (top right panel), and modulus of the magnetic field $|B|$ (bottom panel) at $t = 18.3$ ms. Additionally, we plot the magnetic-field lines on top of these quantities to illustrate the topology of the magnetic field. Because of the symmetries applied in our simulation we only show a quadrant of the black hole–torus system. The figures show a cut through the torus, with the z -axis facing down, in order to be able to see the evacuated region formed along the z -axis and the funnel-wall structure that develops at the interface with the interstellar medium. The domain plotted corresponds to a rectangular grid with dimensions $[0 \text{ km}, 115.8 \text{ km}] \times [-115.8 \text{ km}, 115.8 \text{ km}] \times [0 \text{ km}, 92.16 \text{ km}]$.

relativistic outflow. Instead, it can just be viewed as an almost quasistationary magnetic structure confining the tenuous plasma in the funnel and confining it away from the dynamics of the ultradense plasma in the torus. In fact, despite the resistive losses, the plasma in the funnel does not have yet sufficient internal energy to be able to launch a relativistic outflow. It is possible that the strong magnetic fields in the vicinity of the black hole could provide the conditions for electromagnetic extraction of the black hole’s rotational energy through the Blandford–Znajek mechanism [101] or through a general-

ized Penrose process [102]. Alternatively, the energy required for launching a relativistic outflow could also be efficiently deposited along the baryon-poor funnel by reconnection processes not fully modelled here or by neutrino pair annihilation [103, 104]. Clearly, additional work is needed to assess the robustness of our modelling of the funnel region and to assess whether and how energy can be deposited in the magnetic-jet structure.

A closer look at the magnetic-jet structure is offered in Fig. 5, which shows two-dimensional snapshots on the (x, z)

planes of the rest-mass density (top row) and of the magnetic field. The two columns refer to $t = 11.60$ ms (left column), when the HMNS is just about to collapse, and to $t = 18.54$ ms (right column), when the black hole–torus system is toward reaching a quasistationary equilibrium. The different panels in Fig. 5 should be compared with the corresponding ones in Figs. 3 and 6 but are represented here on much larger spatial scales (see also Appendix A for a closer comparison of the magnetic-field structure in the IMHD simulations). Interestingly, the magnetic-jet structure extends well beyond the scale of direct influence of the black hole and shows a coherent structure on scales of $\gtrsim 250$ km. The scale of the magnetic structure is much larger than that of the accreting torus (i.e., $\sim \pm 60$ km) and of the black hole (i.e., $\sim \pm 5$ km).

Additional information on the magnetic-field topology can be appreciated by considering three-dimensional views of the magnetic-field strength and field lines. This can be seen in Figs. 6, where we show a three-dimensional snapshot of the toroidal (left panel), poloidal (middle panel), and total magnetic field (right panel). It is important to remark that a well-defined magnetic-jet structure has been recently reported also in Ref. [21] from IMHD simulations of the merger of a black hole–neutron-star binary. In addition to a jet structure not very different from the one reported here, the authors in Ref. [21] are also able to produce a sustained outflow from the accretion torus. At the same time, the IMHD simulations reported in Ref. [45], where a very high spatial resolution was used, do not reveal the formation of such a magnetic-jet structure. It is difficult to assess at the moment the origin of these differences, partly because of the limited amount of information provided on the simulations in Ref. [45]. A more extended discussion of the properties of the magnetic-field topology and dynamics, made along the lines suggested in this paper (see also Sec. IVC 1) would be useful to clarify if there are really differences and their origin.

When studying the magnetic-field topology, i.e., whether it is mostly poloidal or toroidal, it is unavoidable to discuss where certain measurements are made, as the magnetic field can be at the same time mostly poloidal and mostly toroidal but in two different regions. A quick inspection of the three panels suggests that the magnetic field in the low-density funnel is predominantly poloidal (clearly shown in the middle panel of Fig. 6). It is quite natural to expect that the magnetic field will be essentially poloidal near the rotation axis, where matter has a specific angular momentum that is intrinsically small. Equally natural is to expect that a toroidal component will start to develop away from the axis and as one approaches the regions filled by the torus. This behavior can be explained by the fact that some of the magnetic-field lines in the funnel are anchored to the highly conducting material at the edges of the torus, which is rotating at nearly Keplerian velocities. Indeed, the left panel of Fig. 6 shows that the magnetic field acquires a toroidal component near the edges of the funnel, and then becomes essentially toroidal in the torus. This is also shown in Fig. 7, which offers three-dimensional snapshots of the rest-mass density ρ (left panel), of the specific internal energy ϵ (middle panel), and of the modulus of the magnetic field $|B|$ (right panel) at $t = 18.3$ ms. Also reported are the

magnetic-field lines, of which the three-dimensional representation confirms that the magnetic field is mostly poloidal in the magnetic-jet structure, acquiring a twist and a kink when it reaches the edges of the funnel, and becoming essentially toroidal inside the torus.

Finally, we note that a vigorous outflow develops at the interface between the magnetic-jet structure and the torus. The resulting shearing boundary layer could be the site for the development of a Kelvin–Helmholtz instability, which unfortunately we cannot investigate at the present resolutions and without a more sophisticated treatment of the conductivity in the transition between large and small values (the instability does potentially develop in a particularly difficult region). It is clear, however, that the dynamics along the torus walls has all the potential of yielding interesting observational features and should be investigated in the future, possibly using advanced numerical techniques such as those presented in Ref. [105].

A word of caution should be spent before concluding this section. While the behavior described above appears reasonable and is possibly the expected one on the basis of rather simple considerations, this behaviour is ultimately the result of our choice for the conductivity profile in Eq. (22) and of our choice of rest-mass density in the atmosphere. The large computational costs associated with these simulations prevent us from presenting at this point a systematic investigation of how sensitive the results are on the choice for σ and ρ_{atm} . We are aware that this represents a limitation of our investigation, which we plan to resolve with future simulations.

C. Comparison with IMHD simulations

As described in the previous section, the dynamics of the RMHD simulations is rather similar, at least qualitatively, to the one observed in IMHD simulations of the same binary presented in Ref. [35]. Yet, there are some important differences, and these can be best appreciated if we perform a careful comparison of the two evolutions. To this scope, we have performed additional simulations of the same binary discussed in the previous section when, however, the set of equations solved are those of general-relativistic IMHD. We note that this was necessary because the simulations in Ref. [35] used a different grid structure and resolution but also investigated quasicircular initial data in contrast to the reduced linear momenta we have considered here.

In the following we present the results of this side-by-side comparison, first using two-dimensional spacetime diagrams and then moving to standard one-dimensional snapshots.

1. Spacetime diagrams

We have found that a very efficient way of carrying out a comparison between two MHD evolutions which are qualitatively similar is to use two-dimensional spacetime diagrams. This technique, which was first introduced in Ref. [106], provides a color-coded evolution of various scalar quantities as measured along principal axes (e.g., the x - and z -directions)

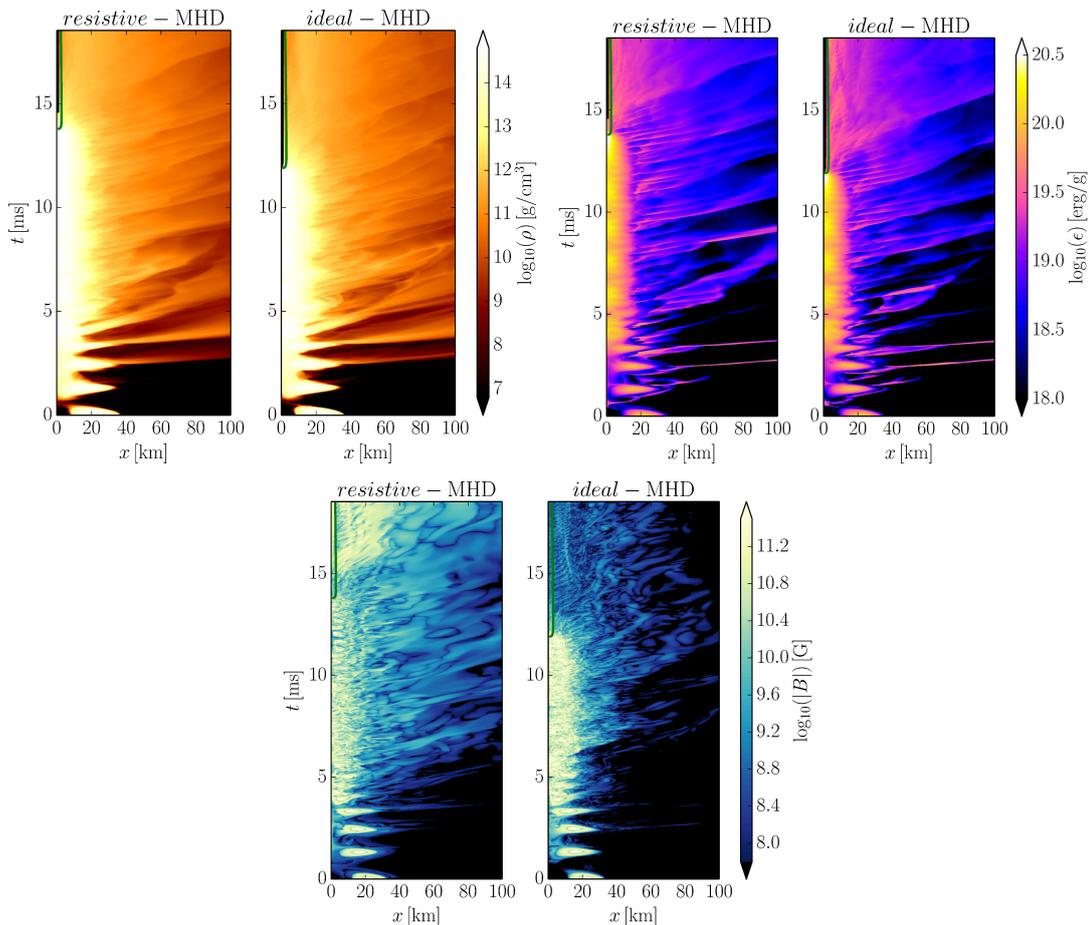


FIG. 8. Two-dimensional spacetime representation of the evolution of the rest-mass density ρ (left panel), of the specific internal energy ϵ (middle panel), and of the magnetic field norm $|B|$ (right panel). All quantities are measured along the x -axis and are therefore representative of motions on the equatorial plane. Indicated with a solid green line is the evolution of the apparent horizon; note that the values in the color bars are saturated and do not correspond to the minimum and maximum values of the corresponding fields.

and has the advantage of summarizing simply even rather complex dynamics.

