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Relativistic encounters in dense stellar systems

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ABSTRACT

Two coalescing black holes (BHs) represent a conspicuous source of gravitational waves (GWs). The merger involves 17 parameters in the general case of Kerr BHs, so that a successful identification and parameter extraction of the information encoded in the waves will provide us with a detailed description of the physics of BHs. A search based on matched-filtering for characterization and parameter extraction requires the development of some 10^{15} waveforms. If a third additional BH perturbed the system, the waveforms would not be applicable, and we would need to increase the number of templates required for a valid detection. In this paper, we calculate the probability that more than two BHs interact in the regime of strong relativity in a dense stellar cluster. We determine the physical properties necessary in a stellar system for three BHs to have a close encounter in this regime and also for an existing binary of two BHs to have a strong interaction with a third hole. In both cases the event rate is negligible. While dense stellar systems such as galactic nuclei, globular clusters and nuclear stellar clusters are the breeding grounds for the sources of GWs that ground-based detectors like Advanced LIGO and Advanced VIRGO will be exploring, the analysis of the waveforms in full general relativity needs only to evaluate the two-body problem. This reduces the number of templates of waveforms to create by orders of magnitude.

Key words: black hole physics – gravitational waves – globular clusters: general – galaxies: kinematics and dynamics.

1 INTRODUCTION

The detection of merging black holes (BHs) is the holy grail of ground-based detectors of gravitational waves (GWs) such as LIGO and VIRGO. For the search for GWs of compact binaries, the availability of accurate waveform models for the full merger is crucial. Thanks to the success of numerical relativity in simulating the late inspiral, merger and ringdown of a binary of two BHs (Pretorius 2005; Campanelli et al. 2006; Baker et al. 2006), we are now able to perform a search with realistic templates. The conjunction of post-Newtonian modelling of the inspiral phase and full numerical relativistic simulations of the merger and ringdown is now a reality for comparable-mass scenarios of mass ratios up to about 10 (see e.g. Buonanno & Damour 1999; Ajith et al. 2007; Buonanno et al. 2007; Santamaría et al. 2010). Nevertheless, the high cost of full numerical relativity simulations constitutes a serious limitation to the development of waveforms.

It has been estimated that we need a bank of 10^{13} waveforms for the identification of a binary of two BHs with an F-statistic based grid search at a *minimal match* (Cornish & Porter 2007). On the other hand, since the BHs are most likely Kerr, the number goes up to $\sim 10^{15}$ to cover the spin parameter. In the case of stellar-mass BHs, since we cannot detect the sky-location of the source, we are limited to a two-dimensional space, the masses of the BHs, so that the number is considerably reduced; we need to create about $\sim 10^5$–10^6 templates. This is the reason why in the last years there has been a significant effort in developing alternative approaches, such as Monte Carlo schemes, genetic algorithms, Metropolis-Hastings methods and Nested Sampling techniques (Cornish & Porter 2006; Lang & Hughes 2006; Gair & Porter 2009; Petiteau, Yu & Babak 2009).

Campanelli, Lousto & Zlochower (2008) addressed for the fist time fully relativistic long-term numerical evolutions of three equal-mass BHs and found that the merger dynamics is very distinct from binaries. In particular, the trajectories were intricate and led to singular waveforms, as e.g. their fig. 4 shows, in which we can see two mergers. Recently there has been an effort in calculating in detail the waveforms of systems of three and four BHs interacting in full GR. Galaviz, Bruegmann & Cao (2010) have developed a knowledgeable scheme to study the waveforms of such configurations and find intricate templates for the waves. Also, Jaramillo & Lousto (2010) have addressed the problem of critical BH separations for

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the formation of a common apparent horizon. The authors study in detail the aligned equal mass cases for up to five BHs.

If we increase the number of BHs involved in the GW, the number of templates to develop increases enormously. Putting it in Neil Cornish' words, 'The sensitive dependence on initial conditions will send the template count through the roof'. It is consequently important to understand the limits imposed by the physical systems which harbour these sources of GWs. Therefore we address the question of the existence of a system with more than two BHs in a relativistic regime. In Section 2 we calculate the probability of having a relativistic three-body encounter in a dense stellar cluster with three BHs initially unbound. In Section 3 we estimate the possibility that an already formed binary of two BHs interacts relativistically with a third BH. In Section 4 we summarize our results and give the conclusions.

