

THE BEAMING PATTERN OF EXTERNAL COMPTON EMISSION FROM RELATIVISTIC OUTFLOWS

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ABSTRACT

The beaming pattern of radiation emitted by a relativistically moving source like jets in microquasars, AGN and GRBs, is a key issue for understanding of acceleration and radiation processes in these objects. In this paper we introduce a formalism based on a solution of the photon transfer equation to study the beaming patterns for emission produced by electrons accelerated in the jet and upscattering photons of low-energy radiation fields of external origin (the so-called External Compton scenario). The formalism allows us to treat non-stationary, non-homogeneous and anisotropic distributions of electrons, but assuming homogeneous/isotropic and non-variable target photon fields. We demonstrate the non-negligible impact of the anisotropy in the electron distribution on angular and spectral characteristics of the EC radiation.

Subject headings: Radiation mechanisms: non-thermal – galaxies: active – gamma rays: theory – Relativistic processes – scattering

1. INTRODUCTION

The inverse Compton scattering (ICS) of relativistic electrons is one of the major radiation mechanisms in high energy astrophysics. Since the Compton cooling time of electrons decreases linearly with energy, this channel becomes especially prolific in the gamma-ray band. The universal presence of dense radiation fields makes ICS an effective gamma-ray production mechanism in various astronomical environments, in particular in sources containing relativistic outflows - microquasars, AGN, GRBs, *etc.* The dramatically enhanced fluxes, the shift of the spectral energy distribution towards higher energies, and the reduction of characteristic timescales are distinct features of the Doppler boosted radiation produced in a relativistically moving source with a Doppler factor $\mathcal{D} \gg 1$. In relativistic outflows, ICS can be realized, depending on the origin of target photon fields, in two different scenarios.

In the so-called Synchrotron-Self-Compton (SSC) model the synchrotron photons of relativistic electrons constitute the main target for Compton scattering of the same electrons. In its simplified homogeneous one-zone version, the SSC model assumes that a spherical source (a “blob”) filled with an isotropic electron population and a random magnetic field, moves towards the observer with a constant Doppler factor \mathcal{D} . The beaming pattern

for emission of a relativistically moving source derived by Lind & Blandford (1985), can be directly applied to the Synchrotron and SSC components of radiation.

In the second scenario, the seed photon field for ICS is dominated by external radiation, i.e. by low-energy photons produced outside the moving source of relativistic electrons. In this, the so-called External Compton (EC) model the beaming pattern of the inverse Compton radiation differs significantly from the beaming pattern of the synchrotron and SSC components (Dermer 1995).

The characteristics of EC radiation of a relativistically moving source can be calculated using two different approaches:

(i) transforming first the external radiation photon field to the jet frame, calculating in the same frame the characteristics of the high energy photon (the outcome of ICS), and finally transforming the latter back to the observer frame;

(ii) transforming the electron distribution from the jet frame to the observer frame, and then calculating the spectrum of IC photons directly in the observer frame.

Using the first approach, Dermer (1995) has derived the EC beaming pattern in the Thomson limit assuming a power-law distribution of electrons. Georganopoulos et al. (2001) using the second approach have extended this result to the general case of Compton scattering, including the Klein-Nishina regime. In both treatments, the distribution of relativistic electrons has been assumed

to be isotropic and homogeneous. In the zeroth approximation, this could be a reasonable assumption, and thus can be in principle applied to the interpretation of gamma-ray observations of many blazars. However, in some other cases one cannot exclude significant deviations of distributions of electrons from homogeneous and isotropic realizations. The non-stationary treatment of the problem is another issue which has not been addressed so far.

In this paper, we develop a new approach which provides a strict formalism for the treatment of the beaming pattern for EC radiation. Namely, we solve the photon transfer equation which allows us to the beaming pattern in a concise way, and, more importantly, to extend the formalism to more general (non-stationary and anisotropic) case of electron distributions, but assuming isotropic, homogeneous and isotropic distribution of the seed (target) photon fields. For demonstration of the potential of the proposed formalism, we examine how anisotropy in the electron distribution affects the the energy spectrum and the angular distribution of EC radiation.

2. THE BEAMING OF EXTERNAL COMPTON EMISSION

2.1. The photon transfer

In two close points located on the line of sight, the specific intensity (the spectral radiance) I and emissivity j are related as (see e.g. Rybicki & Lightman 1979)