As representative examples, we show in Figs. 8 and 9 the differences between the RMHD and IMHD implementations in the evolution of the rest-mass density ρ (top left panel), of the specific internal energy ϵ (top right panel) and of the magnetic field norm $|B|$ (bottom panel). Note that each panel is split into two diagrams, with the left one referring to the RMHD solution and the right one to the IMHD solution. Furthermore, while Fig. 8 refers to quantities measured along the x -axis and hence is representative of motions on the equatorial plane, Fig. 9 refers to the z -axis and is therefore representative of motions in the polar region. In all panels we indicate with a solid green line the evolution of the apparent horizon (see the discussion in Ref. [89] on how to interpret such a line). Note that the values in the color bars are saturated and do not correspond to the minimum and maximum values of the corresponding fields.

It is clear from both figures that the evolution of all quantities is very similar during the inspiral, when indeed the IMHD and RMHD evolutions should be mathematically identical

given that our fields are contained in the stars. As the binary reaches the merger, matter is expelled from the stars mainly in the equatorial plane, as is evident in Fig. 8, which also shows that the ejected matter moving through the low-density medium generates shocks that heat up the plasma and appear as thick lines. The subsequent bursts of matter are mainly associated with the fundamental mode (f -mode) of oscillation of the HMNS [13, 107]. The equatorial ejections are also accompanied by four or five subsequent “bursts” along the z -axis, starting in both simulations at approximately 5 ms, and can be seen in Fig. 9. Clearly, the violent oscillations experienced by the HMNS launch matter essentially isotropically.

We have already mentioned that the differentially rotating HMNS does collapse promptly to a black hole by first rearranging its angular velocity profile and mass distribution through magnetic braking. During this phase, angular momentum is transported outward, with the outer fluid elements moving further away from the star and the inner ones moving toward the center as a result of the angular-momentum losses. Clearly, this redistribution of angular momentum will be different in the RMHD and IMHD evolution, with the latter

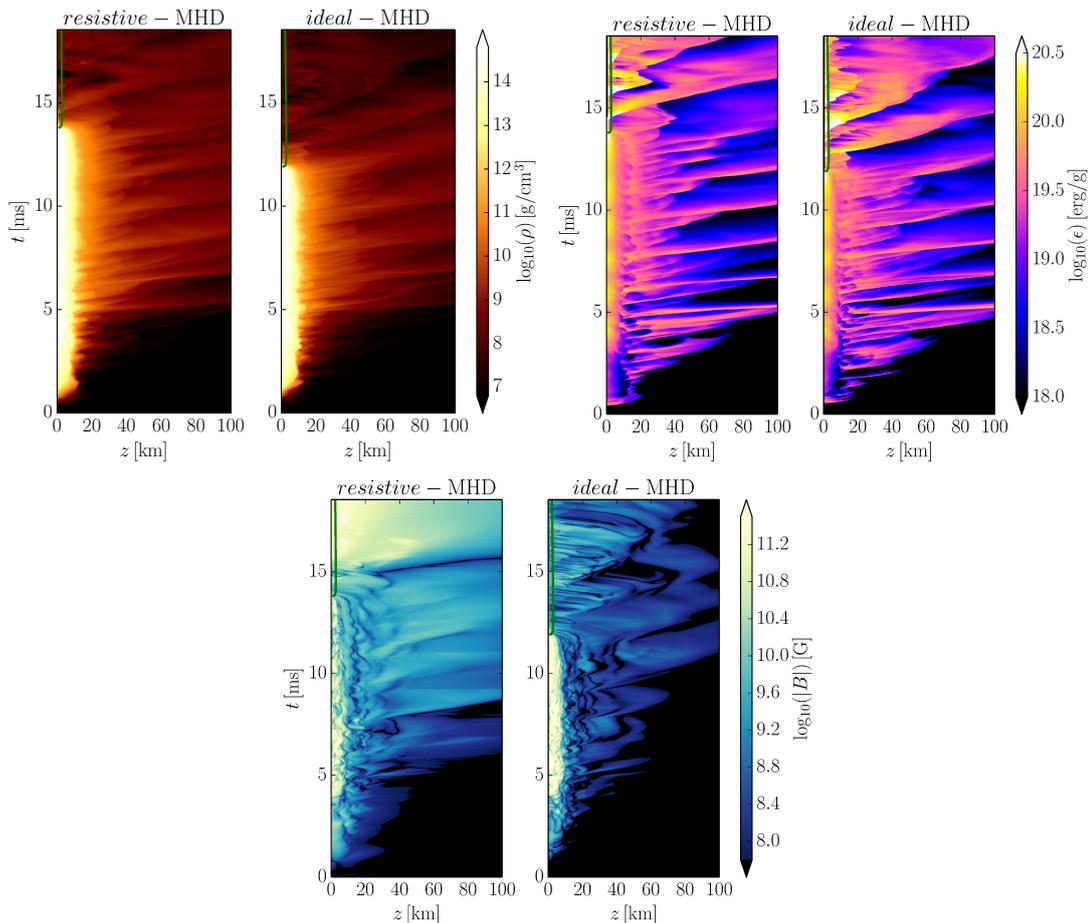


FIG. 9. The same as in Fig. 8 but for quantities measured along the z -axis and therefore representative of motion in the polar regions.

being more efficient in transporting angular momentum outward (in IMHD the fluid can only move along magnetic-field lines, while it can also partially cross them in RMHD). As a result, the collapse can take place slightly earlier, occurring at 11.9 ms in the IMHD evolution and at 13.8 ms in the RMHD simulation (cf., Figs. 8 and 9).

We have already commented that one of the major differences between the IMHD and RMHD simulations is that our implementation of the latter allows for a description of propagating electromagnetic waves in vacuum. By contrast, the atmosphere treatment of the IMHD implementation is such that it does not evolve the magnetic field as the fluid velocities are reset to zero there. As a result, already after the first 0.5 ms the magnetic field manages to diffuse out of the stars, heating up the plasma in the outer layers and forcing it to expand. In this way, magnetic energy is converted into internal energy, and therefore the magnetic field at the center of the stars is (slightly) lower than in the corresponding IMHD simulation. In contrast, the magnetic field along the z -axis in the RMHD simulation is higher than in the IMHD simulation in the first few milliseconds. Indeed, the differences between the IMHD and RMHD simulations become particularly evident after black-hole formation, when the funnel is evacuated and the magnetic-jet structure is built (see the bottom panels of

Figs. 8 and 9). The magnetic diffusivity in the RMHD simulation acts so rapidly that in a bit more than one crossing time the whole computational domain in the RMHD run is filled with electromagnetic fields despite the fact that the magnetic field was initially constrained to the stellar interior.⁹

Note that the modulus of the magnetic field along the z -axis is about 2 orders of magnitude larger in the RMHD simulation (cf., $\sim 10^{10}$ G in IMHD vs $\sim 10^{12}$ G in RMHD). This is due (mainly) to the intense currents produced by the rapidly rotating torus and (partly) to the magnetic-field diffusion of the strong magnetic field in the torus, which diffuses across the walls of the funnel. This behavior of the magnetic field is in fact similar to the one observed in a stable magnetized star with extended magnetic fields which was studied in Ref. [66], where after an initial transient, the system relaxed to a solution consisting of a large-scale, nearly electrovacuum, dipolar magnetic field configuration in the exterior, which was anchored to the highly conducting neutron

⁹ We prefer to be repetitive rather than confusing: the magnetic field diffuses out of the stellar matter because of the finite resistivity at the surface of the neutron stars, of the HMNS, or of the torus. Once in the atmosphere, however, the magnetic fields propagate as electromagnetic waves in vacuum.

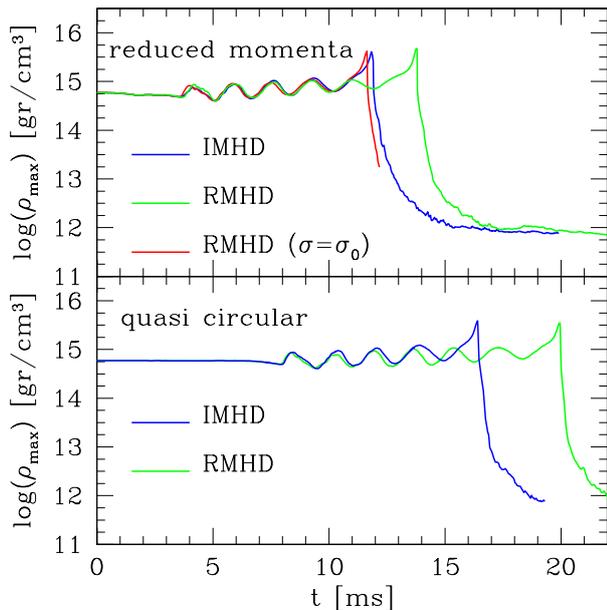


FIG. 10. Evolution of the maximum of the rest-mass density for both the IMHD (blue solid line) and the RMHD simulation (green solid line). The top panel refers to the initial data with reduced linear momenta, while the bottom one refers to data in quasicircular orbits. Also shown in the top panel is the evolution of the RMHD set of equations with a large and uniform conductivity (red solid line); in this case the evolution should mimic the IMHD one, as indeed it does.

star. The larger values of the magnetic field in the funnel are particularly encouraging as strong magnetic fields are necessary to produce a large acceleration along the z -direction. The maximum Lorentz factors achieved after the collapse in both simulations is $W \approx 2.0 - 2.7$, with the highest values occurring at the end of the simulations.

