Higgs triplets at like-sign linear colliders and neutrino mixing

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We study the phenomenology of the type-II seesaw model at a linear e^-e^- collider. We show that the process $e^-e^- \to \alpha^-\beta^-$ ($\alpha, \beta = e, \mu, \tau$ being charged leptons) mediated by a doubly charged scalar is very sensitive to the neutrino parameters, in particular the absolute neutrino mass scale and the Majorana CP-violating phases. We identify the regions in parameter space in which appreciable collider signatures in the channel with two like-sign muons in the final state are possible. This includes Higgs triplet masses beyond the reach of the LHC.

I. INTRODUCTION

The origin of neutrino masses and leptonic flavor mixing emerges as one of the most challenging problems in particle physics. Among various theories of this kind, the seesaw mechanism [1–4] attracts a lot of attention in virtue of its naturalness and simplicity in explaining the smallness of neutrino masses. In the standard type-I seesaw model, the fermion sector of the Standard Model (SM) is extended by adding right-handed neutrinos having large Majorana masses $M_{\rm R}$. In its natural version the neutrino masses are suppressed with respect to typical SM (Dirac) masses $m_{\rm D}$ by a factor $m_{\rm D}/M_{\rm R}$. With $m_{\rm D}$ of weak scale it follows that sub-eV neutrino masses require heavy neutrino masses many orders of magnitude above the center-of-mass energies of realistic colliders. In addition, as right-handed neutrinos possess no gauge couplings, their production is suppressed by a mixing factor of order $m_{\rm D}/M_{\rm R}$, so that all in all the mechanism lacks testability. Only at the price of extreme cancellations [5–8], or by introducing additional gauge groups, one can achieve production of type-I seesaw messengers at colliders.

In contrast, in the type-II seesaw model [9–11] one extends the scalar sector of the SM by introducing an $SU(2)_{\rm L}$ Higgs triplet, which couples to two lepton doublets and thereby gives rise to a Majorana mass term of neutrinos after electroweak symmetry breaking. This mass is given by a vacuum expectation value times a Yukawa coupling. A Higgs triplet can be naturally embedded in many frameworks, e.g., grand unified, left-right symmetric, or little Higgs models. It is important to note that tiny neutrino mass by no means requires that the mass of the Higgs triplet is huge. In addition, Higgs triplets do possess gauge couplings, which facilitates their production at colliders [12–15]. In such a scenario, the bilepton decays of the doubly charged component is firmly connected to the neutrino mass matrix, which opens a very promising link between neutrino parameters and collider signatures [16–24]. Higgs triplets also induce lepton flavor violating (LFV) charged lepton decays, which can be used to constrain the parameters associated with them [25–28]. Moreover, observable non-standard neutrino interaction effects induced by the singly charged component of the Higgs triplet might be discovered in future long-baseline neutrino oscillation experiments [29].

The Large Hadron Collider is shown to be able to discover Higgs triplets up to 600 GeV $\sim$ 1 TeV depending on the neutrino mass hierarchy [14, 15]. Similarly, the doubly charged component of the Higgs triplet may also be pair produced at a linear e^+e^- collider via virtual exchange of Z^* and γ^* [30]. The decay of the Higgs triplet in like-sign lepton pairs is then in analogy to that at the LHC. In this paper, however, we will study the production of doubly charged Higgs triplets at a linear collider in the like-sign lepton mode. A linear e^-e^- collider could provide a substantial and complementary role in identifying new physics beyond the SM. In such a running mode, a number of lepton number violating (LNV) processes mediated by any LNV physics, including Higgs triplets, can be explored to a very good precision, since they are basically free from SM background. One $\Delta L = 2$ process to be investigated is the inverse neutrinoless double beta decay, i.e., $e^-e^- \to W^-W^-$, which is however suppressed by the small vacuum expectation value of the Higgs triplet, and is not very promising unless a very narrow resonance is met [31]. In this work, we focus our attention on the phenomena of bilepton production process mediated by a Higgs triplet at a linear e^-e^- collider:

$$e^-e^- \to \alpha^-\beta^- \, . \quad (1)$$

We demonstrate that the collider signatures are firmly correlated to neutrino parameters, while appreciable signals can be expected without the need of resonant enhancement. Such a process is impossible at the LHC, and proceeds via s-channel exchange, see Fig. 1. The difference to the aforementioned processes is the absence of gauge couplings and the pure s-channel production of basically massless final state particles; the cross section depends on Yukawa couplings (and hence the neutrino flavor structure) only.

This work is organized as follows: In Sec. II, we present the framework of the low-scale type-II seesaw model. In Sec. III, we focus on the characteristic features of the processes mediated by the doubly charged scalar, and summarize the current constraints on the relevant Yukawa

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couplings. In particular, we figure out the intrinsic correlations between neutrino parameters and the bilepton production processes at a linear e^-e^- collider. Numerical analysis and illustrations will be performed in detail in Sec. IV. Finally, in Sec. V, we summarize our results and conclude.