$$I(s) - I(s - ds) = j ds, \quad (1)$$

where s is the distance along the line of sight. It is assumed than one can neglect the scattering and absorption of radiation during its propagation. It is convenient to consider I and j as functions of the radius-vector \mathbf{r} , the time t , and the wave-vector of the photon \mathbf{k} , i.e. $I \equiv I(\mathbf{k}, \mathbf{r}, t)$, and \mathbf{k} , i.e. $s \equiv s(\mathbf{k}, \mathbf{r}, t)$. For simplicity, hereafter we will use the system of units in which the speed of light $c = 1$ and the Planck constant $\hbar = 1$. Also, instead of I and j we will use the distribution function of photons $g = I/\epsilon^3$ and the source function $Q = j/\epsilon^3$, where $\epsilon = |\mathbf{k}|$ is the photon energy. The function $g(\mathbf{k}, \mathbf{r}, t) d^3k d^3r$ describes the number of photons in the volume element $d^3k d^3r$ of the phase space at the moment t , while $Q(\mathbf{k}, \mathbf{r}, t) d^3k d^3r dt$ is the number of photons in the momentum interval d^3k , emitted during the time interval $(t, t + dt)$ from the volume d^3r of the source located at the point \mathbf{r} . With these new notations, Eq. (1) can be written in the form

$$g(\mathbf{k}, \mathbf{r}, t) - g(\mathbf{k}, \mathbf{r} - \mathbf{n}ds, t - ds) = Q(\mathbf{k}, \mathbf{r}, t) ds, \quad (2)$$

where $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$ is a unit vector along the photon momentum. Using Eq. (2), it is easy to derive the following relation

$$g(\mathbf{k}, \mathbf{r}, t) = \int_0^\infty ds Q(\mathbf{k}, \mathbf{r} - \mathbf{n}s, t - s), \quad (3)$$

which expresses g through Q . For calculations of the flux one should integrate g over the directions of the vector \mathbf{n} . This determines the total number of photons of a given energy which reach the point \mathbf{r} at the moment t ,

$$\tilde{g}(k, \mathbf{r}, t) = \int g(\mathbf{k}, \mathbf{r}, t) d\Omega_{\mathbf{n}}. \quad (4)$$

For integration over $d\Omega_{\mathbf{n}}$, it is convenient to write Eq. (3) in the form

$$g(\mathbf{k}, \mathbf{r}, t) = \int_0^\infty ds \int d^3r_0 Q(\mathbf{k}_r, \mathbf{r}_0, t - s) \delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{n}s), \quad (5)$$

where $\delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{n}s)$ is the three-dimensional δ -function, and $\mathbf{k}_r = \epsilon(\mathbf{r} - \mathbf{r}_0)/|\mathbf{r} - \mathbf{r}_0|$. After a simple integration of Eq. (5) over d^3r_0 , we apparently would return to Eq. (3). In the integrand of Eq. (5), the vector \mathbf{n} enters only in the argument of the δ -function, therefore for determination of \tilde{g} , one should calculate the integral

$$X \equiv \int \delta(\mathbf{r} - \mathbf{r}_0 - \mathbf{n}s) d\Omega_{\mathbf{n}}. \quad (6)$$

Using the equation

$$\delta(\mathbf{r} - \mathbf{r}_1) = \frac{1}{r^2 \sin \theta} \delta(r - r_1) \delta(\theta - \theta_1) \delta(\phi - \phi_1), \quad (7)$$

where r, θ, ϕ (r_1, θ_1, ϕ_1) are the spherical coordinates of the point \mathbf{r} (\mathbf{r}_1), we find

$$X = \frac{\delta(|\mathbf{r} - \mathbf{r}_0| - |s|)}{|\mathbf{r} - \mathbf{r}_0|^2}. \quad (8)$$

Then the function \tilde{g} becomes

$$\tilde{g}(k, \mathbf{r}, t) = \int_0^\infty ds \int d^3r_0 \frac{Q(\mathbf{k}_r, \mathbf{r}_0, t - s)}{|\mathbf{r} - \mathbf{r}_0|^2} \times \delta(|\mathbf{r} - \mathbf{r}_0| - s), \quad (9)$$

and the integration over s results in

$$\tilde{g}(k, \mathbf{r}, t) = \int d^3r_0 \frac{Q(\mathbf{k}_r, \mathbf{r}_0, t - |\mathbf{r} - \mathbf{r}_0|)}{|\mathbf{r} - \mathbf{r}_0|^2}. \quad (10)$$

This equation has a simple physical meaning and can be also obtained from heuristic considerations. From the definition of the function Q , it follows that

$$\frac{Q d^3r_0}{|\mathbf{r} - \mathbf{r}_0|^2} \epsilon^2 d\epsilon \quad (11)$$

is the number of photons in the unit volume at the point \mathbf{r} and in the energy interval $d\epsilon$, emitted by a point-like source located at the point \mathbf{r}_0 . Note that due to the delay, the number of photons at the instant t is determined by the source function at the moment $t - |\mathbf{r} - \mathbf{r}_0|$. In the case of an extended source, for calculation of the number of photons one should use the sum of Eq. (11) type expressions. The result will be the quantity $\tilde{g} \epsilon^2 d\epsilon$. Replacing the sum by the integral, we would arrive at Eq. (10).