The magnetic field in the torus is also stronger in the RMHD simulation than the corresponding field in the IMHD run, although the differences in this case are only of 1 order of magnitude. The reason for this has to be found in the fact that in the RMHD simulation the torus is more massive and therefore able to sustain larger amounts of magnetic fields (as for isolated stars, also a self-gravitating torus in MHD equilibrium will be able to sustain stronger magnetic fields for increasing masses; cf., Table I).

2. Angular-momentum transfer and HMNS lifetime

The top panel of Fig. 10 reports the evolution of the maximum of the rest-mass density ρ_{\max} for both the IMHD (blue solid line) and the RMHD simulation (green solid line). After the merger takes place at $t \simeq 3.5$ ms, the maximum rest-mass density oscillates at the f -mode frequency and experiences a sudden drop when an apparent horizon is found since matter inside the apparent horizon is excluded from the calculation of ρ_{\max} . It is quite obvious that the behavior of ρ_{\max} is different

in the two simulations, with the HMNS in the RMHD surviving for a longer time. Note, however, that the first increase in the central rest-mass density in the top panel of Fig. 10 is slightly larger in the RMHD case, most likely because of the resistive increase of the internal energy at the merger.

Also shown in the top panel of Fig. 10 is the evolution of a simulation in which the set of RMHD equations is used together with a large and uniform conductivity $\sigma = \sigma_0 = 10^6$ (red solid line). In this case, despite the very different set of equations solved and the different approach for enforcing the divergence-free condition of the magnetic field, the RMHD evolution should mimic the IMHD one. This is indeed the case, and it provides considerable confidence on the robustness of our RMHD approach. It suggests that the delayed collapse is an effect associated with the choice of physical resistivity and not due to numerical artifacts.

Despite the complex dynamics, the differences between the IMHD and RMHD runs are not difficult to explain. As mentioned in the previous section, in fact, an important difference between the IMHD and RMHD simulations is that in the latter the magnetic field cannot be perfectly locked with the plasma. As a result, the IMHD evolution is more efficient in redistributing the angular momentum in the system and, in particular, in transporting it outward. This magnetic-braking process deprives the HMNS core of the angular-momentum support, and this leads to an earlier collapse. Clearly, since the conductivity in the HMNS interior is very high also in the RMHD simulation (although not infinite), magnetic flux freezing is very efficient here as well and the differences in the dynamics of the IMHD and RMHD simulations can only be small. This explains why overall the time of collapse varies by only ~ 1.9 ms. In addition, the important differences in this dynamics are also expected to take place in the outer layers of the HMNS, where the conductivity decreases as a response to the conductivity profile (22). We should note that the difference in the lifetime of the HMNS is not related to the use of initial data with modified momenta, but is present also for a binary of which the initial data is on a quasicircular orbit. This is shown in the bottom panel of Fig. 10, where the corresponding quantities are shown, and where it is clear that also in this case the HMNS collapses earlier to a black hole in the IMHD simulation. We also recall that for the binary in quasicircular orbit no additional refinement level is added after the merger. Hence, when comparing in Fig. 10 the RMHD evolution of the binaries with reduced momenta and on quasicircular orbits, one is also effectively comparing the evolution of the HMNS at two different resolutions, i.e., 148 and 296 m, respectively. The fact that they both yield a longer lifetime of the HMNS provides an indirect validation of the numerical consistency of the RMHD solution.

Because the differences in the magnetic braking between the RMHD and IMHD implementations are intrinsically small, it is not easy to show that it is exactly these differences that are responsible for the earlier collapse of the HMNS in the IMHD simulations. However, such evidence is offered in Fig. 11, which reports the spacetime diagrams along the x -direction of the specific angular momentum $\ell := -u_\phi/u_t$

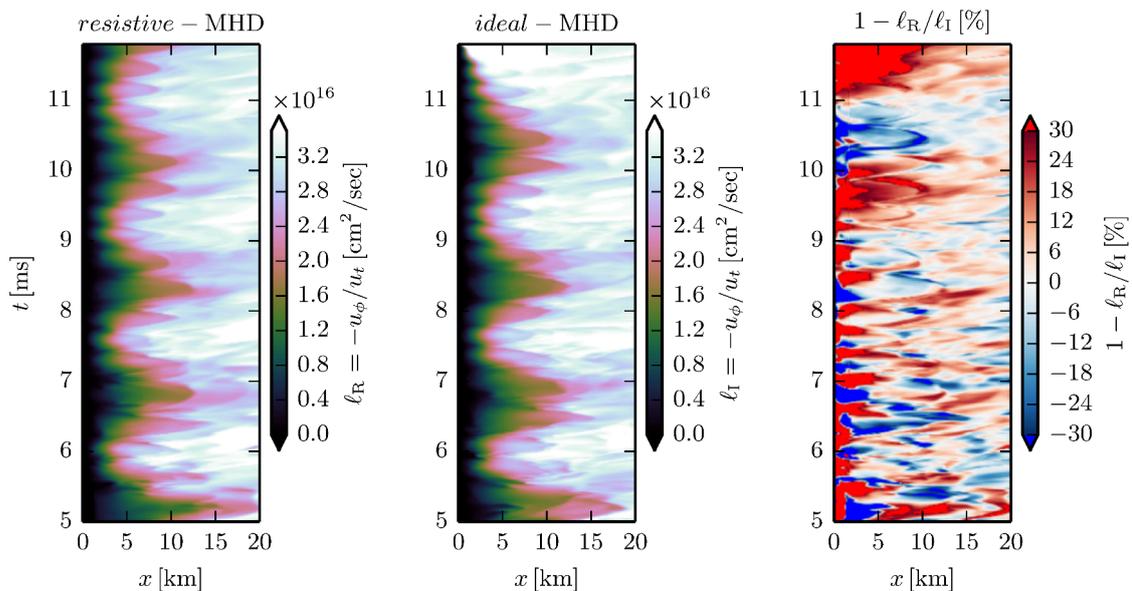


FIG. 11. Spacetime diagrams of the specific angular momentum $\ell := -u_\phi/u_t$ for both the RMHD (left panel) and IMHD (middle panel) simulations. In addition, we show the relative difference in the specific angular momenta between the RMHD and IMHD implementations (right panel).

[68].¹⁰ The left panel refers to ℓ_R , the specific angular momentum of the RMHD run, while the middle panel refers to the corresponding quantity for the IMHD simulation, ℓ_I , and the right panel refers to the relative difference: $1 - \ell_R/\ell_I$.

A rapid inspection of the left and middle panel reveals that in both the RMHD and IMHD simulations the specific angular momentum increases outward (as it should for a rotating fluid satisfying the Rayleigh stability criterion), but also that the profiles are not constant in time and show instead periodic variations that are in phase in the two simulations. These variations reflect the large oscillations of the HMNS, and, indeed, the oscillations in ℓ_R, ℓ_I take place at the same frequency as those in the rest-mass density and shown in the other spacetime diagrams in Fig. 8 (note the different scale in the x -axis). However, there are important small differences in the dynamics of ℓ_R and ℓ_I , which are apparent in the right panel of Fig. 11, where regions in red indicate that the specific angular momentum in these regions is higher (of $\sim 20 - 30\%$) in the IMHD simulation than in the RMHD simulation, while blue regions are exactly the opposite. It is apparent that the differences are larger in the central parts of the HMNS, while the specific angular momenta are very similar in the outer layers, i.e., for $x \gtrsim 5$ km. We recall that an excess of specific angular momentum at a given position on the x -axis reflects fluid elements that are rotating at a larger frequency ($\ell = \Omega x^2$ in the Newtonian limit) and this is indeed what one would expect if the magnetic fields and the fluid are tightly coupled. Hence,

the red regions in the right panel Fig. 11 can be taken to signal a more efficient transfer of angular momentum from the inner regions of the HMNS. A direct consequence of this transfer of angular momentum is the appearance of blue regions adjacent to the red ones and signalling therefore fluid elements that have slowed down.

Although the differences in ℓ_R and ℓ_I are small, the transfer continues steadily and with an increased rate up to $t \approx 11$ ms, when a much larger transfer of angular momentum takes place. This signals the onset of the instability to gravitational collapse in the IMHD simulation, which effectively takes place soon after, i.e., at $t \approx 12$ ms.

Three remarks should be made before concluding this section. First, the fact that an RMHD simulation with a uniform conductivity yields the same collapse time as an IMHD simulation gives us great confidence about the correctness of the RMHD evolution with nonuniform conductivity. Second, although the difference in the survival time between the RMHD and IMHD evolution is here rather small, it can be much larger if smaller values of the resistivity are chosen for the stellar interior and for less massive HMNSs; unfortunately present astronomical observations do not set any stringent constraint on the values of the conductivity at these temperatures, rest-mass densities, and magnetic fields. More importantly, however, a longer survival time is a useful new result in the modelling of binary neutron stars, as it points out that the HMNS can survive on comparatively longer time scales than those computed so far in pure hydrodynamics or in IMHD simulations. This is not a minor detail as the most recent modelling of SGRBs with an extended x-ray emission invokes the existence of a magnetized HMNS that is able to survive on time scales of the order of $10^3 - 10^4$ sec before collapsing to a black hole

¹⁰ A very similar behaviour is observed if the specific angular momentum is shown in terms of $\ell := hu_\phi$ (see [68] for a discussion in the differences in the two definitions).

[57–62]. Finally, computational constraints have prevented us from extending much past black-hole formation the evolution of the RMHD/IMHD simulations with initial data on quasicircular orbits. Nevertheless, the fact that already the dynamics of the HMNS is unaffected by the initial reduction in linear momenta and that the HMNS collapses to a black hole earlier in both cases, provides us with confidence that the magnetic field dynamics discussed in Sec. IV B will be very similar also for binaries on quasicircular orbits. This will also be the focus of our future work.

3. magnetic field growth

In Secs. IV B and IV C 1, we have already discussed the properties of the evolution of the magnetic fields, but have not quantified in detail how the magnitude of the magnetic field changes with time and how this evolution varies in the RMHD and IMHD simulations. This is done now in Fig. 12, where we report the evolution of the maximum of the modulus of the magnetic field (black lines) but also of its toroidal (red lines) and poloidal (blue lines) components, either in the RMHD simulation (left panel) or in the IMHD simulations (middle panel).