II. NEUTRINO MASSES FROM THE TYPE-II SEESAW

In the simplest type-II seesaw framework, one heavy Higgs triplet with hypercharge $Y = 2$ is introduced besides the SM particle content. Apart from the SM interactions, one can further write down a gauge invariant coupling between the Higgs triplet and two lepton doublets as

$$\mathcal{L}_\Delta = h_{\alpha\beta}\overline{L}_\alpha i\tau_2 \Delta L^c_\beta + \text{H.c.}, \qquad (2)$$

where $L_\alpha = (\nu_\alpha, \ell_\alpha)^T$ (for $\alpha = e, \mu, \tau$) denote the lepton doublet, $h_{\alpha\beta}$ is a symmetric Yukawa coupling matrix, and Δ is a 2×2 representation of the Higgs triplet

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \qquad (3)$$

We further expand Eq. (2) and express the interactions in terms of component fields, i.e.,

$$\begin{aligned}\mathcal{L}_\Delta \; = \; & h_{\alpha\beta}\left[\Delta^0 \overline{\nu_\alpha} P_{\rm L} \nu^c_\beta - \frac{1}{\sqrt{2}}\Delta^+\left(\overline{\ell_\alpha} P_{\rm L}\nu^c_\beta + \overline{\nu_\alpha} P_{\rm L} \ell^c_\beta\right)\right. \\ & \left. -\Delta^{++}\overline{\ell_\alpha} P_{\rm L} \ell^c_\beta\right] + \text{H.c.} \end{aligned} \qquad (4)$$

In the language of effective theory, the heavy Higgs triplet should be integrated out from the full theory, and, at tree level, a Majorana mass term of neutrinos is given by [32]

$$\mathcal{L}_\nu = h_{\alpha\beta}\frac{v_{\rm L}}{\sqrt{2}}\overline{\nu_{{\rm L}\alpha}}\nu^c_{{\rm L}\beta} + \text{H.c.} = \frac{1}{2} m_{\alpha\beta}\overline{\nu_{{\rm L}\alpha}}\nu^c_{{\rm L}\beta} + \text{H.c.}, \quad (5)$$

where $v_{\rm L}$ is the vacuum expectation value (VEV) of the Higgs triplet, i.e., $\langle\Delta^0\rangle = v_{\rm L}/\sqrt{2}$. The main constraint on $v_{\rm L}$ stems from the electroweak ρ parameter, and roughly gives $v_{\rm L} \lesssim 8$ GeV. How exactly $v_{\rm L}$ is related to the model parameters depends on details of the underlying physics, it may be that $v_{\rm L} = v^2\mu/m^2_\Delta$, where μ is a dimensionful coupling of the triplet with two Higgs doublets, or $v_{\rm L} \propto v^2/v_{\rm R}$ in left-right symmetric theories, where $v_{\rm R}$ is the scale of right-handed physics, i.e., the VEV of an $SU(2)_{\rm R}$ triplet. Here we will not speculate on the origin of the magnitude of m_Δ, but assume in a model-independent way that it is not above TeV scale, and that the smallness of $v_{\rm L}$ is due to other parameters in the underlying physics.

As usual, m can be diagonalized by means of a unitary transformation, viz.

$$m = U\ \text{diag}(m_1, m_2, m_3)\ U^T\,, \qquad (6)$$

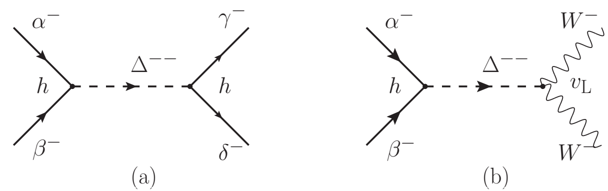

FIG. 1: Feynman diagrams for the doubly charged Higgs mediated bilepton channel (left) and WW channel (right).

where m_i ($i = 1, 2, 3$) denote neutrino masses, and U the leptonic mixing matrix. In the standard (CKM-like) parametrization one has

$$U = R_{23}P_\delta R_{13}P^{-1}_\delta R_{12}P_M\,, \qquad (7)$$

where R_{ij} correspond to the elementary rotations in the $ij = 23$, 13, and 12 planes (parametrized by three mixing angles $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$), $P_\delta = \text{diag}(1, 1, {\rm e}^{{\rm i}\delta})$, and $P_M = \text{diag}({\rm e}^{{\rm i}\phi_1}, {\rm e}^{{\rm i}\phi_2}, 1)$ contain the Dirac and Majorana CP-violating phases, respectively.

According to Eq. (5), the Yukawa coupling matrix h is related to the light neutrino mass matrix as $h = m/(\sqrt{2}v_{\rm L})$. Hence, the flavor structure of the Yukawa coupling h is identical to that of the neutrino mass matrix, which allows for a direct test of neutrino parameters via measurements of h. For example, if kinematically accessible, the doubly charged component of the Higgs triplet can be on-shell produced at colliders, and its subsequent LNV or LFV decays may bring in significant signatures allowing the determination of flavor structure of h, and therefore m, at current and forthcoming colliders. In particular, the decay branching ratios of Δ are shown to be highly sensitive to the neutrino mixing parameters and the neutrino mass hierarchy.

The possibility of testing the neutrino mass matrix by studying the decays of Higgs triplet at hadron colliders, e.g., the LHC, has been discussed intensively [16–24]. In what follows, we will concentrate on the production of the doubly charged Higgs Δ^{--} at linear e^-e^- colliders, and in particular, investigate the LFV processes $e^-e^- \to \alpha^-\beta^-$ in detail.