Eq. (10) is correct for any \mathbf{r} and t . In order to calculate the photon distribution at a distance that significantly exceeds the characteristic size of the source R_0 , let's define the origin of coordinates somewhere inside the source. Then for $r \gg R_0$, one can set $\mathbf{n}_r = \mathbf{r}/r$ and $|\mathbf{r} - \mathbf{r}_0| = r - (\mathbf{n}_r \mathbf{r}_0)$. Neglecting in the denominator $(\mathbf{n}_r \mathbf{r}_0)$ compared to r , we obtain

$$\tilde{g}(k, \mathbf{r}, t) = \frac{1}{r^2} \int d^3r_0 Q(\mathbf{k}, \mathbf{r}_0, t - r + (\mathbf{n}_r \mathbf{r}_0)), \quad (12)$$

where $\mathbf{k} = k\mathbf{n}_r$. This means that all photons detected by a distant observer travel essentially in the direction of

\mathbf{r} . Therefore below we will not distinguish between the vectors $\mathbf{n} = \mathbf{k}/k$ and $\mathbf{n}_r = \mathbf{r}/r$, i.e. $\mathbf{n}_r = \mathbf{n}$. On the other hand, we should note that in general one cannot neglect the last term in the argument $t - r + (\mathbf{n}\mathbf{r}_0)$. The possibility of such an approximation is determined not by the relation of r to $(\mathbf{n}\mathbf{r}_0)$, but by the speed of variation of the function Q during the time $(\mathbf{n}\mathbf{r}_0)$.

2.2. The beaming pattern

Let us consider the collision of ultrarelativistic electrons and soft photons. It is convenient to describe the ensemble of electrons by the distribution function in the phase space $f(\mathbf{p}, \mathbf{r}, t)$. By definition, the quantity $f_e(\mathbf{p}, \mathbf{r}, t) d^3p d^3r$ is the number of electrons located at the moment of time t in the phase volume element $d^3p d^3r$. Let's define $n_{\text{ph}}(\epsilon_{\text{ph}}) d\epsilon_{\text{ph}}$ as the number of photons of energy ϵ_{ph} within the interval $d\epsilon_{\text{ph}}$ in the unit volume. We assume that the distribution of these target photons is homogeneous, isotropic and stationary. Then the high energy photons appearing due to ICS are described by the source function

$$Q(\mathbf{k}, \mathbf{r}, t) = \int w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k}) f_e(\mathbf{p}, \mathbf{r}, t) n_{\text{ph}}(\epsilon_{\text{ph}}) d^3p d\epsilon_{\text{ph}}, \quad (13)$$

where $w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k})$ is the probability of scattering averaged over the directions of the soft photons. According to Eq. (12), the corresponding expression for \tilde{g} is given by

$$\tilde{g}(\mathbf{k}, \mathbf{r}, t) = \frac{1}{r^2} \int d^3r_0 d^3p d\epsilon_{\text{ph}} w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k}) \times f_e(\mathbf{p}, \mathbf{r}_0, t - r + \mathbf{n}\mathbf{r}_0) n_{\text{ph}}(\epsilon_{\text{ph}}). \quad (14)$$

All quantities in Eq. (14) are relevant to a reference system K (the observer system). Let's assume that the accelerated electrons belong to a blob which moves with a relativistic speed $V \sim 1$ and Lorentz factor $\Gamma = 1/\sqrt{1-V^2}$. In order to determine the impact of the blob's bulk motion on the energy distribution of secondary photons, we introduce a co-moving coordinate system K' , and define $f'(\mathbf{p}', \mathbf{r}', t')$ as the distribution function of electrons in the blob, i.e. in the K' system. Below all quantities in the K' system will be indicated by the prime symbol. Note that the distribution function in the phase space is a relativistic invariant (Landau & Lifshitz (1975), Rybicki & Lightman (1979)), i.e.

$$f(\mathbf{p}, \mathbf{r}, t) = f'(\mathbf{p}', \mathbf{r}', t'), \quad (15)$$

where the "primed" and "unprimed" variables are connected via Lorentz transformations.

If we assume (without a loss of generality) that the source is moving along the z -axis, then Eq. (15) can be written in the form

$$f(\mathbf{p}, x, y, z, t) = f'(\mathbf{p}', x, y, \Gamma(z - Vt), \Gamma(t - Vz)), \quad (16)$$

where

$$p'_x = p_x, \quad p'_y = p_y, \quad p'_z = \Gamma(p_z - VE_e). \quad (17)$$

In Eq. (14) the function f is given at an instant delayed in time. Replacing in Eq. (16) t by $t - r + \mathbf{n}\mathbf{r}_0$, one finds

$$f(\mathbf{p}, x_0, y_0, z_0, \tau + n_x x_0 + n_y y_0 + n_z z_0) =$$

$$f'[\mathbf{p}', x_0, y_0, \Gamma(z_0 - V(\tau + n_x x_0 + n_y y_0 + n_z z_0)), \Gamma(\tau + n_x x_0 + n_y y_0 + n_z z_0 - Vz_0)], \quad (18)$$

where n_x, n_y, n_z are the components of the unit vector \mathbf{n} . Note that for simplicity we have set the retarded time as $\tau \equiv t - r$.