Note that the evolution of the magnetic field in the two implementations is very similar during the inspiral, but also that this changes considerably after the merger. While in both cases the toroidal magnetic field grows exponentially, the growth is of about 1 order of magnitude in the IMHD simulation but of a factor 2 smaller for the RMHD simulation. Furthermore, the magnetic field reaches values of 3×10^{13} G just before the collapse in the IMHD simulation and 8.3×10^{12} G only just after the merger in the RMHD simulation. This different behavior is not difficult to explain and is simply due to the fact that the shearing of magnetic-field lines is less efficient in RMHD because of the finite conductivity of the matter. Because the growth at the merger is mostly due to the shear layer between the two impacting stars, it is quite natural that a resistive calculation will lead to a smaller magnetic field, quite independently of how well the instability is resolved.

As for the rest-mass density (cf., Fig. 10), the collapse of the HMNS to a black hole leads to a rapid decrease of the maximum value of the magnetic field, as shown in Fig. 12 where the vertical lines signal the first appearance of the apparent horizon. Also in this case, the strongest magnetic fields are hidden inside the horizon, and the maximum values reported are those relative to the magnetic field in the torus. As remarked already when commenting on the spacetime diagrams in Sec. IV C 1, the larger values of the magnetic field in the RMHD simulation are the result of a more massive torus produced in this case (cf., Table I).

In the right panel of Fig. 12 we complement the evolution of the magnetic fields with the evolution of the L_2 -norm of the divergence of the magnetic field for the RMHD simulation (green dashed line) and for the IMHD one (blue solid line) after it is multiplied by 10^{13} . We recall that the IMHD simulation makes use of a constrained transport scheme [108],

and hence it is able to maintain the violations of this constraint down to machine precision. The RMHD simulation, on the other hand, makes use of a divergence-cleaning scheme [67], which is widely known to be less efficient in suppressing the violations. Yet, the purpose of making this comparison is mostly that of highlighting that the divergence-cleaning approach used here may not be as efficient as the constrained transport, but it yields nevertheless very small violations. Indeed, the evolution of the L_2 -norm of the ratio between the divergence and the magnetic field strength, i.e., $\|\nabla_i B^i / \sqrt{B_i B^i}\|_2$ and not shown in Fig. 12, is of the order of $\sim 10^{-2}$, thus similar to the values reported by similar works in the literature [87]. Note that the late-time moderate growth of the divergence in the RMHD simulation is due to the amplification of the magnetic fields in the torus, and it would be interesting to investigate if a dynamically adapted dissipation parameter for the divergence cleaning method could help reduce such a growth.

4. Magnetically driven wind and bursting activity

In Secs. IV A and IV C 1, we have already anticipated that a wind is produced after the merger, either as a result of shock heating at the merger or because of magnetic winding and consequent pressure imbalance in the outer layers of the HMNS or because of neutrino losses [109, 110]. In addition to an almost quasistationary and quasi-isotropic wind, both the RMHD and the IMHD simulation show the existence of mildly anisotropic and quasiperiodic launching of low rest-mass density, high internal energy blobs of matter that we will refer to as bursts. Overall, seven bursts are launched during the total time of the simulation, with five bursts relative to the HMNS stage and two being produced after black-hole formation.

In particular, the first two bursts eject material that is moving through the low-density atmosphere with an average speed of $\approx 0.4 - 0.6$ but that decelerate as they move away from the HMNS, reaching a final outward velocity of $\sim 0.16 - 0.18$. This behavior is in part due to the natural conversion of kinetic energy to binding energy but also to the interaction of the bursting material with the slow isotropic wind. This interaction, which is obviously accompanied by shocks, provides a damping mechanism on the propagation of the ejected material. However, the slow down of the baryon rich material could be revived later on if the slow wind is impacted by a faster, baryon poor wind, as suggested in the “two-winds” model of Ref. [61].

We have investigated the properties of the bursts by studying the evolution of the specific internal energy as measured by an observer on the z -axis. This is reported in Fig. 13 for an observer at $(x, y, z) = (0, 0, 86.82)$ km, for both the RMHD simulation (green dashed line) and the IMHD one (blue solid line). Clearly, after the merger the specific internal energy exhibits a quasiperiodic behavior in both simulations, with a very rapid increase. The increase in the specific internal energy is of slightly less than 2 orders of magnitude and is followed by a slower decay.

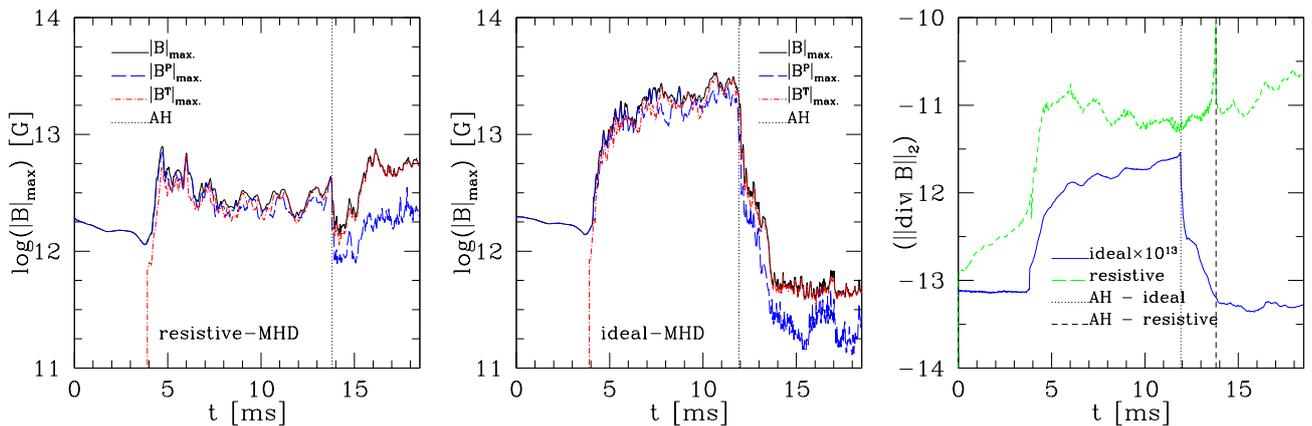


FIG. 12. Evolution of the maximum of the magnetic-field strength $|B|_{\max}$ for both the RMHD (left panel) and the IMHD simulation (middle panel). The black solid lines correspond to the time series of the maximum magnetic-field modulus; the blue dashed lines correspond to the time series of the maximum poloidal magnetic-field norm, $|B^P|_{\max}$; and the red dotted-dashed lines correspond to the maximum toroidal magnetic field, $|B^T|_{\max}$. Additionally, we show the L_2 -norm of the divergence of the magnetic field, for the RMHD simulation (green dashed line) and IMHD one (blue solid line). The collapse times are depicted with black dotted or black dashed lines.

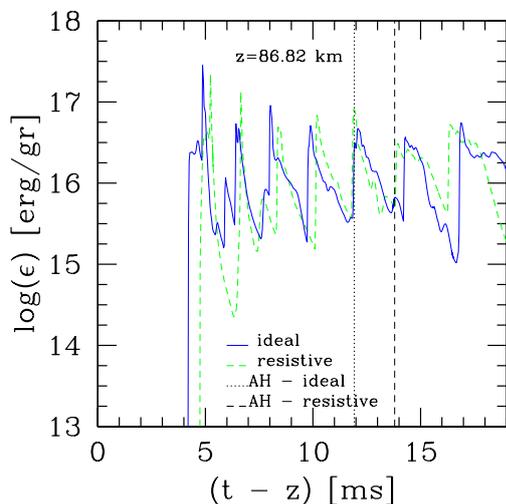


FIG. 13. The time series of the specific internal energy measured at $(x, y, z) = (0, 0, 86.82)$ km are shown for the RMHD simulation with a green dashed line and for the IMHD simulation with a solid blue line. The time of collapse for the RMHD case is depicted with a black dashed line, while it is depicted with a black dotted line for the IMHD one.

As already mentioned in Sec. [IV C 1](#), this behavior can be associated with the oscillations of the HMNS and is observed also in the evolution of the rest-mass density (cf., the five peaks in Fig. 10 and in Fig. 9, top left panel). The characteristic frequency of these peaks is related to the f -mode frequencies of the bar-deformed HMNS [[13](#), [107](#)] and thus these bursts occur every $\sim 1.7 - 2.0$ ms. Similar peaks (although less marked) can be observed also in the magnetic field along the z -axis (cf., Fig. 9, bottom panel). The difference is that they occur slightly later than the specific internal energy ones, possibly indicating a conversion of kinetic energy into

magnetic energy and vice versa. The rest-mass density in the blobs of low-density material ejected in the bursts depends on height, but, at a distance of $z = 86.82$ km along the z -axis, it is $6 \times 10^6 \text{ gr cm}^{-3} - 6 \times 10^8 \text{ gr cm}^{-3}$, while the magnetic field, at the same location, has a strength of $\sim 10^9 - 5 \times 10^{10}$ G.

The bursting activity continues also after the collapse, but with somewhat different properties. First, the frequency is now set by the radial epicyclic frequencies of the oscillating torus as deduced, for instance, when analyzing the time series of the specific internal energy at a ~ 60 km on the x -axis (see Refs. [[106](#), [111](#)] for an introduction to these frequencies in rotating tori). Second, the rest-mass density of the blobs ejected is smaller and of the order of $5 \times 10^7 \text{ gr cm}^{-3}$, while the magnetic field oscillates between $10^8 - 10^9$ G and is stronger probably as a result of the magnetic field increase in the funnel.

With only two bursts observed after black-hole formation, the time span is too short to reach a firmer conclusion on the origin of the bursts after the collapse. However, all present evidence seems to suggest that the postmerger bursts are triggered by an increased mass-accretion rate as the torus approaches the black hole during the inward phase of its epicyclic oscillation. Of course also resistive reconnection processes could lead to the conversion of magnetic energy into internal energy and thus may be invoked to explain this phenomenology. We find this not a likely explanation mostly because of the ordered magnetic field structure that builds up in the funnel and which seems rather stationary. Clearly, a more detailed study of long-term evolutions with different prescriptions for the conductivity profiles is necessary before reaching more robust conclusions.