III. DOUBLY CHARGED HIGGS AT A LINEAR e^-e^- COLLIDER

Among various interesting physics possibilities at a lepton-lepton collider, one of the most promising processes to be investigated is the bilepton production mediated by a doubly charged scalar Δ^{--}. The production of Higgs triplets in like-sign lepton collisions has been discussed in Refs. [31, 33–47]. The relevant diagram is shown in the left-hand panel of Fig. 1. In the framework under discussion, the general cross section for

$\alpha^-\beta^- \to \gamma^-\delta^-$, where $\alpha, \beta, \gamma, \delta = e, \mu, \tau$, reads

$$\begin{aligned} \sigma &= \frac{|h_{\alpha\beta} h_{\gamma\delta}|^2}{4\pi(1+\delta_{\gamma\delta})} \frac{s}{(s-m_\Delta^2)^2 + m_\Delta^2 \Gamma_\Delta^2} \\ &= \frac{|h_{\alpha\beta} h_{\gamma\delta}|^2}{(1+\delta_{\gamma\delta})} \sigma_0 \,, \end{aligned} \tag{8}$$

where Γ_Δ is the decay width of Δ^{--}, and the bare cross section σ_0 is independent of the flavor indices. Note that, if there are two electrons in the final states, the above formula does not apply, since the SM contributions mediated by γ and Z may play the dominating role.

In addition to the bilepton channel, the inverse neutrinoless double beta decay process $e^-e^- \to W^-W^-$ can also be used as a probe of LNV [31, 33–47]. The corresponding diagram is shown in the right-hand panel of Fig. 1. The cross section is given by

$$\begin{aligned} \sigma &= \frac{G_F^2 v_{\rm L}^2}{2\pi} |h_{\alpha\beta}|^2 \frac{(s-2m_W^2)^2 + 8m_W^4}{(s-m_\Delta^2)^2 + m_\Delta^2\Gamma_\Delta^2} \sqrt{1-4\frac{m_W^2}{s}} \\ &\simeq \frac{G_F^2 v_{\rm L}^2}{2\pi} |h_{\alpha\beta}|^2 \frac{s^2}{(s-m_\Delta^2)^2 + m_\Delta^2\Gamma_\Delta^2} \,, \end{aligned} \tag{9}$$

where the mass of W-boson m_W is neglected in the second line. Depending on the mass splitting within the Higgs triplet, the decays $\Delta^{--} \to W^-\Delta^-$ and $\Delta^{--} \to \Delta^-\Delta^-$ may also take place, which may be dominating as they are driven by a gauge coupling. Here we assume a degeneracy among the Higgs triplet components, viz., the above two channels are kinematically suppressed. Accordingly, the relative ratio of the cross sections can be estimated by

$$\begin{aligned} \frac{\sigma(\alpha^-\beta^- \to W^-W^-)}{\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)} &\simeq \frac{2G_F^2 v_{\rm L}^2 (1+\delta_{\gamma\delta}) s}{|h_{\gamma\delta}|^2} \\ = \frac{\Gamma(\Delta^{--} \to W^-W^-)}{\Gamma(\Delta^{--} \to \gamma^-\delta^-)} \frac{s}{m_\Delta} \,. \end{aligned} \tag{10}$$

The requirement $\sigma(\alpha^-\beta^- \to W^-W^-) \ll \sigma(\alpha^-\beta^- \to \gamma^-\delta^-)$ implies that the bilepton decays dominate the decay of Δ^{--}, which occurs when $v_{\rm L} < 10^{-4}$ GeV for a TeV scale Higgs triplet [14].

Inserting Eq. (5) into Eq. (8), one can rewrite the cross section in terms of the neutrino mass matrix elements as

$$\sigma(\alpha^-\beta^- \to \gamma^-\delta^-) = \frac{|m_{\alpha\beta} m_{\gamma\delta}|^2}{4v_{\rm L}^4(1+\delta_{\gamma\delta})} \sigma_0 \,. \tag{11}$$

One can then observe that the cross sections are correlated to both the neutrino mass matrix and $v_{\rm L}$. If the neutrino mass scale is fixed, a smaller $v_{\rm L}$ generally corresponds to larger Yukawa couplings and hence larger cross sections. In case of the e^-e^- collider, the above equation reduces to

$$\sigma(e^-e^- \to \gamma^-\delta^-) = \frac{|m_{ee}|^2 |m_{\gamma\delta}|^2}{4v_{\rm L}^4(1+\delta_{\gamma\delta})} \sigma_0 \,, \tag{12}$$

Decay	Constraint on	Bound (90 % C.L.)
$\mu^- \to e^-e^+e^-$	$\lvert h_{ee}h_{e\mu}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	2.1×10^{-12}
$\tau^- \to e^-e^+e^-$	$\lvert h_{ee}h_{e\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	4.4×10^{-7}
$\tau^- \to \mu^-\mu^+\mu^-$	$\lvert h_{\mu\mu}h_{\mu\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	3.9×10^{-7}
$\tau^- \to e^-\mu^+e^-$	$\lvert h_{ee}h_{\mu\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	2.4×10^{-7}
$\tau^- \to \mu^-e^+\mu^-$	$\lvert h_{e\mu}h_{e\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	2.8×10^{-7}
$\tau^- \to e^-\mu^+\mu^-$	$\lvert h_{e\mu}h_{\mu\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	2.3×10^{-7}
$\tau^- \to e^-e^+\mu^-$	$\lvert h_{\mu\mu}h_{e\tau}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	1.7×10^{-7}
$\mu^- \to e^-\gamma$	$\lvert (hh^\dagger)_{e\mu}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	6.5×10^{-9}
$\tau^- \to e^-\gamma$	$\lvert (hh^\dagger)_{e\mu}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	1.0×10^{-4}
$\tau^- \to \mu^-\gamma$	$\lvert (hh^\dagger)_{e\mu}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	1.4×10^{-4}
$\mu^+e^- \to \mu^-e^+$	$\lvert h_{ee}h_{\mu\mu}\rvert^2 \left(\frac{250~\text{GeV}}{m_\Delta}\right)^4$	9.5×10^{-6}