Let's introduce the new variables of integration in a way that the space arguments of the function f' can be written in the form (x'_0, y'_0, z'_0) . Apparently, for this we should set

$$x'_0 = x_0, \quad y'_0 = y_0, \quad z'_0 = \Gamma(z_0 - V(\tau + n_x x_0 + n_y y_0 + n_z z_0)). \quad (19)$$

As it follows from Eq. (19), $dz'_0 = \Gamma(1 - Vn_z) dz_0$; therefore the volume element is transformed according to

$$d^3r_0 = \frac{d^3r'_0}{\Gamma(1 - Vn_z)} = \mathcal{D} d^3r'_0, \quad (20)$$

where

$$\mathcal{D} = \frac{1}{\Gamma(1 - Vn_z)} = \frac{1}{\Gamma(1 - \mathbf{n}\mathbf{V})} \quad (21)$$

is the Doppler factor. Note that the transformation given by Eq. (20) differs from the standard Lorentz transformation. At $\mathcal{D} > 1$, we have an increase (but not a contraction as in the case of Lorentz transformation) of the volume by a factor of \mathcal{D} .

The time-argument in the right part of Eq. (18) with respect to the new variables (x'_0, y'_0, z'_0) becomes

$$\mathcal{D} \cdot (\tau + n_x x'_0 + n_y y'_0 + \Gamma(n_z - V) z'_0). \quad (22)$$

Let's introduce a unit vector along \mathbf{k}' . Using Eq.(17), we obtain the following well-known expressions for the aberration of light:

$$n'_x = \mathcal{D} n_x, \quad n'_y = \mathcal{D} n_y, \quad n'_z = \mathcal{D} \Gamma(n_z - V). \quad (23)$$

Then the time-argument in Eq. (18) can be written as

$$\mathcal{D} \tau + (\mathbf{n}' \mathbf{r}'_0). \quad (24)$$

As a result, we find that the function \tilde{g} in the system K is expressed via f' in the system K' as

$$\tilde{g}(\mathbf{k}, \mathbf{r}, t) = \frac{\mathcal{D}}{r^2} \int d^3r'_0 d^3p d\epsilon_{\text{ph}} w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k}) \times f'_e(\mathbf{p}', \mathbf{r}'_0, \mathcal{D} \tau + \mathbf{n}' \mathbf{r}'_0) n_{\text{ph}}(\epsilon_{\text{ph}}). \quad (25)$$

Remarkably, the function f' is not constrained by any condition; the distribution of electrons in the comoving frame can be *non-stationary*, *non-homogeneous*, and *anisotropic*. We should note that, in the case of a homogeneous, isotropic and stationary target photon field, the homogeneity of electrons becomes irrelevant, i.e. Eq. (25) does not depend on the spatial distribution of electrons.

Eq. (25) can be significantly simplified after the following approximations. Let's assume that the distribution function of electrons in the comoving frame K' is stationary and isotropic, i.e. $f' = f'(E'_e, \mathbf{r}'_0)$, where E'_e is the electron energy in the K' frame. The energy E_e in the K frame and E'_e are related as $E'_e = E_e \Gamma(1 - \mathbf{v}_e \mathbf{V})$ where \mathbf{v}_e is the electron speed. Furthermore, we propose that the Lorentz factor of electrons significantly exceeds the Lorentz factor of the bulk motion, $\gamma \gg \Gamma$, and take

into account that the up-scattered photon moves practically in the direction of the electron (the accuracy of this approximation is of the order of $1/\gamma$). Therefore, $\mathbf{v}_e \approx \mathbf{n}$ and, consequently, $E'_e \approx E_e/\mathcal{D}$.

Integration of w over all directions gives

$$W(E_e, \epsilon_{\text{ph}}, \epsilon) = \epsilon^2 \int w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k}) d\Omega_e$$

$$= \frac{8\pi r_e^2}{E_e \eta} \left[2q \ln q + (1-q) \left(1 + 2q + \frac{\eta^2 q^2}{2(1+\eta q)} \right) \right], \quad (26)$$

where

$$\eta = \frac{4\epsilon_{\text{ph}} E_e}{m^2}, \quad q = \frac{\epsilon}{\eta(E_e - \epsilon)}. \quad (27)$$

For the given values of ϵ_{ph} and E_e , the maximum energy of the upscattered photon is

$$\epsilon_{\text{max}} = \frac{E_e}{1 + 1/\eta}. \quad (28)$$

Usually Eq. (26) is obtained by integration over the directions of the momentum of the upscattered photon (Jones (1968) and Aharonian & Atoyan (1981)). However, the argument that depends on the angle enters in the integrand as $(\mathbf{k}\mathbf{p})$, therefore there is no difference over the directions of which vectors, \mathbf{k} or \mathbf{p} , the integration is performed. The above formulae are applicable under the conditions

$$\epsilon_{\text{ph}} \ll m \ll E_e. \quad (29)$$

Let's denote by $\widetilde{W}(E_e, \epsilon) = \frac{dN_\gamma}{dt d\epsilon}$ the upscattered photon energy spectrum per electron:

$$\widetilde{W}(E_e, \epsilon) = \int W(E_e, \epsilon_{\text{ph}}, \epsilon) n_{\text{ph}}(\epsilon_{\text{ph}}) d\epsilon_{\text{ph}}. \quad (30)$$