We finally note that the outflows produced either by shock-heating, magnetically driven winds or by the periodic bursts, eject a substantial amount of matter. More specifically, the total rest-mass flux across a spherical surface located at $r = 295.4$ km is measured to be $0.5 - 2.0 M_{\odot} \text{ sec}^{-1}$, which amounts to a total of $\sim 0.01 M_{\odot}$ ejected from the beginning

	$M (M_{\odot})$	J/M^2	$M_{\text{tor.}} (M_{\odot})$	$r_{\text{tor.}} (\text{km})$
RMHD	2.88	0.873	0.095	105.9
IMHD	2.91	0.884	0.075	88.9

TABLE I. Properties of the black hole–torus system at $t = 4.74$ ms after the appearance of the apparent horizon for both the resistive and IMHD simulations. Shown are the mass and dimensionless spin of the black hole, as well as the rest mass and size of the torus as estimated with a cutoff on the rest-mass density at $\rho = 10^{10} \text{g cm}^{-3}$.

of the simulation and over the survival time of the HMNS (i.e., 13.8 ms). We also note that the mass-ejection rates reported here are larger than those obtained in Ref. [43], where mass fluxes of $\sim 10^{-3} - 10^{-2} M_{\odot} \text{sec}^{-1}$ were reported. However, this is not particularly surprising and for a number of reasons. First, the HMNS considered here is the self-consistent result of a binary merger, while the one studied in Ref. [43] was built using an axisymmetric differentially rotating equilibrium with a standard (but somewhat arbitrary) law of differential rotation. Second, as mentioned in Sec. III D, the linear momenta in our initial data are artificially reduced to accelerate the inspiral. This also leads to a more violent merger and to larger mass losses at least till black-hole formation. After the HMNS collapses, in fact, the mass flux saturates at $0.2 M_{\odot} \text{sec}^{-1}$. Finally, the initial magnetic field in Ref. [43] is about 2 orders of magnitude larger, and this facilitates substantially the loss of MHD equilibrium at the surface of the HMNS and thus the mass loss.

5. Black hole–torus properties

Because the magnetic field is not strong enough to alter the torus dynamics significantly [35], it is natural to expect the two solutions (IMHD and RMHD) to be very similar in terms of dynamical properties once a nearly quasistationary state is established after the collapse to a black hole. This expectation is confirmed in the upper panel of Fig. 14, which shows the rest-mass density distribution along the x -direction at about 4.74 ms after the formation of the apparent horizon. Clearly, the two distributions are very similar but not identical. The RMHD simulation, in particular, yields larger rest-mass densities in the outer portions of the torus, which, in turn, are responsible for larger rest masses (cf., Table I) and stronger magnetic fields (see the discussion in Sec. IV C 1). Furthermore, the RMHD simulation also yields a larger torus as measured from the average position on the x -axis where the rest-mass density falls below $\rho = 10^{10} \text{g cm}^{-3}$.

Additionally, the bottom panel of Fig. 14 illustrates the angular velocity profile along the x direction $\Omega := u^{\phi}/u^t$ for the RMHD implementations (green dashed line) and for the IMHD one (blue solid line), at the same time as the upper panel. Also shown as a black dashed line is a reference Keplerian profile, i.e., scaling like $x^{-3/2}$ and which is well matched by both distributions. As remarked in Ref. [106], the fact that the outer parts of the torus have a quasi-Keplerian behavior

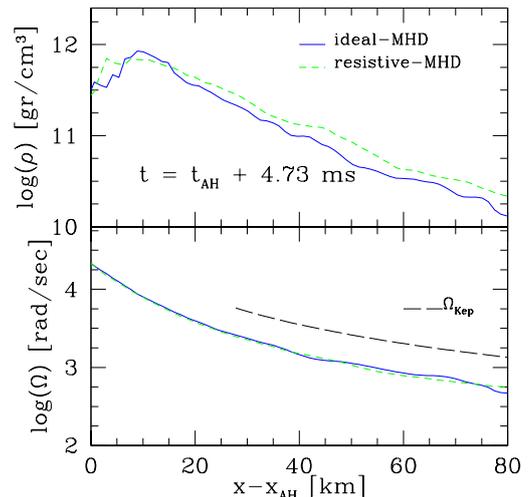


FIG. 14. *Upper panel:* rest-mass density profile along the x -axis for the RMHD (green dashed line) and IMHD (blue solid line) simulations at time $t = 4.74$ ms after the apparent-horizon formation. *Bottom panel:* the same as above but for the angular velocity; also shown as a reference is a Keplerian profile (black dashed line).

has two important implications. First, it suggests that the tori will be stable and not subject to dynamical instabilities that would lead to their rapid destruction (see, e.g., Refs. [112–114]). Second, a quasi-Keplerian profile also provides optimal conditions for the development of an MRI in the torus, thus opening the possibility of further amplification of the magnetic fields that are present after the collapse of the HMNS.

6. Electromagnetic luminosities

Despite the exploratory nature of the simulations carried out here, we have computed for both the RMHD and IMHD implementations the total electromagnetic luminosity L_{EM} emitted. This has been estimated as a surface integral of the Poynting flux over spherical surfaces placed at representative coordinate radii r_E , where r_E has been varied to guarantee that the measurement is an asymptotic one and is not affected by the local plasma dynamics. We recall, in fact, that, because in the IMHD approximation the magnetic fields are locked with the plasma, the electromagnetic luminosity estimates can be heavily influenced by the presence of matter in the outer regions and thus not correspond to a genuine amount of electromagnetic energy flux leaving the system. Unfortunately, there is no simple way within the IMHD approximation of determining whether the integral of the Poynting flux computed on the numerical grid is genuinely asymptotic. However, it is certainly reassuring if the values of L_{EM} are independent from the extraction radius.

The evolution of the electromagnetic luminosity is illustrated in Fig. 15, where we report it as computed at three different extraction radii, $r_E = \{147.71, 221.57, 295.43\}$ km. Note that the luminosity during the inspiral phase of the

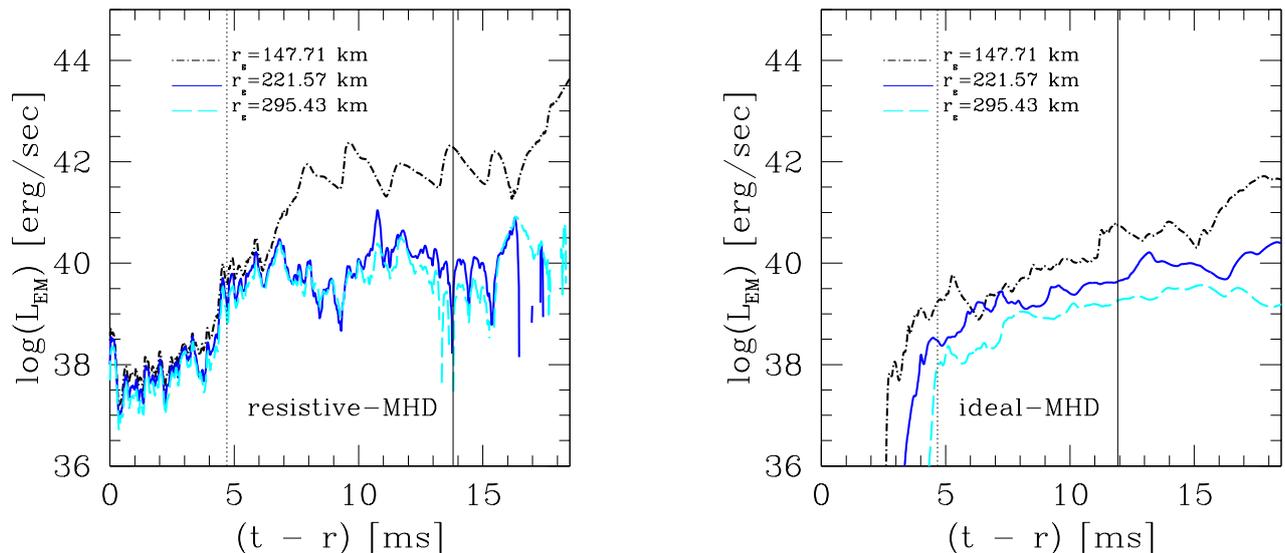


FIG. 15. Time series of the Poynting flux computed at different extraction radii $r_E = \{147.71, 221.57, 295.43\}$ km are shown here for the RMHD implementation (left panel) and the IMHD one (right panel). The dotted line represents the time of the merger, and the black solid line represents the time of horizon formation.

RMHD simulation is much larger than the corresponding one in the IMHD simulation because of the diffusion of the magnetic field across the stars' surfaces, that fills the entire domain with vacuum electromagnetic fields. As remarked already in Sec. III C 1 and IV C 1, these magnetic fields are in areas which are treated as atmosphere from a hydrodynamical point of view, but where the Maxwell equations are solved with zero conductivity, so that the electromagnetic fields can propagate freely.

We note that a nominal value of $\sigma = 0$ does not imply that the electromagnetic luminosity will be zero, since the motion of the compact stars will introduce perturbations in the external electric and magnetic fields, and thus a net Poynting flux (see Ref. [115] for the electromagnetic emission of inspiralling binary black holes in electrovacuum). Indeed, we have found that the electrovacuum luminosity before the merger is $L_{\text{EM}} \sim 10^{38}$ erg sec $^{-1}$, which is smaller than the one reported in Ref.[56] (i.e., $L_{\text{EM}} \sim 10^{41}$ erg sec $^{-1}$), where the stellar exteriors were modelled in the force-free approximation. Although it has already been found that the electrovacuum luminosity is slightly smaller than the force-free one for the same system (see Refs. [116–118]), the differences found here are larger than expected and this may be due to the rather different way in which the exterior regions of stars are treated. By contrast, the electromagnetic luminosity before the merger in the IMHD simulation, where the magnetic fields are always contained inside the stars, is essentially zero.