2 THREE-BODY RELATIVISTIC ENCOUNTERS OF UNBOUND BHs

Rough estimates will be sufficient to show how unlikely triple relativistic encounters are. Therefore, for simplicity, we assume that all BHs in a given stellar system have the same mass, m. Let us assume that we have two of them fly by with a periapsis distance of a few Schwarzschild radii, $R_{\rm peri} = \hat{d} Gm/c^2$. Therefore, $\hat{d}$ is the periapsis distance in units of Gm/c^2, with G the gravitational constant and c the speed of light. If the relative velocity between the two objects at large separation is $V_{\rm rel}$, the cross-section for an encounter with periapsis distance within $\hat{d}$ can be estimated as

$$S = \pi \hat{d}^2 \frac{G^2 m^2}{c^4} \left[1 + \frac{2}{\hat{d}} \left(\frac{c}{V_{\rm rel}} \right)^2 \right] \approx 2\pi \hat{d} \frac{G^2 m^2}{c^2 \sigma^2}. \quad (1)$$

We have assumed that the BHs are in a non-relativistic stellar cluster, as observed, $V_{\rm rel} \approx \sigma \ll c$. Here, σ is the 3D velocity dispersion in the cluster, defined such that the total kinectic energy in the cluster is $(1/2) M\sigma^2$, where M is the total mass of the cluster.

Hence, the encounter rate for one BH is $1/t_{\rm enc} = nSV_{\rm rel} \cong 2\pi \hat{d} G^2 m^2 n/(c^2 \sigma)$, where n is the number density of BHs. In order to obtain the rate of relativistic three-body encounters, one multiplies by the probability of having a third object within a volume $\sim R^3_{\rm peri}$, $P_3 = n R^3_{\rm peri}$ (see p. 201 of Heggie & Hut 2003)

$$\frac{1}{t_3} \cong P_3 \frac{1}{t_{\rm enc}} \cong n \hat{d}^3 \frac{G^3 m^3}{c^6} 2\pi \hat{d} \frac{G^2 m^2 n}{c^2 \sigma} \cong 2\pi \hat{d}^4 \frac{G^5 m^5 n^2}{c^8 \sigma}. \quad (2)$$

This is the rate per object. We now calculate the rate for a whole cluster of N BHs,

$$\frac{1}{t_{3,\,\rm tot}} = \frac{N}{t_3} \cong 2\pi \hat{d}^4 \frac{G^5 m^5 N^3}{c^8 \sigma R^6} = 2\pi \hat{d}^4 \frac{G^5 M^5}{c^8 \sigma R^6 N^2}, \quad (3)$$

with $M = Nm$ and $n \approx N/R^3$. We ascertain now that the cluster is self-gravitating, so that $GM/R \cong \sigma^2$, where R is a typical length-scale of the cluster, for instance its half-mass radius, i.e. the radius of sphere containing one half of the BHs (see for instance Binney & Tremaine 2008)

$$\begin{aligned} \frac{1}{t_{3,\,\rm tot}} &\cong 2\pi \hat{d}^4 \frac{(Gm)^{9/2} N^{5/2}}{c^8 R^{11/2}} \\ &\cong 2 \cdot 10^{-18}\ {\rm Gyr}^{-1} \\ &\quad \times \left(\frac{\hat{d}}{100} \right)^4 \left(\frac{m}{10\,{\rm M}_\odot} \right)^{\frac{9}{2}} \left(\frac{N}{10^6} \right)^{\frac{5}{2}} \left(\frac{R}{0.1\,{\rm pc}} \right)^{-\frac{11}{2}}. \end{aligned} \quad (4)$$

This equation makes it clear that the probability of even just one triple encounter in a relativistic regime (small value of $\hat{d}$) is extremely low, even if one manages to pack a million stellar BHs within a sphere with a radius of 0.1 pc.