For the above simplifications, the observed energy flux ($F_e = \epsilon^3 \tilde{g}$) becomes

$$F_e = \frac{\epsilon \mathcal{D}^3}{r^2} \int N'_e \left(\frac{E_e}{\mathcal{D}} \right) \widetilde{W}(E_e, \epsilon) dE_e, \quad (31)$$

where $N'_e(E'_e) = E_e'^2 \int f'(E'_e, \mathbf{r}'_0) d^3 r'_0$ is the differential number of electrons in the comoving frame per energy and per solid angle. In the Thomson limit ($\eta \ll 1$), $\epsilon \widetilde{W}$ becomes a function of a single argument, ϵ/E_e^2 . Writing $\epsilon \widetilde{W}(E_e, \epsilon) = \Phi(\epsilon/E_e^2)$, and performing an integration over $E'_e = E_e/\mathcal{D}$, we obtain

$$F_e = \frac{\mathcal{D}^4}{r^2} \int N'_e(E'_e) \Phi \left(\frac{\epsilon}{\mathcal{D}^2 E_e'^2} \right) dE'_e. \quad (32)$$

This implies that if the EC flux of a source at rest is described by some function $S(\epsilon)$, the relativistic bulk motion of the source results in

$$F_e = \mathcal{D}^4 S(\epsilon/\mathcal{D}^2). \quad (33)$$

In a log-log plot the function F_e is obtained from S by moving the latter up by a factor of $\log_{10}(\mathcal{D}^4)$, and shifting it to the right by a factor of $\log_{10}(\mathcal{D}^2)$. The total intensity is enhanced by a factor of \mathcal{D}^6 :

$$\int_0^\infty F_e d\epsilon = \mathcal{D}^6 \int_0^\infty S(\epsilon) d\epsilon. \quad (34)$$

Integrating over all angles, we obtain the luminosity detected by the observer (the apparent luminosity):

$$L = \frac{1}{5} (16\Gamma^4 - 12\Gamma^2 + 1) L_0, \quad (35)$$

where L_0 is the luminosity of the source at rest (the intrinsic luminosity).

The above results are obtained for an arbitrary distribution of relativistic electrons. For electrons with a power-law distribution, $N'_e(E'_e) \propto E_e'^{-p}$, in the Thomson limit we arrive at the well known result for the beaming pattern, $F_{e,\gamma} \propto \mathcal{D}^{3+p}$ (Dermer 1995).

3. NON-ISOTROPY

In this section we study the impact of a possible anisotropy of the electron distribution inside the moving source on the angular and energy distributions of EC radiation. Non-isotropic distribution of electrons might be caused by different reasons. In particular, anisotropic particle distributions are expected within different acceleration scenarios, including the particle acceleration by relativistic shocks (see e.g. Dempsey & Duffy 2007), by the converter mechanism (Derishev et al. 2003) or due to magnetic reconnection (Cerutti et al. 2012).

We will use the general Eq. (25) assuming that the source function (distribution of electrons) in the comoving system is not isotropic, but stationary. Let's introduce anisotropy (in K') in the form

$$f'(\mathbf{p}', \mathbf{r}') = \Psi(\mathbf{n}'_e) f'(E'_e, \mathbf{r}'), \quad (36)$$

where $\mathbf{n}'_e = \mathbf{p}'/|\mathbf{p}'|$. The calculations for anisotropy in a general form are quite complex, therefore, for the purpose of demonstration, we will use an empirical approach, namely, adopt the specific form of elongated ellipsoid of revolution described by the function

$$\Psi(\mathbf{n}'_e) = \frac{\sin \alpha}{\alpha \sqrt{1 - (\mathbf{s}' \mathbf{n}'_e)^2 \sin^2 \alpha}}. \quad (37)$$

Here \mathbf{s}' denotes a constant unit vector, which could have an arbitrary direction, and the parameter α is confined within the interval $0 < \alpha < \pi/2$. The function Ψ is normalized by the condition

$$\int \Psi(\mathbf{n}'_e) \frac{d\Omega'_e}{4\pi} = 1. \quad (38)$$

The angular distribution given by Eq. (37) is azimuthally symmetric relative to \mathbf{s}' . If the axis z is directed along \mathbf{s}' , the function $\Psi(\mathbf{n}'_e)$ can be considered as an equation of the surface of the ellipsoid of revolution written in spherical coordinates. The semi-major axis of the ellipsoid is directed along \mathbf{s}' and is equal to $a_{>} = \frac{\tan \alpha}{4\pi\alpha}$; the another two semi-axes are $a_{<} = \frac{\sin \alpha}{4\pi\alpha}$.