After the merger, the electromagnetic luminosity grows rapidly of about 2 orders of magnitude, essentially as a result of the growth of the magnetic field already discussed in the left panel of Fig. 12 (we recall that the electromagnetic luminosity should scale quadratically with the magnetic field). During the postmerger phases, the luminosity ranges from $\sim 10^{39}$ to

10^{41} erg sec $^{-1}$, to reach values up to 10^{42} erg sec $^{-1}$ after the collapse of the HMNS to a black hole.

In the left panel of Fig. 15, the luminosity computed on a surface of radius $r_E = 147.71$ km (black dot-dashed line) does not overlap with those computed on larger radii (dark-blue solid line and light-blue dashed solid lines), signalling that this radius is too close to the central object and contaminated by the presence of matter. Fortunately, however, the luminosities at $r_E = 221.57$ km and $r_E = 295.43$ km are very close to each other, confirming the robustness of these measurements. By contrast, the three luminosities in the IMHD simulation reported in the right panel of Fig. 15 provide three different values for the luminosity, indicating that at least two of them (i.e., those at the smaller extraction radii) are probably contaminated by the presence of bound matter and hence not reasonable.

V. CONCLUSIONS

We have presented general-relativistic simulations of the inspiral and merger of binary neutron stars when evolved solving the coupled set of the Einstein equations and those of RMHD. Our main interest here has been to assess the impact that resistive effects have on the dynamics of these binaries, and which are usually investigated in the more idealized framework of IMHD.

Because the differences with an IMHD description could be rather small in certain stages of the process (e.g., during the inspiral), we have carried out a close comparison between two simulations evolving the same binary, either in the context of RMHD or in that of IMHD. More specifically, we have studied the dynamics of an equal-mass binary system of neu-

tron stars with a total ADM mass $M_{\text{ADM}} = 3.25 M_{\odot}$ and an initial orbital separation of 45 km. The stars are initially irrotational and with zero magnetic field. A dipolar magnetic field is therefore added before the evolution is started, which is entirely contained inside the stars, at least initially. Furthermore, to reduce computational costs and “accelerate” the inspiral, we have slightly reduced the linear momenta of the initial data as done in Ref. [95], so that the merger occurs after approximately one orbit.

A crucial goal of our RMHD approach has been that of attaining a smooth resistive description from the highly conducting, high-density stellar interior, out to regions of very low-density plasma, where the electromagnetic fields decouple from the fluid. Falling between these two regimes is the large amount of high-density, small-velocity material that is ejected during and after the merger by the HMNS and, once the latter collapses to a black hole, by the accreting torus. This material, occupies a large portion of the computational domain and is produced either by the spiral arms launched at the merger, or by the magnetic winding and consequent pressure imbalance in the outer layers of the HMNS. Neutrino losses can also be a source of a wind, but we do not model this here.

While there are several ways of potentially reaching a smooth transition between the IMHD limit in the stellar interior and an electrovacuum behavior, we have here adopted the same approach we have extensively investigated with isolated neutron stars in Ref. [66]. In essence, this matching is achieved through a carefully chosen conductivity profile, where the conductivity is directly related to the conserved rest-mass density and is set to zero once the latter reaches a value close to the atmospheric floor. This prescription has at least two free parameters. First, they ensure that the transition region covers only a thin layer close to the surface of the star. Second, they guarantee that this layer remains “thin” even in the first steps of the evolution, when the outer layers of the star expand due to a nonzero pressure in the atmosphere. While we have set these two parameters to sensible values, their influence on the results still needs to be fully explored.

Overall, we have found that there are many similarities between the RMHD and IMHD evolutions, but also one important difference, namely, that the survival time of the hypermassive neutron star, which increases in a RMHD simulation. The increased lifetime of the HMNS appears to be due to a less efficient magnetic-braking mechanism in the resistive regime, in which matter can move across magnetic-field lines, so that the outward transport of angular momentum is reduced. This interpretation is supported by the analysis of the evolution of the specific angular momentum, and it shows that the transport is more efficient in the IMHD simulation. An extended lifetime of the HMNS could have intriguing astrophysical consequences, since a longer-lived magnetized hypermassive neutron star brings support to the recent modelling of SGRBs in terms of long-lived magnetarlike objects produced by the merger [57, 58, 60, 61].

Another important result of these simulations is the confirmation that a magnetic-jet structure is formed in the low-density funnel produced by the black hole–torus system. We note that these simulations have been carried out at higher res-

olutions and with a different grid structure than those in Ref. [36]. In the RMHD simulations the magnetic-jet structure is far more regular, essentially axisymmetric, and extending out to the largest scale in our system. This is most likely the result of the effective decoupling established between the dynamics of the plasma and that of the electromagnetic fields. In the IMHD simulations, a magnetic-jet structure is still present, but on the scale of the torus. This difference is due to the fact that a decoupling of the electromagnetic fields from the plasma is not possible in the IMHD approximation, and the magnetic field follows tightly the turbulent dynamics of the matter. In this case, the magnetic-field lines are almost parallel to the z -axis (in analogy with what was shown in Ref. [36]) and the topology becomes more turbulent on large scales. In both regimes, the magnetic field is predominantly toroidal in the highly conducting torus and predominantly poloidal in the nearly evacuated funnel, although in the IMHD simulation, this happens near the rotation axis. The matter in the funnel does not have an internal energy sufficiently large to launch a relativistic outflow. However, it is reasonable to expect that reconnection processes or neutrino annihilation occurring in the funnel, none of which we model here, could potentially increase the internal energy in the funnel.

The final comment of this work is in fact a caveat. While the dynamics of the magnetic-field results presented here appears reasonable, matching the expectations for this type of system as well as previous simulations, we should remark that our results are ultimately dependent on the choice made for Ohm’s law and for the conductivity profile. Again, while our choice is a very conservative and a plausible one, it represents a choice nevertheless. The large computational costs associated with these simulations have prevented us from presenting a systematic investigation of how sensitive the results are on the choices for Ohm’s law, for the conductivity profile, or on the treatment of the atmosphere. All of these issues deserve further investigation and will be the focus of our future work.

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Appendix A: Magnetic-jet structure in the IMHD simulations

Although the IMHD simulations are not the focus of this paper, for completeness we provide in this Appendix a rapid overview of the properties of the magnetic-jet structure as obtained within this approximation. The essence of the results is shown in Fig. 16, which represents the equivalent of Fig. 5 but within IMHD. The different panels show large-scale, two-dimensional snapshots on the (x, z) planes of the rest-mass density (top row), and of the magnetic field (bottom row). The left column of Fig. 16 refers to $t = 10.25$ ms (left column), when the HMNS has not yet collapsed to a black hole, while the right column refers to $t = 18.89$ ms, when a black hole has already been formed. Because in the IMHD approximation the magnetic fields are tightly locked with the matter, it does not come as a surprise that no ordered magnetic field structure seems to develop before the HMNS collapses and forms a black hole. This is because the dynamics

of the plasma is quite turbulent at the merger and during the HMNS stage. However, after a black hole is formed, a well-ordered magnetic-field structure appears as the system reaches a quasistationary state. We note again that the formation of a magnetic-jet structure occurs around the black-hole rotation axis.

Differently from the corresponding RMHD simulation, the magnetic-jet structure here is not very regular on large scales, and it is necessary to go down to the length scale of the torus, as shown in Fig. 17, for the magnetic-jet structure to become evident. Note that the magnetic-field lines are almost parallel to the z -axis, in analogy with what was shown in Ref. [36]. Finally, although we are here using only the projection of the magnetic-field lines on the (x, z) plane, the magnified view in Fig. 17 reveals that the magnetic field in the low-density funnel is still predominantly poloidal, although not as ordered as in the RMHD simulation (cf., Fig. 3, left and right panels). At the same time, and not shown here for compactness, the magnetic field is essentially toroidal in the torus.