So far, we have considered only a self-gravitating cluster of compact objects (COs), such as the stellar BHs, that have accumulated at the centre of a globular cluster as a result of mass segregation (Baumgardt et al. 2003a,b; Baumgardt, Makino & Ebisuzaki 2004). However, a more promising environment for the accumulation of a large number of COs in a small volume is the centre of a galactic nucleus. There the gravitational force is likely dominated by a massive BH with a mass $M_{\rm MBH} \gtrsim 10^5\,{\rm M}_\odot$ (Kormendy & Richstone 1995; Magorrian et al. 1998; Gültekin et al. 2009). In that case, $\sigma \cong \sqrt{GM_{\rm MBH}/R}$, and equation 4 must be modified with a factor $\sqrt{M/M_{\rm MBH}}$. This shows that relativistic triple interactions are even less probable within COs orbiting MBHs than in a self-gravitating cluster.

The lifetime of an isolated self-gravitating cluster is limited by two-body relaxation, i.e. the exchange of energy between stars during hyperbolic two-body encounters. Relaxation drives the overall expansion and evaporation of the cluster (see for instance Heggie & Hut 2003).[1] We set an optimistic upper bound for the lifetime of the cluster, $t_{\rm life} \lesssim 100\, t_{\rm rel}$. The relaxation time is approximately (Binney & Tremaine 2008)

$$t_{\rm rel} \cong \frac{N}{\ln(0.1\,N)} \frac{R}{\sigma}. \quad (5)$$

Hence, we can estimate the total number of relativistic three-body encounters over the lifetime of a self-gravitating cluster,

$$\begin{aligned} N_{3,\,\rm singles} &\cong \frac{t_{\rm life}}{t_{3,\,\rm tot}} \lesssim 2\pi \frac{100}{\ln(0.1\,N)} \hat{d}^4 \frac{G^4 m^4 N^3}{c^8 R^4} \\ &\cong 3 \cdot 10^{-18} \\ &\quad \times \left(\frac{\hat{d}}{100} \right)^4 \left(\frac{m}{10\,{\rm M}_\odot} \right)^4 \left(\frac{N}{10^6} \right)^3 \left(\frac{R}{0.1\,{\rm pc}} \right)^{-4}. \end{aligned} \quad (6)$$

3 THREE-BODY INTERACTIONS BETWEEN A BINARY AND A SINGLE BH

The possibility of a relativistic encounter between three single objects being ruled out, we turn to the possibility of achieving a three-body relativistic interaction through the encounter between a binary and a single object. Here the difficulty is that the binary itself needs to be already in a relativistic regime. This implies that it is emitting gravitational radiation at a high rate, hence his lifetime is necessarily very limited. Consider a binary of two BHs of semimajor axis a and masses m_1 and m_2 and a single BH of mass m_3 passing at a distance of d from the centre of mass of the binary.

Approximately, the binary will need a time $t_{\rm GW}$ to shrink its orbit due to the emission of GWs as estimated by Peters (1964),

$$t_{\rm GW} = \frac{5}{256} \frac{a^4 c^5}{G^3\, m_1 m_2\, (m_1 + m_2)} \quad (7)$$

[1] Energetically, the overall cluster expansion can occur only because a very small number of stars, at the centre of the cluster, are getting more and more bound together. A single tight binary can drive this expansion by releasing energy to passing-by objects until it is ejected from the cluster or merges. In the next section, we consider whether relativistic encounters can occur during interaction between a binary and a single object.

Let us now estimate the time-scale t_3 for the system to interact with a third BH that flies by at a distance of $\leq d$ of the centre-of-mass of the binary. If the mass of the third BH is m_3 and the relative velocity to the binary $V_{\rm rel}$, the cross-section taking gravitational focusing into account is

$$S = \pi d^2 \left[1 + \frac{G(m_1+m_2+m_3)}{V_{\rm rel}^2\, d}\right] \cong \pi d \frac{G(m_1+m_2+m_3)}{V_{\rm rel}^2} \tag{8}$$

Therefore, the time-scale is

$$t_3 = \frac{1}{n V_{\rm rel} S} \cong \frac{V_{\rm rel}}{n\pi d G(m_1+m_2+m_3)}. \tag{9}$$

We assume that the three objects have a similar mass m; i.e. $m_1 \approx m_2 \approx m_3 = m$. Hence,

$$t_{\rm GW} \cong \frac{5}{512}\frac{c^5 a^4}{G^3 m^3}, \quad t_3 \cong \frac{V_{\rm rel}}{3\pi n d G m}. \tag{10}$$