It is convenient to introduce the asymmetry parameter λ ,

$$\lambda = \frac{a_{>}}{a_{<}} - 1 = \frac{1}{\cos \alpha} - 1. \quad (39)$$

In the limit of $\lambda \rightarrow 0$, the ellipsoid degenerates into a sphere, i.e. the angular distribution becomes isotropic ($\Psi|_{\lambda \rightarrow 0} = 1$). At $\lambda \gg 1$, the angular distribution is strongly extended in the directions of \mathbf{s}' and $-\mathbf{s}'$. We

allow λ to be function of E'_e , i.e. the anisotropy can be energy-dependent.

In the case of anisotropic angular distribution, the integration of Eq. (26) over directions of the electron momentum results in

$$W_{\text{anis}} = \epsilon^2 \int \Psi(\mathbf{n}'_e) w(\mathbf{p}, \epsilon_{\text{ph}}, \mathbf{k}) d\Omega_e. \quad (40)$$

Since the function w is different from zero when the directions of vectors \mathbf{p} and \mathbf{k} practically coincide, the argument of the function Ψ in Eq. (40) can be replaced by \mathbf{n}' , and $\Psi(\mathbf{n}'_e)$ can be taken out under the sign of integral. This gives

$$W_{\text{anis}} \approx \Psi(\mathbf{n}') W(E_e, \epsilon_{\text{ph}}, \epsilon), \quad (41)$$

where \mathbf{n}' in the argument of Ψ should be expressed via \mathbf{n} according to relations in Eq. (23). Therefore, for the anisotropic angular distribution of electrons the observed flux is

$$F_\epsilon = \frac{\epsilon \mathcal{D}^3}{r^2} \int \Psi(\mathbf{n}') N'_e \left(\frac{E_e}{\mathcal{D}} \right) \widetilde{W}(E_e, \epsilon) dE_e, \quad (42)$$

It is interesting to compare this equation with Eq. (31). In the Thomson regime of scattering, Eq. (42) is simplified,

$$F_\epsilon = \frac{\mathcal{D}^4}{r^2} \int \Psi(\mathbf{n}') N'_e(E'_e) \Phi \left(\frac{\epsilon}{\mathcal{D}^2 E'_e} \right) dE'_e. \quad (43)$$

In the case of an energy-independent anisotropy ($\lambda = \text{const}$), for the total (integrated over energy) intensity of radiation in the given direction, we have

$$\int F_\epsilon d\epsilon \propto \Psi(\mathbf{n}') \mathcal{D}^6. \quad (44)$$

For illustration of the results, we fix the orientations of the axes in the following way. As before, the axis z is directed along the velocity V ; the axis y we choose from the condition that the vector \mathbf{s}' becomes parallel to the (y, z) plane. Then the components of the vector \mathbf{s}' can be written in the following form

$$\mathbf{s}' = (0, \sin \theta'_*, \cos \theta'_*). \quad (45)$$

Here the angle θ'_* determines the direction of the axis of symmetry of the angular distribution of electrons in the comoving frame.

To simplify the analysis, we assume that the radiation is detected in the plane (y, z) , and introduce polar coordinates in this plane. Then $(\mathbf{s}' \mathbf{n}') = \cos(\theta' - \theta'_*)$, where θ' varies in the interval from $-\pi$ to π ($\theta' = 0$ for the points on the axis z). Expressing in this equation θ' through the viewing angle θ in the system K , we get

$$(\mathbf{s}' \mathbf{n}') = \mathcal{D} (\sin \theta \sin \theta'_* + \Gamma (\cos \theta - V) \cos \theta'_*). \quad (46)$$

This expression is to be substituted into Eq. (37).

In Fig 1 we show the dependence of the intensity of the total EC radiation (the flux integrated over the photon energies) on θ - the angle between the line of sight and the direction of the jet. The calculations are performed for different combinations of the bulk motion Lorentz factor Γ and the asymmetry parameter λ . The curves shown in these figures correspond to five different directions of the axis of symmetry of the electrons angular distribution

described by the angle θ'_* . For comparison we show also the intensity for isotropically distributed electrons. One can see that except for $\theta'_* = 0$ and $\theta'_* = \pi/2$, the maximum of the observed radiation appears not at $\theta = 0$, as it happens in the isotropic case. Instead, it is shifted to larger angles as the level of anisotropy increases. For the chosen parameter, the shift can be as large as several degrees. This implies that in the case of anisotropic distributions of electrons in the source, we should be able to see misaligned jets as it has been indicated by Derishev et al. (2007).