TABLE I: Constraints (at 90 % C.L.) on h from $\ell \to \ell\ell\ell$, $\ell \to \ell\gamma$, and $\mu^+e^- \to \mu^-e^+$ processes. The experimental bounds have been obtained from Refs. [48, 49].

where $|m_{ee}|$ is the effective mass of the neutrinoless double beta decay process[1], and constrained by current experiments as $|m_{ee}| \lesssim 1$ eV. If the effective mass of neutrinoless double beta decay turns out to be very small, there will be no visible collider signatures either. Then the process might be observed in running a future muon collider in the $\mu^-\mu^-$ mode, or in an $e^-\mu^-$ collider.

The most relevant experimental constraints on the Yukawa coupling matrix h come from the LFV decays $\mu \to 3e$ and $\tau \to 3\ell$ (which occur a tree level), the radiative lepton decays $\ell_\alpha \to \ell_\beta\gamma$ (one-loop), and the muonium to antimuonium conversion. Bounds from Bhabha scattering and universality tests of weak interactions are relatively weaker and will be not elaborated on in our calculations. The constraints at 90 % C.L. are summarized in Table I (see also Refs. [25, 26, 28]).

An interesting feature is that, among all the relevant experimental bounds, at least one off-diagonal element in h is involved except for the constraint from muonium-antimuonium conversion. Note further that the most stringent experimental constraint comes from the rare muon decays $\mu \to 3e$ and $\mu \to e\gamma$. In order to generate observable collider signatures and avoid large LFV processes at the same time, one requires the flavor non-

[1] Note that neutrinoless double beta decay is also triggered by Δ^{--} via the reverse diagram of Fig. 1b. However, the contribution from the Higgs triplet is suppressed by a factor q^2/m_Δ^2 (q represents the momentum transfer carried by the exchanged neutrinos, and is typically of order 10 MeV) compared to the standard contribution from a Majorana mass term of light neutrinos.

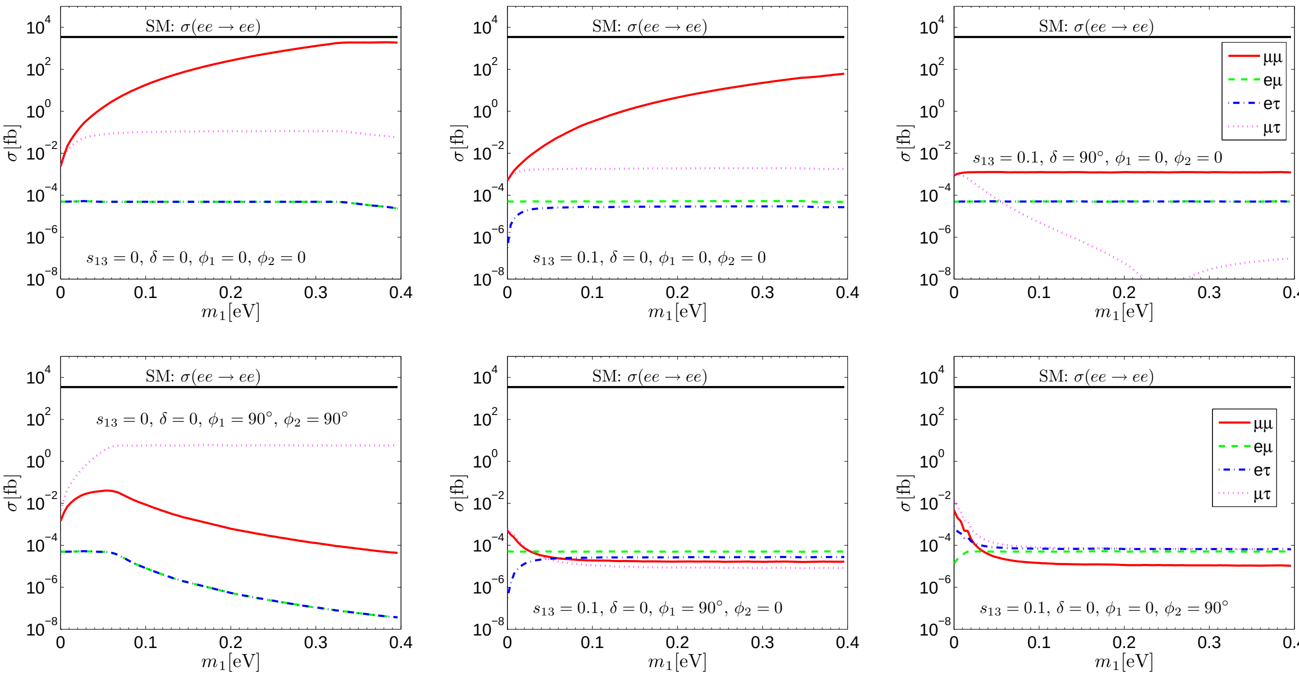

FIG. 2: Upper limits of cross sections $e^-e^- \to \alpha^-\beta^-$ with respect to m_1 at a linear collider with $\sqrt{s} = 1$ TeV. The Higgs triplet mass $m_\Delta = 800$ GeV has been assumed as an example, while the choices of $\sin\theta_{13}$ are labeled on each plot. Furthermore, we take the Majorana CP-violating phases ϕ_1 and ϕ_2 to be zero for the plots on the upper panel, and the Dirac CP-violating phase $\delta = 0$ for the plots on the lower panel. For comparison, the SM Møller scattering cross section is also shown on the plots with black solid lines. The luminosity one can assume is 80 fb^{-1}.

