The Tortuous Route of Confinement Prediction near Operational Boundary
Improvement of Analysis based on ITERH.DB4/L.DB3 Database

Motivation. For practical purposes, empirical confinement time scalings, e.g. ITER-89P (L–mode) and ITERH-98(y,2)
(ELMy H–mode, thermal), have been used and are used to predict the confinement time in future fusion experiments
such as ITER and accompanying satellite devices. For a variety of reasons, prediction intervals, especially for reactor-scale
(excepted to be in L–mode and with hydrogen or helium working gas. The isotope effect is of such scalings turns out to be a long and even tortuous path. This approach, based on multi-machine databases, is
motivated by reactor design-issues and provides a benchmark for various plasma-physical theories. The isotope effect is
interesting for physical reasons and also motivated by the fact that during the first years of ITER operation the discharges
are expected to be in L–mode and with hydrogen or helium working gas.

Several types of nonlinear models have been tried to describe more accurately than ITERH-98(y,2) the
confinement degradation near operational boundaries, such as the L/H transition and the density limit,
and also to express the confinement time in dimensionless plasma parameters. One such approach is
described in [4(b)]. The current version of the international H–mode database DB 4.3 (N = 2934, 14
tokamaks (including unconventionally shaped PBX–M, MAST and NSTX), using the standard dataset,
compatible with DB 2.8 [1], but now for deuterium only and pre-whitened by (0.67 < H_G(2, y) < 1.5),
gives the following scaling, for a motivation of the LHS see [1] Ch.2.7:

\[ \Omega_{p,c} = \frac{1}{2} (R/L)^{3/2} \sim \frac{I_p}{B_t^{1.4}} \frac{1}{D_i^{1.2}} \frac{P_{L'}}{R_{geo}^{1.7}} \frac{1}{A^{0.51}} \frac{1}{(q_{95}/q_{cyl})^{0.37}} \frac{1}{(n/n_G)^{0.11}} \delta_{95}^{0.08} \delta_{98}^{0.22} \log(n/n_G) \]

with a normalised rmse of 36.0/2.5 = 14.4%. (For JFT-2M the isotope effect has been estimated by a factor
\( \Omega_{E,th}(DD)/\Omega_{E,th}(HH) \) varying between 1.08 and 1.17, according to the interaction term 0.9(\( P_L/\pi a V \))^{0.107},
see [4(a)].) Here, \( \tau_{E,th} \) is the thermal energy confinement time, \( n \) is the plasma density, \( A = R/a \) is
the aspect-ratio, \( \Omega_{p,c} = (e/m)B_{tp} \) is the cyclotron frequency and \( \rho_s \) is the normalised Larmor radius,
both for the ions and in the poloidal magnetic field \( B_{tp} = I_p/L \) with \( I_p \) the plasma current and \( L \) the
separatrix contour length. \( P_{L'} \) is the loss power not corrected for radiation [1, 4(a)]. One can see an
effect of the plasma shape, described by \( q_{95}/q_{cyl} \) related to the triangularity, a nonlinear aspect-ratio
dependence and a roll-over as a function of \( n/n_G \) with \( n_G = I_p/\pi a^2 \) the Greenwald limit. All quantities
have been normalised with respect to ITER reference discharge parameters described in [6]. Based on
the same dataset, the scaling for the plasma stored energy is

\[ W_{th} = 290 \text{[MJ]} \quad I_p^{1.40} D_t^{0.12} R_{geo}^{1.7} P_{L'}^{0.26} A^{0.51 - 0.90 \log(n/n_G)}^{0.37} (q_{95}/q_{cyl})^{0.77} (n/n_G)^{0.11 - 0.22 \log(n/n_G)} \]

with an rmse of 13.7%, according to which the point prediction of the confinement time in ITER FEAT
equals \( \tau_{E,th} = 3.3 \) s. Physically, it is interesting to compare the difference between L–mode and H–mode
confinement scaling. Compared to H–mode, in L–mode a different Larmor radius and aspect-ratio
dependence of \( \tau_{E,th} \) exists, since we have estimated from the L–mode database

\[ \frac{\tau_{E,th,H_{98y}}^{0.10 + 0.15 \ln M (R/a)^{0.20} \nu_s^{0.08} M^{0.48} (n/n_G)^{-0.36 - 0.20 \ln(n/n_G)}}{\tau_{E,th,L}} \]

*MPI für Plasmaphysik, Boltzmannstraße 2, D-85748 Garching. e-mail: otto.kardaun@ipp.mpg.de.
Author abbreviation: O. Kardaun for the confinement database working group.
with a residual rmse of 13.4%, based on 14 tokamaks (N = 922), NSTX, with high L-mode confinement, not included. Standard discharges performed at AUG [3] and scans at JT–60U show a reproducibility of about 5% to 7%, while the root-mean squared error from a simple power law (interaction-type scaling) is about 15%, which is substantially higher. This means that non-linear effects and/or hidden variables still play an important role, which might possibly lead to interesting effects [4,5] for the plasma performance in next-step devices [6]. We investigate here several complementary steps to improve upon this situation.

(A) Using a recent extension of the database especially from the spherical tokamaks (MAST, NSTX) the aspect-ratio dependence of confinement, see [8], is re-investigated by using the (aspect-ratio dependent) absolute magnetic field in the midplane, \( |B_{\text{out}}| = (B^2_{\text{tor, out}} + B^2_{\text{pol, out}})^{1/2} \) and also by using the fraction of trapped particles, \( f_{\text{trap}} = \frac{2}{\pi} \sqrt{\frac{2}{(A + 1)}} \), see [9,14]. Perhaps more importantly, we try to assess the difference in confinement scaling between small (or type-III ELMs) and large (or type-I) ELMs since they have an impact on the divertor heat load of future tokamaks such as ITER. We must leave the determination of their operational region by discriminant analysis as a topic for separate work. (B) In [5], it has been suggested that the classical errors-in-variables (EIV) method could partly explain a mismatch between the beta dependence of global scaling and several dedicated scans (DIII-D/JET vs AUG/JT–60U). Therefore we look in more detail into the influence of measurement error propagation. In general, standard error-in-variable (EIV) techniques are indicated when the confinement time is expressed in the temperature (a plasma response variable) or in dimensionless physical variables like \( \rho^*, \beta, \nu_* \). On the other hand, Berkson-type errors [7] (which should be handled by ordinary least squares) constitute the appropriate framework if \( \tau_E \) is expressed as a function of heating power and a prediction of the performance \( nT\tau_E \) of future devices is needed. For physical scalings, classical EIV is appropriate, but analytical formulae and principal-component based methods are applicable under quite restricted assumptions only, see [5, 7, 13]. For the more complicated error structure that occurs in practice, we apply the more recently developed SIMEX method. (C) As originally shown in [10], the triangularity plays an important role for high density discharges near the Greenwald/Borrass limit [11]. This effect has been described by interaction-type scalings that include the shape factor \( q_{95}/q_{97} \) which is related to the triangularity, in combination with a shape dependent roll-over near the Greenwald limit, see [12, 4(b)]. In order to reduce the residual variation of normalised \( \tau_E \) between the tokamaks (especially, PBX-M, JT–60U, MAST, NSTX compared to ITER shaped devices), one can consider the fuelling method (pellet, NBI) and density peaking, and also attempt to characterise more fully the magnetic shape, in particular the mean curvature of the separatrix contour on the high field side (HFS) compared to that on the low field side (LFS) as well as the peakedness (‘curvature variance’) of the top and bottom half contours. Such curvatures are related to plasma stability limits, and ELM-type, which in practice co-determine the range of attainable confinement times.