We again measure distances in units of GM/c^2, with $\hat{a} = a/(Gm/c^2)$ and $\hat{d} = d/(Gm/c^2)$. Typical interesting values are $\hat{a} \approx$ a few and $\hat{d} \approx$ fewtens. Therefore,

$$t_{\rm GW} \cong \frac{1}{100}\frac{Gm}{c^3}\hat{a}^4 \tag{11}$$

and

$$t_3 \cong \frac{1}{10}\frac{V_{\rm rel}c^2}{n\hat{d}(Gm)^2}. \tag{12}$$

We then obtain the ratio of the two timescales,

$$\begin{aligned}\frac{t_{\rm GW}}{t_3} &\cong \frac{1}{10}\frac{G^3 m^3 n \hat{d}\hat{a}^4}{c^5 V_{\rm rel}} \\ &\cong 3\cdot 10^{-14} \\ &\quad\times \left(\frac{V_{\rm rel}}{10\,{\rm km\,s^{-1}}}\right)^{-1}\left(\frac{\hat{a}^4\hat{d}}{(100)^5}\right)\left(\frac{m}{10\,{\rm M_\odot}}\right)^3\left(\frac{n}{10^{10}\,{\rm pc^{-3}}}\right).\end{aligned} \tag{13}$$

For a self-gravitating cluster, we can rewrite this in terms of N and R, using $V_{\rm rel} \approx \sigma$,

$$\begin{aligned}\frac{t_{\rm GW}}{t_3} &\cong \frac{1}{10}\frac{(Gm)^{5/2}\hat{d}\hat{a}^4 N^{1/2}}{c^5 R^{5/2}} \\ &\cong 5\cdot 10^{-17} \\ &\quad\times \left(\frac{\hat{a}^4\hat{d}}{(100)^5}\right)\left(\frac{m}{10\,{\rm M_\odot}}\right)^{\frac{5}{2}}\left(\frac{N}{10^6}\right)^{\frac{1}{2}}\left(\frac{R}{0.1\,{\rm pc}}\right)^{-\frac{5}{2}}.\end{aligned} \tag{14}$$

This quantity can be interpreted as the probability that a relativistic binary has a close encounter with a single object before it merges. The total number of mergers that can occur in the evolution of a cluster of N objects is $N_{\rm merg} \leq N$, with $N_{\rm merg} = N$ only possible if all objects merge together (a scenario to form a MBH from a cluster of stellar BHs, see Lee 1993; Kupi, Amaro-Seoane & Spurzem 2006). Hence, the total number of relativistic single-binary interactions over the lifetime of a cluster (evolving through a succession of binary mergers) is

$$\begin{aligned}N_{3,\,\rm bin} &\lesssim N\frac{t_{\rm GW}}{t_3} \cong 5\cdot 10^{-11} \\ &\quad\times \left(\frac{\hat{a}^4\hat{d}}{(100)^5}\right)\left(\frac{m}{10\,{\rm M_\odot}}\right)^{\frac{5}{2}}\left(\frac{N}{10^6}\right)^{\frac{3}{2}}\left(\frac{R}{0.1\,{\rm pc}}\right)^{-\frac{5}{2}}.\end{aligned} \tag{15}$$

4 DISCUSSION

In this work we have addressed the formation of systems of three BHs in the strong gravity regime. For that we have first studied the probability that three BHs interact in a dense stellar cluster and we conclude that it is totally negligible. We have then addressed the possibility that a binary of BHs which has previously formed in the cluster interacts relativistically with a third BH. We judge that the stellar system harbouring the BHs needs to have unachievable densities.

Based on simple physical arguments, we have established that the time scales for a triple relativistic encounter to occur in a cluster of stellar-mass COs is extremely long. We recall that we consider two extreme cases. In the first situation, we neglect the existence (and formation) of binaries. Hence, we consider that three single objects have to find themselves, by chance, within a few (tens of) Schwarzschild radii of each other. In this case, we have admitted that the cluster cannot survive (with a high stellar density) for more than about 100 relaxation time. Indeed, in such a long time-scale, most of the cluster would expand to lower and lower densities and a very significant fraction of the object would escape, with only a very small number of objects getting closer and closer to provide energy for this evolution (Heggie & Hut 2003). The second (much idealized) case is that of a cluster made mostly of binaries. In that case, one can hope that it would suffice for an object to interact closely with a binary but in order for the interaction to be relativistic for the three objects, the binary must be so tight that its lifetime is limited by emission of GWs. Accordingly, we consider that the lifetime of the whole cluster is limited by the successive merger of binaries. Any real cluster would present a situation which is somewhat in between these two extremes. In particular, the evolution of a cluster made of single objects would naturally lead to the formation of binaries during core collapse (see e.g. Heggie & Hut 2003). We stress that we have made a large number of simplifications in our estimates but always in such a way as to overestimate the rate of triple relativistic events. For instance, we have assumed that all the binaries in a cluster are relativistic. This limits their lifetime but non-relativistic binaries are useless for our purpose.