In a more general approach, the λ -parameter can depend on the electron energy. This could be realized, for example, in the case of diffusive shock acceleration of electrons when the low-energy particles can be effectively isotropised in the downstream region due to pitch-angle scattering, whereas higher-energy particles radiate away their energy before being fully isotropised (see, e.g. Derishev et al. 2007). To demonstrate this effect, let's assume a simple energy-dependence of the asymmetry parameter, $\lambda = \lambda' \gamma'^{p_1}$, and consider a distribution of electrons in the standard "power-law with an exponential cutoff" form:

$$N'_e(\gamma', \mathbf{n}') = A \Psi(\mathbf{n}') \gamma'^{-p} e^{-\gamma'/\gamma'_0}, \quad (47)$$

where $A = \text{const}$. Here instead of the electron energy E'_e , we use its Lorentz factor, $\gamma' = E'_e/m_e$. From the condition of normalization in Eq. (38) it follows that $\int N'_e(\gamma', \mathbf{n}') d\Omega_{\mathbf{n}'}$ does not depend on the level of anisotropy.

In Fig. 2 we show the spectral energy distribution of EC radiation assuming that a relativistic jet propagates through the 2.7 K microwave background radiation. This might be relevant to the inverse Compton X-ray emission of extended jets of AGN which could be (still) relativistic on kpc scales (see, e.g. Sambruna et al. 2002). The radiation spectra are calculated for the viewing angle $\theta = 0$, thus $\mathcal{D} \approx 2\Gamma$, and for the following combination of model parameters: $p = 2$, $\lambda' = 0.1$, $p_1 = 1/2$ and $\gamma'_0 = 10^4$. Since $\eta \sim 4kT \gamma'_0/m \approx 2 \times 10^{-5} \ll 1$, the Compton scattering proceeds in the Thomson limit. One can see from Fig. 2 that if the photon energy is expressed in units of $\mathcal{D}^2 \epsilon_* = 4kT \gamma_0'^2 \mathcal{D}^2$, the shape of the energy spectrum of EC radiation does not depend on the jet's Doppler factor. On the other hand, it depends on the angle θ'_* . The apparent reason is the dependence of the electron energy distribution on θ'_* . For example, for $\theta = 0$ and $\lambda' \gamma'^{p_1} \gg 1$, the spectrum given by Eq. (47) for $\theta'_* = 0$ can be written in the form

$$N'_e(\gamma', \mathbf{n}') = \frac{\lambda' A}{2\pi^2} \gamma'^{-p+p_1} e^{-\gamma'/\gamma'_0}, \quad (48)$$

For $\theta'_* = \pi/2$, Eq. (47) differs from the spectrum corresponding to the isotropic distribution of electrons, by a constant factor of $2/\pi$.

4. SUMMARY

In various astrophysical objects, like microquasars, AGN, and GRBs, the observed fluxes of radiation emerge from relativistically moving jets. The Doppler boosting caused by this motion can significantly enhance (by orders of magnitude!) the emitted absolute flux and shift the spectrum towards higher energies. Therefore, the beaming pattern of radiation is a key issue for proper