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- [1] G. M. Harry *et al.*, *Class. Quantum Grav.* **27**, 084006 (2010).
 - [2] T. Accadia *et al.*, *Class. Quantum Grav.* **28**, 114002 (2011).
 - [3] Y. Aso, Y. Michimura, K. Somiya, M. Ando, O. Miyakawa, T. Sekiguchi, D. Tatsumi, and H. Yamamoto, *Phys. Rev. D* **88**, 043007 (2013).
 - [4] J. Abadie *et al.*, *Class. Quantum Grav.* **27**, 173001 (2010).
 - [5] N. Andersson, V. Ferrari, D. I. Jones, K. D. Kokkotas, B. Krishnan, J. Read, L. Rezzolla, and B. Zink, *Gen. Rel. Grav.* **43**, 409 (2011).
 - [6] A. J. Weinstein, *Classical and Quantum Gravity* **29**, 124012 (2012).
 - [7] J. S. Read, L. Baiotti, J. D. E. Creighton, J. L. Friedman, B. Giacomazzo, K. Kyutoku, C. Markakis, L. Rezzolla, M. Shibata, and K. Taniguchi, *Phys. Rev. D* **88**, 044042 (2013).
 - [8] W. Del Pozzo, T. G. F. Li, M. Agathos, C. Van Den Broeck, and S. Vitale, *Phys. Rev. Lett.* **111**, 071101 (2013).
 - [9] A. Bauswein and H.-T. Janka, *Phys. Rev. Lett.* **108**, 011101 (2012).
 - [10] A. Bauswein, H.-T. Janka, K. Hebeler, and A. Schwenk, *Phys. Rev. D* **86**, 063001 (2012).
 - [11] K. Hotokezaka, K. Kiuchi, K. Kyutoku, T. Muranushi, Y.-i. Sekiguchi, M. Shibata, and K. Taniguchi, *Phys. Rev. D* **88**, 044026 (2013).
 - [12] K. Takami, L. Rezzolla, and L. Baiotti, *Phys. Rev. Lett.* **113**, 091104 (2014).
 - [13] K. Takami, L. Rezzolla, and L. Baiotti, arXiv:1412.3240, PRD in press (2014).
 - [14] E. Berger, *Annual Review of Astron. and Astrophys.* **52**, 43 (2014).
 - [15] D. Eichler, M. Livio, T. Piran, and D. N. Schramm, *Nature* **340**, 126 (1989).
 - [16] R. Narayan, B. Paczynski, and T. Piran, *Astrophysical Journal, Letters* **395**, L83 (1992).
 - [17] M. Shibata and K. Uryū, *Phys. Rev. D* **61**, 064001 (2000).
 - [18] L. Baiotti, B. Giacomazzo, and L. Rezzolla, *Phys. Rev. D* **78**, 084033 (2008).
 - [19] M. Anderson, E. W. Hirschmann, L. Lehner, S. L. Liebling, P. M. Motl, D. Neilsen, C. Palenzuela, and J. E. Tohline, *Phys. Rev. D* **77**, 024006 (2008).
 - [20] S. Bernuzzi, M. Thierfelder, and B. Brügmann, *Phys. Rev. D* **85**, 104030 (2012).
 - [21] V. Paschalidis, M. Ruiz, and S. L. Shapiro, arXiv:1410.7392 (2014).
 - [22] B. Giacomazzo, R. Perna, L. Rezzolla, E. Troja, and D. Lazdazi, *Astrophys. J.* **762**, L18 (2013).
 - [23] J.-P. Lasota, P. Haensel, and M. A. Abramowicz, *Astrophys. J.* **456**, 300 (1996).
 - [24] K. Belczynski, R. O’Shaughnessy, V. Kalogera, F. Rasio, R. E. Taam, and T. Bulik, *Astrophys. J.* **680**, L129 (2008).
 - [25] D. Radice and L. Rezzolla, *Astron. Astrophys.* **547**, A26 (2012).
 - [26] D. Radice, L. Rezzolla, and F. Galeazzi, *Mon. Not. R. Astron. Soc. L.* **437**, L46 (2014).
 - [27] D. Radice, L. Rezzolla, and F. Galeazzi, arXiv:1502.00551 (2015).
 - [28] K. Kyutoku, M. Shibata, and K. Taniguchi, *Phys. Rev. D* **90**, 064006 (2014).
 - [29] S. Bernuzzi, A. Nagar, T. Dietrich, and T. Damour, arXiv:1412.4553 (2014).
 - [30] Y. Sekiguchi, K. Kiuchi, K. Kyutoku, and M. Shibata, *Phys. Rev. Lett.* **107**, 051102 (2011).
 - [31] D. W. Neilsen, S. L. Liebling, M. Anderson, L. Lehner, E. O’Connor, and C. Palenzuela, *Phys. Rev. D* **89**, 104029 (2014).
 - [32] M. Anderson, E. W. Hirschmann, L. Lehner, S. L. Liebling, P. M. Motl, D. Neilsen, C. Palenzuela, and J. E. Tohline, *Phys. Rev. Lett.* **100**, 191101 (2008).
 - [33] Y. T. Liu, S. L. Shapiro, Z. B. Etienne, and K. Taniguchi, *Phys. Rev. D* **78**, 024012 (2008).
 - [34] B. Giacomazzo, L. Rezzolla, and L. Baiotti, *Mon. Not. R. Astron. Soc.* **399**, L164 (2009).
 - [35] B. Giacomazzo, L. Rezzolla, and L. Baiotti, *Phys. Rev. D* **83**, 044014 (2011).
 - [36] L. Rezzolla, B. Giacomazzo, L. Baiotti, J. Granot, C. Kouveliotou, and M. A. Aloy, *Astrophys. J. Letters* **732**, L6 (2011).
 - [37] D. M. Siegel, R. Ciolfi, A. I. Harte, and L. Rezzolla, *Phys. Rev. D* **87**, 121302 (2013).
 - [38] E. P. Velikhov, *Sov. Phys. JETP* **36**, 995 (1959).

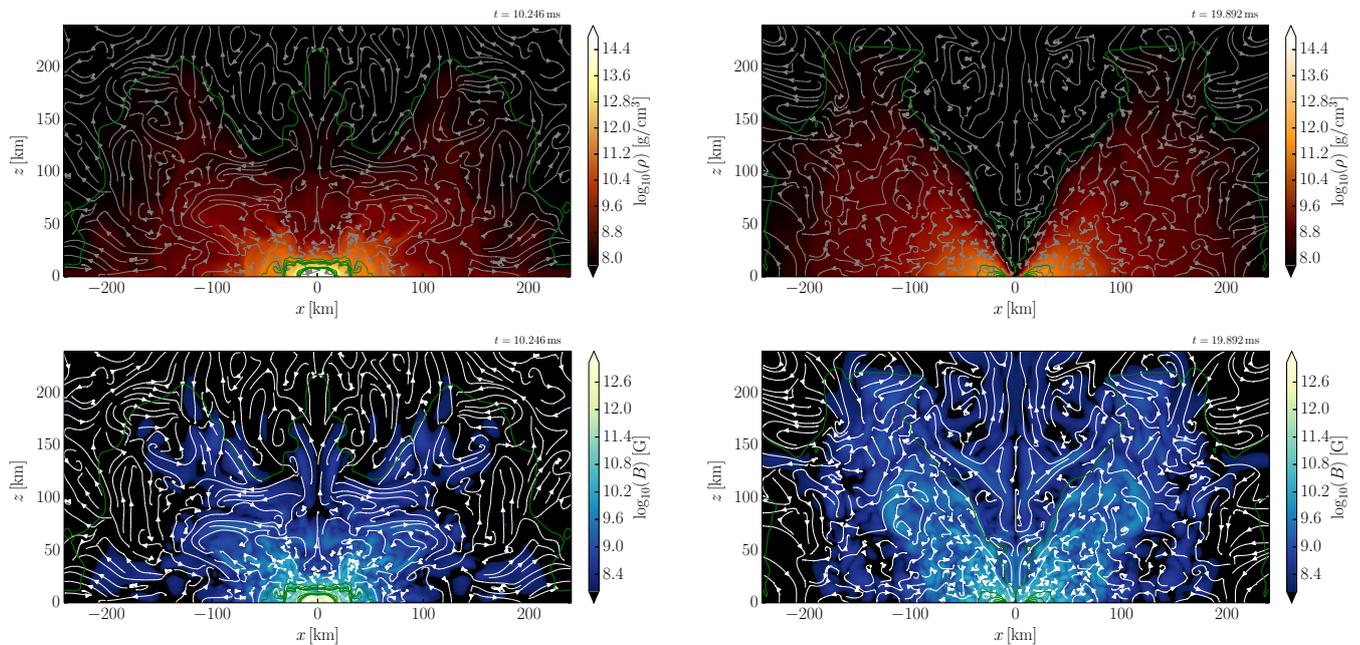


FIG. 16. The same as Fig. 5, but for the IMHD simulation. The two columns refer to $t = 10.25$ ms (left column), when the HMNS has not yet collapsed, and to $t = 18.92$ ms (right column), when a black hole has already been formed. Note again the formation of a magnetic-jet structure around the black-hole rotation axis, which however is far less regular on large scales than the one produced in the RMHD simulation. See also Fig. 17 for a view on smaller scales.

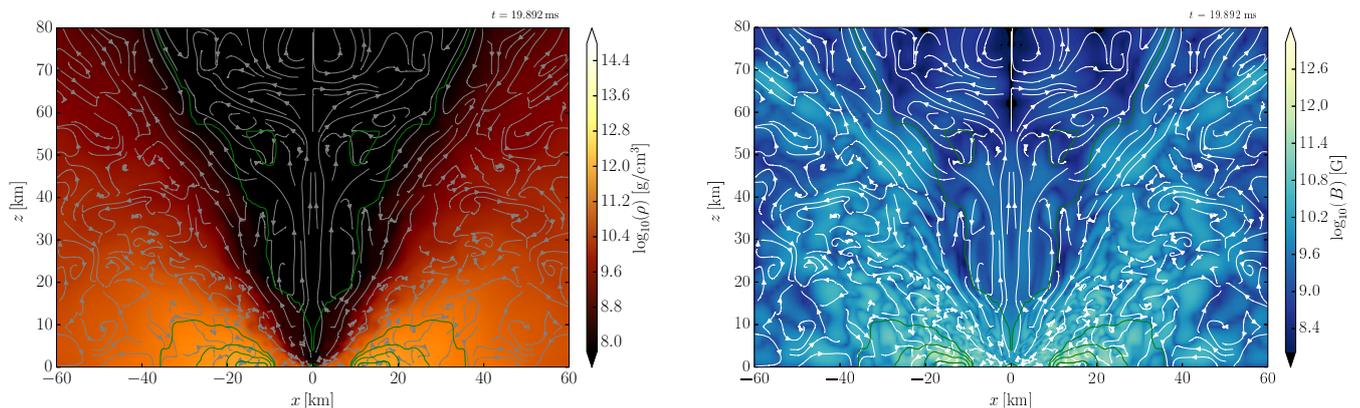


FIG. 17. The same as Fig. 16, but on a scale of $[-60, 60]$ km on the x -axis and of $[0, 80]$ km on the z -axis. Note that the magnetic-jet structure becomes more evident on these scales. See also Fig. 3 for the corresponding quantities in the RMHD simulation.

- [39] S. Chandrasekhar, *Proc. Natl. Acad. Sci.* **46**, 253 (1960).
- [40] M. D. Duez, Y. T. Liu, S. L. Shapiro, M. Shibata, and B. C. Stephens, *Phys. Rev. D* **73**, 104015 (2006).
- [41] M. D. Duez, Y. T. Liu, S. L. Shapiro, M. Shibata, and B. C. Stephens, *Phys. Rev. Lett.* **96**, 031101 (2006).
- [42] K. Kiuchi, K. Kyutoku, and M. Shibata, *Phys. Rev. D* **86**, 064008 (2012).
- [43] D. M. Siegel, R. Ciolfi, and L. Rezzolla, *Astrophys. J.* **785**, L6 (2014).
- [44] B. Giacomazzo, J. Zrake, P. Duffell, A. I. MacFadyen, and R. Perna, arXiv:1410.0013 (2014).
- [45] K. Kiuchi, K. Kyutoku, Y. Sekiguchi, M. Shibata, and T. Wada, *Phys. Rev. D* **90**, 041502 (2014).
- [46] Z. B. Etienne, V. Paschalidis, Y. T. Liu, and S. L. Shapiro, *Phys. Rev. D* **85**, 024013 (2012).
- [47] C. Palenzuela, L. Lehner, O. Reula, and L. Rezzolla, *Mon. Not. R. Astron. Soc.* **394**, 1727 (2009).
- [48] M. Dumbser and O. Zanotti, *Journal of Computational Physics* **228**, 6991 (2009).
- [49] S. S. Komissarov, *Mon. Not. R. Astron. Soc.* **382**, 995 (2007).
- [50] S. Zenitani, M. Hesse, and A. Klimas, *Astrophys. J. Lett.* **716**, L214 (2010).
- [51] O. Zanotti and M. Dumbser, *Mon. Not. R. Astron. Soc.* **418**, 1004 (2011).
- [52] M. Takamoto and T. Inoue, *Astrophys. J.* **735**, 113 (2011).
- [53] Y. Mizuno, *Astrophysical Journal, Supplement* **205**, 7 (2013).
- [54] N. Bucciantini and L. Del Zanna, *Mon. Not. R. Astron. Soc.* **428**, 71 (2013).