diagonal parts of h to be relatively small, in particular, the $e\mu$ component[2]. One way to achieve this goal requires that m takes an approximately diagonal form, which is the case for a nearly degenerate (ND) neutrino mass spectrum, i.e., $m_1 \simeq m_2 \simeq m_3 = m_0$, together with vanishing CP-violating phases. For example, in the ND case and neglecting the smallest mixing angle θ_{13}, $h_{e\mu}$ is given by

$$|h_{e\mu}|^2 \simeq \frac{2m_0^2}{v_{\rm L}^2} s_{12}^2 c_{12}^2 c_{23}^2 \sin^2(\phi_1 - \phi_2) \ . \qquad (13)$$

Therefore, in the limit $\phi_1 - \phi_2 \simeq 0$, the most stringent constraint in Table I disappears. This situation holds no matter whether the neutrino mass ordering is normal ($m_1 < m_2 < m_3$) or inverted ($m_3 < m_1 < m_2$). Actually, to be precise we should note that $h_{e\mu}$ cannot vanish exactly if $\theta_{13} = 0$ [50], but can be sufficiently small for our purposes.

In the case of a hierarchical neutrino mass spectrum, i.e., $m_1 \simeq 0$ or $m_3 \simeq 0$, there still exists parameter space ensuring the (close-to) vanishing of $h_{e\mu}$. In the extreme case with $m_1 = 0$, one can expand $h_{e\mu}$ according to small quantities s_{13} and $r = m_2/m_3$. Taking $\theta_{23} = \pi/4$, we obtain approximately

$$|h_{e\mu}|^2 \simeq \frac{m_2 m_3 s_{12} c_{12}}{4v_{\rm L}^2}\left[r s_{12} c_{12} + 2 s_{13} c_{\delta+2\phi_2}\right] , \qquad (14)$$

where higher order terms in proportion to s_{13}^2 or r^2 have been neglected. Thus, the severe constraint from the $\mu \to 3e$ process is evaded if the relation

$$r s_{12} c_{12} \simeq -2 s_{13} c_{\delta+2\phi_2} , \qquad (15)$$

holds. As for the IH case, in the limit $m_3 = 0$, we obtain an estimate

$$|h_{e\mu}|^2 \simeq \frac{m_2^2 s_{12}^2 c_{12}^2 s_{\phi_1-\phi_2}^2}{v_{\rm L}^2} , \qquad (16)$$

which again suggests degenerate Majorana CP-violating phases in favor of observable collider signatures. We will see later that typically the di-muon channel $e^-e^- \to \mu^-\mu^-$ is of interest, and thus we want that both h_{ee} and $h_{\mu\mu}$ are sizable. This implies again that nearly degenerate neutrinos, and to some extent inversely hierarchical mass schemes will be favored over a normal hierarchy, for which the ee entry of the mass matrix is typically much smaller than the $\mu\mu$ element.

IV. NUMERICAL RESULTS

In our numerical illustrations, we consider a linear e^-e^- collider with center-of-mass energy $\sqrt{s}$ =

[2] Single texture zeros in the neutrino mass matrix have been studied in Ref. [50].