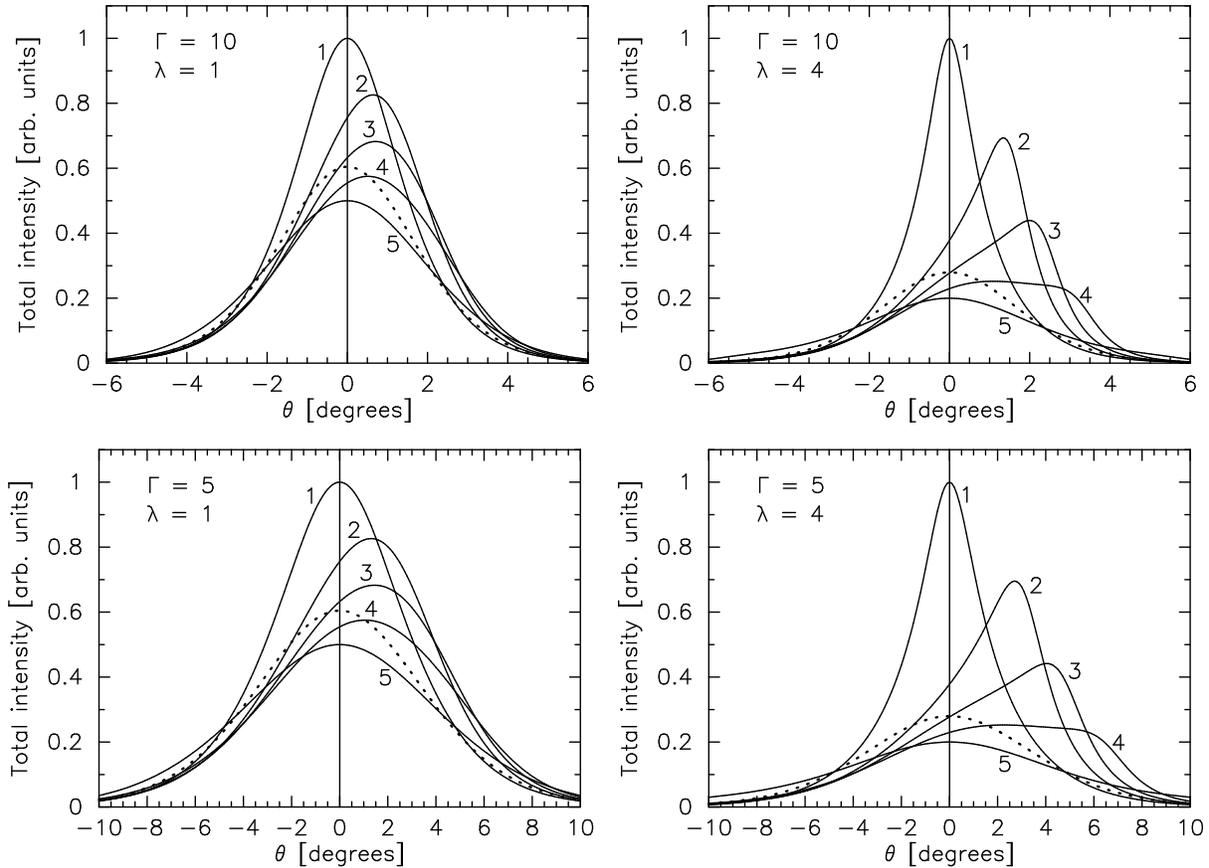


FIG. 1.— The intensity of the EC radiation detected by an observer as a function of the angle between the line of sight and the direction of the relativistically moving source θ . Calculations are performed for four different combinations of the jet's Lorentz factor Γ and the asymmetry parameter λ characterizing the angular distribution of electrons. The curves correspond to different directions of the axis of symmetry of the angular distribution of electrons in the comoving system θ'_e : 0° (curve 1), 30° (curve 2), 45° (curve 3), 60° (curve 4), and 90° (curve 5). For comparison the intensity corresponding to the isotropically distributed electrons is also shown (dotted curves).

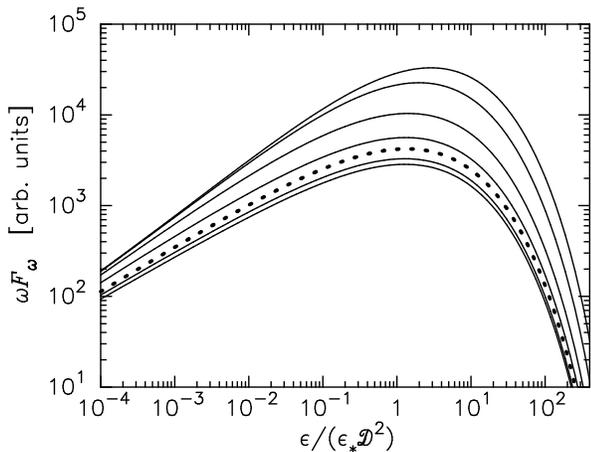


FIG. 2.— The spectral energy distribution of the EC radiation for different orientations of the axis of symmetry of angular distribution of electrons. The solid curves correspond to the angles $\theta'_e = 0^\circ, 5^\circ, 15^\circ, 30^\circ, 60^\circ, 90^\circ$ (from top to bottom). The dotted line is the spectrum of radiation corresponding to the isotropic distribution of electrons. The energy is expressed in the units $\epsilon_* = 4kT\gamma_0'^2$.

understanding of acceleration and radiation processes in these objects.

In this paper we derived the energy distribution of the EC radiation by solving the photon transfer equation for an optically thin source in a rather general case. It is

described by Eq. (25) which allows non-stationary and non-isotropic distribution of electrons in the frame of a relativistically moving source. Eq. (25) does not specify the energy distribution of electrons either, but requires isotropic, homogeneous and non-variable fields of seed photons for ICS. The latter condition makes the solution independent of the spatial distribution of electrons. For power-law energy distribution of isotropically distributed electrons Eq. (25) is reduced to previously derived results (Dermer 1995; Georganopoulos et al. 2001).

Anisotropic distribution of electrons in a relativistically moving source can be realized in some acceleration scenarios, therefore it is of a special practical interest. The formalism developed in this paper has been used to study the impact of the electron anisotropy on the EC emission. The calculations show that the anisotropy of emitting particles can significantly modify the beaming pattern. Most notably, the emission peak can be significantly shifted relative to the line of sight. This implies that, thanks to the anisotropic distribution of electrons in the source, modestly mis-aligned jets may become detectable. And vice versa, while the energy of strongly anisotropic distribution of electrons in a source at rest can be radiated away from the observer, the relativistic motion of the source would make the radiation detectable, even in the case of most unfavorable anisotropy of electrons.

Generally, the electron anisotropy is expected to be energy-dependent. In this case the anisotropy could result in harder spectra of EC emission compared to the isotropic distribution of electrons. The effects related to

the anisotropic distribution of electrons in general, and in the context of the EC scenario, in particular, are quite strong. They cannot be ignored when interpreting the high energy emission from highly relativistic jets in AGN and GRBs.

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