- [55] C. Palenzuela, L. Lehner, M. Ponce, S. L. Liebling, M. Anderson, D. Neilsen, and P. Motl, *Phys. Rev. Lett.* **111**, 061105 (2013).
- [56] M. Ponce, C. Palenzuela, L. Lehner, and S. L. Liebling, *Phys. Rev. D* **90**, 044007 (2014).
- [57] B. Zhang and P. Mészáros, *Astrophys. J.* **552**, L35 (2001).
- [58] B. D. Metzger, E. Quataert, and T. A. Thompson, *Mon. Not. R. Astron. Soc.* **385**, 1455 (2008).
- [59] B. D. Metzger, D. Giannios, T. A. Thompson, N. Bucciantini, and E. Quataert, *Mon. Not. R. Astron. Soc.* **413**, 2031 (2011).
- [60] N. Bucciantini, B. D. Metzger, T. A. Thompson, and E. Quataert, *Mon. Not. R. Astron. Soc.* **419**, 1537 (2012).
- [61] L. Rezzolla and P. Kumar, arXiv:1410.8560 (2014).
- [62] R. Ciolfi and D. Siegel, arXiv:1411.2015 (2014).
- [63] B. Giacomazzo and L. Rezzolla, *Classical Quantum Gravity* **24**, S235 (2007).
- [64] L. Baiotti, B. Giacomazzo, and L. Rezzolla, *Classical and Quantum Gravity* **26**, 114005 (2009).
- [65] L. Baiotti, M. Shibata, and T. Yamamoto, *Phys. Rev. D* **82**, 064015 (2010).
- [66] K. Dionysopoulou, D. Alic, C. Palenzuela, L. Rezzolla, and B. Giacomazzo, *Phys. Rev. D* **88**, 044020 (2013).
- [67] A. Dedner, F. Kemm, D. Kröner, C. D. Munz, T. Schnitzer, and M. Wesenberg, *Journal of Computational Physics* **175**, 645 (2002).
- [68] L. Rezzolla and O. Zanotti, *Relativistic Hydrodynamics* (Oxford University Press, Oxford, UK, 2013).
- [69] L. Pareschi and G. Russo, *Journal of Scientific Computing* **25**, 129 (2005).
- [70] E. G. Blackman and G. B. Field, *Phys. Rev. Lett.* **72**, 494 (1994).
- [71] R. Khanna, *Mon. Not. R. Astron. Soc.* **294**, 673 (1998).
- [72] A. Kandus and C. G. Tsagas, *Mon. Not. R. Astron. Soc.* **385**, 883 (2008).
- [73] N. Andersson, *Phys. Rev. D* **86**, 043002 (2012).
- [74] M. Tsamparlis, *Special Relativity: An Introduction with 200 Problems and Solutions* (Springer-Verlag Berlin Heidelberg, 2010).
- [75] T. Nakamura, K. Oohara, and Y. Kojima, *Progress of Theoretical Physics Supplement* **90**, 1 (1987).
- [76] M. Shibata and T. Nakamura, *Phys. Rev. D* **52**, 5428 (1995).
- [77] T. W. Baumgarte and S. L. Shapiro, *Phys. Rev. D* **59**, 024007 (1999).
- [78] D. Pollney, C. Reisswig, L. Rezzolla, B. Szilágyi, M. Ansorg, B. Deris, P. Diener, E. N. Dorband, M. Koppitz, A. Nagar, and E. Schnetter, *Phys. Rev. D* **76**, 124002 (2007).
- [79] E. F. Toro, *Riemann Solvers and Numerical Methods for Fluid Dynamics* (Springer-Verlag, 1999).
- [80] P. Colella and P. R. Woodward, *Journal of Computational Physics* **54**, 174 (1984).
- [81] A. Harten, P. D. Lax, and B. van Leer, *SIAM Rev.* **25**, 35 (1983).
- [82] F. Galeazzi, W. Kastaun, L. Rezzolla, and J. A. Font, *Phys. Rev. D* **88**, 064009 (2013).
- [83] K. Takami, L. Rezzolla, and L. Baiotti, *Phys. Rev. Lett.* **113**, 091104 (2014).
- [84] URL <http://www.cactuscode.org>.
- [85] E. Schnetter, S. H. Hawley, and I. Hawke, *Class. Quantum Grav.* **21**, 1465 (2004).
- [86] L. Lehner, C. Palenzuela, S. L. Liebling, C. Thompson, and C. Hanna, *Phys. Rev. D* **86**, 104035 (2012).
- [87] C. Palenzuela, *Mon. Not. R. Astron. Soc.* **431**, 1853 (2013).
- [88] V. Paschalidis and S. L. Shapiro, *Phys. Rev. D* **88**, 104031 (2013).
- [89] L. Baiotti, I. Hawke, P. J. Montero, F. Löffler, L. Rezzolla, N. Stergioulas, J. A. Font, and E. Seidel, *Phys. Rev. D* **71**, 024035 (2005).
- [90] V. Paschalidis, Z. B. Etienne, and S. L. Shapiro, *Phys. Rev. D* **88**, 021504 (2013).
- [91] B. Giacomazzo, L. Rezzolla, and L. Baiotti, *Phys. Rev. D* **83**, 044014 (2011).
- [92] D. A. Uzdensky, *Space Science Reviews* **160**, 45 (2011).
- [93] E.ourgoulhon, P. Grandclément, K. Taniguchi, J.-A. Marck, and S. Bonazzola, *Phys. Rev. D* **63**, 064029 (2001).
- [94] URL <http://www.lorene.obspm.fr>.
- [95] W. Kastaun, F. Galeazzi, D. Alic, L. Rezzolla, and J. A. Font, *Phys. Rev. D* **88**, 021501 (2013).
- [96] S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich, and M. Ujevic, *Phys. Rev. Lett.* **112**, 201101 (2014).
- [97] R. H. Price and S. Rosswog, *Science* **312**, 719 (2006).
- [98] M. Obergaulinger, M. A. Aloy, and E. Müller, *Astron. Astrophys.* **515**, A30 (2010).
- [99] O. Dreyer, B. Krishnan, D. Shoemaker, and E. Schnetter, *Phys. Rev. D* **67**, 024018 (2003).
- [100] J. Thornburg, *Classical Quantum Gravity* **21**, 743 (2004).
- [101] R. D. Blandford and R. L. Znajek, *Mon. Not. R. Astron. Soc.* **179**, 433 (1977).
- [102] J.-P. Lasota, E.ourgoulhon, M. Abramowicz, A. Tchekhovskoy, and R. Narayan, *Phys. Rev. D* **89**, 024041 (2014).
- [103] M. Ruffert, H.-T. Janka, and G. Schäfer, *Astron. Astrophys.* **311**, 532 (1996).
- [104] M. A. Aloy, H. Janka, and E. Müller, *Astron. Astrophys.* **436**, 273 (2005).
- [105] P. C. Duffell and A. I. MacFadyen, *Astrophys. J.* **775**, 87 (2013).
- [106] L. Rezzolla, L. Baiotti, B. Giacomazzo, D. Link, and J. A. Font, *Class. Quantum Grav.* **27**, 114105 (2010).
- [107] N. Stergioulas, A. Bauswein, K. Zagkouris, and H.-T. Janka, *Mon. Not. R. Astron. Soc.* **418**, 427 (2011).
- [108] G. Toth, *J. Comput. Phys.* **161**, 605 (2000).
- [109] B. D. Metzger and R. Fernández, *Mon. Not. R. Astron. Soc.* **441**, 3444 (2014).
- [110] A. Perego, S. Rosswog, R. M. Cabezón, O. Korobkin, R. Käppeli, A. Arcones, and M. Liebendörfer, *Mon. Not. R. Astron. Soc.* **443**, 3134 (2014).
- [111] O. Zanotti, L. Rezzolla, and J. A. Font, *Mon. Not. Roy. Soc.* **341**, 832 (2003).
- [112] O. Zanotti, J. A. Font, L. Rezzolla, and P. J. Montero, *Mon. Not. R. Astron. Soc.* **356**, 1371 (2005).
- [113] K. Kiuchi, M. Shibata, P. J. Montero, and J. A. Font, *Phys. Rev. Lett.* **106**, 251102 (2011).
- [114] O. Korobkin, E. Abdikamalov, N. Stergioulas, E. Schnetter, B. Zink, S. Rosswog, and C. D. Ott, *Mon. Not. R. Astron. Soc.* **431**, 349 (2013).
- [115] P. Mösta, C. Palenzuela, L. Rezzolla, L. Lehner, S. Yoshida, and D. Pollney, *Phys. Rev. D* **81**, 064017 (2010).
- [116] C. Palenzuela, T. Garrett, L. Lehner, and S. L. Liebling, *Phys. Rev. D* **82**, 044045 (2010).
- [117] P. Moesta, D. Alic, L. Rezzolla, O. Zanotti, and C. Palenzuela, *Astrophys. J. Lett.* **749**, L32 (2012).
- [118] D. Alic, P. Moesta, L. Rezzolla, O. Zanotti, and J. L. Jaramillo, *Astrophys. J.* **754**, 36 (2012).