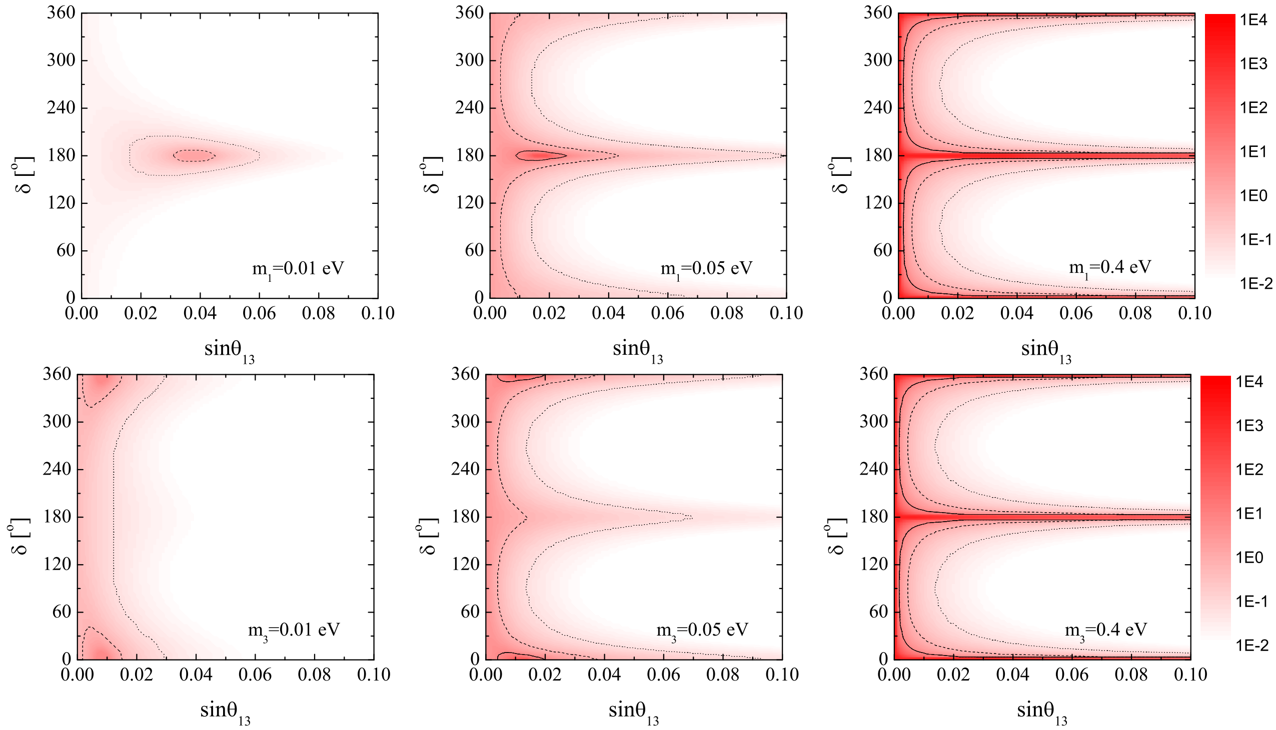

FIG. 3: Upper bounds of the total cross section $\sigma(e^- e^- \to \mu^- \mu^-)$ (in units of fb) for $\sqrt{s} = 1$ TeV in the $\sin\theta_{13} - \delta$ plane with the experimental setup and the mass of Higgs triplet are the same as these in Fig. 2. For the plots in the upper row, we consider the normal mass ordering, while in the lower row the inverted mass ordering is assumed. The lightest neutrino mass can be read off from the plots, and for the sake of simplicity, we take all the Majorana phases to be zero. For an integrated luminosity 80 fb^{-1}, the solid, dashed and dotted curves show the regions with expected number of events greater than 5, 50 and 500, respectively.

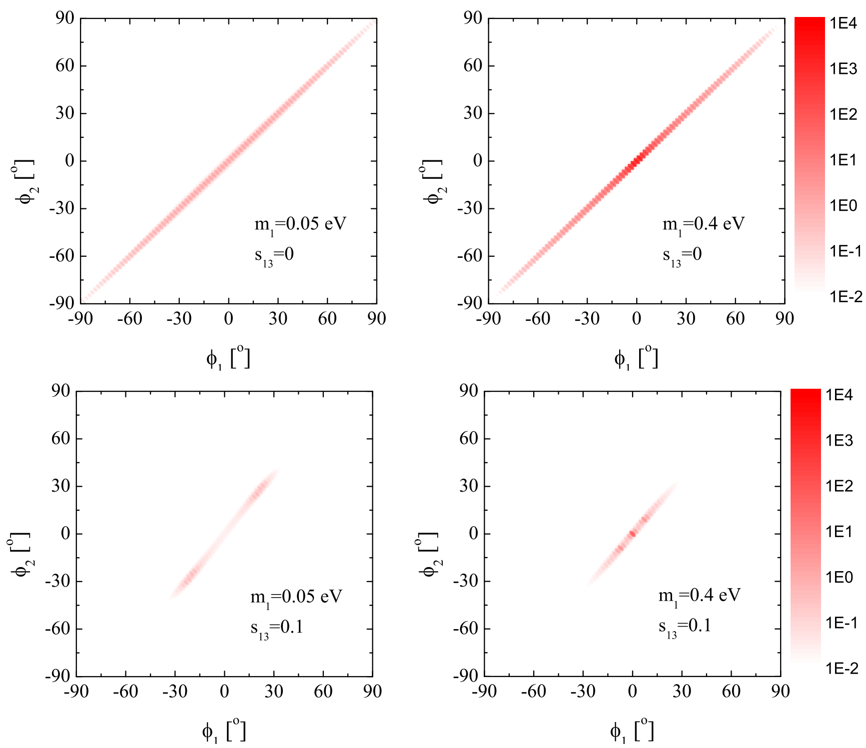

FIG. 4: Upper bounds of the total cross section $\sigma(e^- e^- \to \mu^- \mu^-)$ (in units of fb) in the $\phi_1 - \phi_2$ plane with the same experimental setting of Fig. 3. The input values of s_{13} and m_1 are labeled on the plots, while δ is taken to be zero. The luminosity one can assume is 80 fb^{-1}.

1 TeV, while the mass of Higgs triplet is assumed to be $m_\Delta = 800$ GeV. The typical luminosity is $\mathcal{L} = 80\ (\sqrt{s}/\mathrm{TeV})^2\ \mathrm{fb}^{-1}$. We also make use of the neutrino parameters from a global-fit of the current neutrino oscillation experiments [51]. In the numerical calculations, we fix the neutrino parameters and m_Δ. We let v_L vary between 10^{-10} GeV and 10^{-6} GeV (in order to have the bilepton decays dominate over the W^-W^- mode) and vary the Yukawa coupling matrix h until one of the upper bounds in Table I is saturated. Perturbativity of h is satisfied, and the Yukawas do not exceed 0.35 in all of the results we will present in what follows.

In Fig. 2 we illustrate the upper limits of the cross sections $\sigma(e^-e^- \to \alpha^-\beta^-)$ with respect to the lightest neutrino mass m_1 given the constraints from Table I. The choices for the neutrino mixing parameters are labeled on each plot, and the cross section of the SM process $e^-e^- \to e^-e^-$ is also shown for the purpose of comparison[3]. Due to the experimental difficulty in tau reconstruction, we do not show channels with two tau. As we discussed in the previous section, the upper bounds of the cross sections are very sensitive to the neutrino parameters. Especially, in the ND case and vanishing CP-violating phases, e.g., the left upper plot, the cross section of the channel with two muons in the final states could be more than 1000 fb. It turns out that for $m_1 \gtrsim 0.3$ eV the bound coming from $\mu^+e^- \to \mu^-e^+$ is the dominating one[4], which then sets an upper limit on the di-muon channel. Moreover, a non-zero s_{13} will slightly decrease the maximal cross sections. One also observes from plots in the right column that non-vanishing CP-violating phases can suppress the di-muon cross section, mostly because h_{ee} and $h_{\mu\mu}$ cannot simultaneously be large in those cases. As shown in the left lower plot, a dominating and observable cross section in the $\mu\tau$ channel arises for the Majorana phases $\phi_1 = \phi_2 = \pi/2$. One may check that in this case h takes a special form in which the h_{ee} and $h_{\mu\tau}$ are large and the other entries small. This corresponds to the approximate conservation of the $L_\mu - L_\tau$ flavor symmetry [53].

We conclude from Fig. 2 that the di-muon channel looks most promising among the possible final states, and therefore study it further. The upper bounds of the cross section $\sigma(e^-e^- \to \mu^-\mu^-)$ with respect to $\sin\theta_{13}$ and δ are illustrated in Fig. 3. One reads from the plots that sizable cross sections could be expected in the case $\delta \sim n\pi$ or $\theta_{13} \sim 0$ if the neutrino mass spectrum is nearly degenerate. Furthermore, even if the light neutrino mass spectrum is hierarchical, there are still certain parameter regions, i.e., $(\delta, \sin\theta_{13}) \sim (\pi, 0.04)$ for the normal mass hierarchy and $(\delta, \sin\theta_{13}) \sim (0, 0.01)$ for the inverted mass hierarchy, that allow for identifying the Higgs triplet since the expected number of events would be greater than 50 for an integrated luminosity 80 fb^{-1}. This is in agreement with our the analytical expressions. For example, in the normal mass ordering case, for $\delta = \pi$, Eq. (15) yields $\sin\theta_{13} \simeq 0.04$. In what regards the inverted hierarchy, in the limit $m_3 \sim 0$ the leading order contributions in $h_{e\mu}$ disappear for $\phi_1 = \phi_2 = 0$ as shown in Eq. (16). Thus, one should take into account the "next-to-leading order" contributions, which is given by

$$|h_{e\mu}|^2 \simeq \frac{m_2^2}{4v_\mathrm{L}^2}\left(\varepsilon^2 s_{12}^2 c_{12}^2 - 2\varepsilon s_{12}c_{12}s_{13}\cos\delta + s_{13}^2\right), \quad (17)$$

where we have expanded the formula based on small parameters $\varepsilon = (m_2 - m_1)/m_2 \simeq$ and s_{13}. For $\cos\delta = 0\,(2\pi)$, $h_{e\mu} = 0$ in Eq. (17) requires roughly $s_{13} \simeq s_{12}c_{12}(m_2 - m_1)/m_2$, which corresponds to $\sin\theta_{13} \simeq 0.007$. This result again confirms the numerical analysis.

As discussed before, the cross section relies heavily on the configuration of Majorana CP-violating phases. In particular, in order to obtain observable signatures, one would expect that ϕ_1 and ϕ_2 are almost degenerate. In Fig. 4, we show the upper bounds of the cross section $\sigma(e^-e^- \to \mu^-\mu^-)$ in the $\phi_1 - \phi_2$ plane in the normal ordering case. The interesting parameter region is, as expected, the diagonal line of $\phi_1 - \phi_2 \simeq 0$. Consequently, collider experiments may play a crucial role in determining the Majorana CP-violating phases. Since h is not sensitive to neutrino mass ordering in the ND limit, e.g., $m_i > 0.05$ eV, similar parameter region for ϕ_1 and ϕ_2 also exists in the inverted mass ordering case. If the mass spectrum is hierarchical, it is difficult to acquire visible cross sections unless some fine tuning exists as shown in Eqs. (14) and (17).

Finally, one may wonder if the above estimate strongly depends on the assumption on the mass of the doubly charged scalar, since resonant production of Δ^{--} requires its mass to be known with reasonable accuracy in order to tune the center-of-mass energy of the colliding electrons [41, 44, 45]. In Fig. 5, we give the bare cross section σ_0 [see Eq. (8)] as a function of $\sqrt{s}$ for different masses of the Higgs triplet. One reads from the plot that remarkable cross sections can be naturally obtained without the need of a severe fine-tuning of the colliding energy. To be concrete, we also present in Fig. 6 the upper limits of the cross section $\sigma(e^-e^- \to \mu^-\mu^-)$ for the triplet masses $m_\Delta = (0.5, 0.8, 1.5)$ TeV and $\sqrt{s} = (1, 4)$ TeV. Recall that with the scaling behavior $\mathcal{L} = 80\ (\sqrt{s}/\mathrm{TeV})^2\ \mathrm{fb}^{-1}$ one requires for 100 events cross sections of $\sigma = 1.25$ fb and $\sigma = 0.078$ fb, if $\sqrt{s} = 1$ TeV and 4 TeV, respectively. Note that the cross sections for $m_\Delta = 0.8$ TeV and $m_\Delta = 1.5$ TeV are almost identical despite that σ_0 is different. This can be understood from the LFV constraints on the Yukawa coupling h, which becomes less severe for a larger triplet mass. This fea-

[3] In computing the total cross section in the SM framework, $|\cos\theta| < 0.8$ is used with θ being the scattering angle between the initial and final electron in the center-of-mass frame.

[4] The next generation muon factory may improve this constraint by one order of magnitude [52], which however does not seriously change the main conclusion addressed in this work.

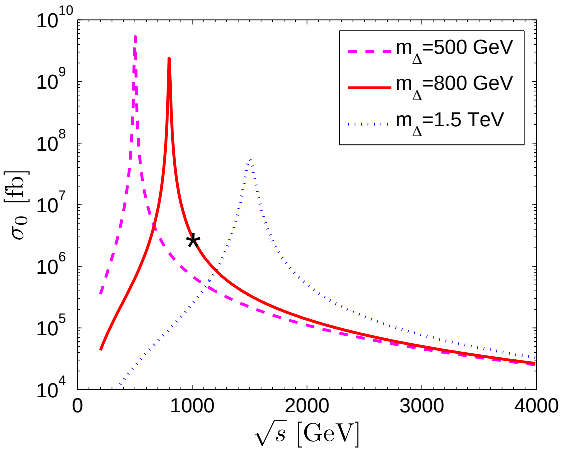

FIG. 5: The bare cross section σ_0 (in units of fb) as a function of $\sqrt{s}$, with solid, dashed and dotted lines corresponding to $m_\Delta = 0.8$ TeV, 0.5 TeV and 1.5 TeV, respectively. The black star on the plot indicates the values of the parameters used in plotting Figs. (2-5). For the definition of σ_0, see Eq. (8).

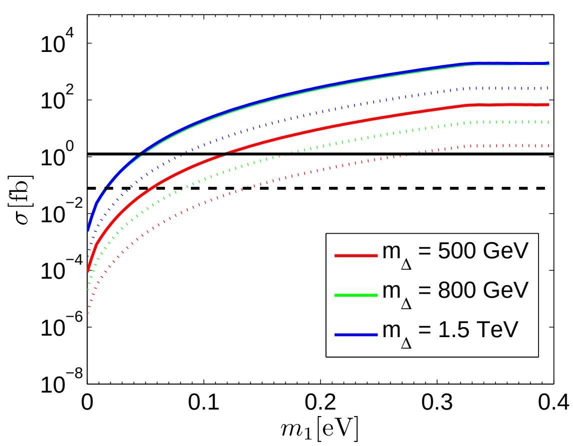

FIG. 6: Upper limits of the cross section $e^-e^- \to \mu^-\mu^-$ with $\sqrt{s} = 1$ TeV (solid lines) and $\sqrt{s} = 4$ TeV (dashed lines). The Higgs triplet masses are labeled on the plot. Here we consider the normal mass ordering case, and assume all the CP-violating phases and s_{13} to be zero for simplicity. The blue and green solid lines for $\sqrt{s} = 1$ TeV and $m_\Delta = 800$ and 1500 GeV are lying basically on top of each other and are hardly distinguishable. 100 events correspond to $\sigma = 1.25$ fb (black solid line) and $\sigma = 0.078$ fb (black dashed line), respectively.

ture is compensated by a reduced cross section. Recall that triplet masses larger than 1 TeV cannot be probed at the LHC. Hence a future linear collider running at the TeV scale can be a probe for such particles. The bilepton channel at a linear collider is so spectacular that, Δ^{--} with mass around TeV scale can be easily probed together with much better sensitivity than that of the LHC.

V. SUMMARY

We have studied the Higgs triplet mediated processes $e^-e^- \to \alpha^-\beta^-$ at a future linear collider run in a like-sign lepton mode. The strong dependence on neutrino parameters and hence the flavor structure of the Majorana mass matrix m was emphasized and the di-muon channel was identified as the most promising one. In order to avoid strong constraints from lepton flavor violation, suppressing the $e\mu$ element of m is required. The largest cross sections occur for a near-diagonal mass matrix, which implies nearly degenerate neutrino masses. However, even in the limit of a hierarchical neutrino spectrum observable signatures could still be expected for certain choices of neutrino mixing parameters. Therefore, measurements of bilepton channels at a linear collider could be quite helpful in order to provide valuable information on the neutrino parameters. In addition, triplet masses beyond the reach of the LHC can be probed.

Finally, we would like to note that similar analyses could be performed for a muon-collider (e.g., the process $\mu^-\mu^- \to e^-e^-$), or an electron-muon facility, where in addition interesting and characteristic processes like $e^-\mu^+ \to e^+\mu^-$ could be studied.

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