With Fig. 1, we can estimate what conditions are required for at least one such encounter to take place during the lifetime of the cluster. For this figure, we have assumed $m = 10\,{\rm M_\odot}$ and $\hat{d} = \hat{a} = 10^4$. The latter values correspond to encounters that are only weakly relativistic. Even with such values, the figure shows that most of the parameter space for $N_{3,\,\rm single} \gtrsim 1$ or $N_{3,\,\rm bin} \gtrsim 1$ is excluded, either because the cluster, as a whole, would be a massive BH (for large N and small R) or because the cluster would have such a small two-body relaxation time that it wouldn't have time to form (for smaller N and small R). Indeed, the evolution of massive stars into COs (neutron stars or BHs) requires at least 3 Myr. Because of these constraints, it appears that clusters hosting triple relativistic encounters have to be made of at least 10^8 COs concentrated within a region smaller than 10^{-3} pc. In fact, for $N \approx 10^8$, the size has to be of order 10^{-4} pc. The corresponding number density is comprised between 10^{17} and 10^{19} pc^{-3}.

Such values are beyond observed ones by many orders of magnitude. For instance, the stellar density in Galactic globular clusters is, at most, of the order of 10^5 pc^{-3} (see Harris 1996, and the 2003 on-line update http://www.physics.mcmaster.ca/ harris/mwgc.dat). Already the necessary number of COs is much larger than what one can expect in the kind of clusters that exist. A globular cluster might contain up to 10^7 stars but only a very small fraction of them

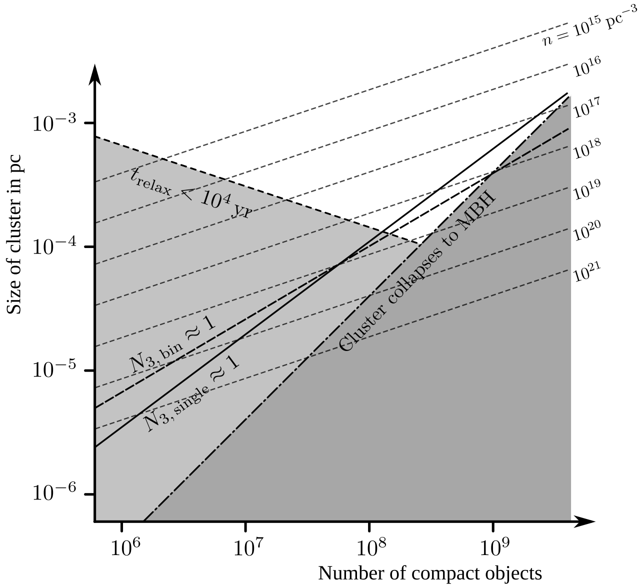

Figure 1. Parameter space for the occurrence of triple relativistic encounters. For this figure, we consider a self-gravitating cluster made of a given number of identical CO with an individual mass of $10\,\mathrm{M}_\odot$. The line labeled $N_{3,\,\rm bin} \approx 1$ is based on equation 15 with $\hat{d} = \hat{a} = 10^4$. Below this line, in our simple estimate, at least one binary–single encounter in which the three objects are relativistic should occur during the lifetime of the cluster (which is limited by successive mergers, see text). The line labeled $N_{3,\,\rm single} \approx 1$ is based on equation 6 with $\hat{d} = 10^4$. Below this line, one can expect at least one triple relativistic encounter between single objects to occur during the lifetime of the cluster (limited by two-body relaxational processes). The large grey area in the bottom-right section is excluded from the parameter space because the whole cluster would be smaller than its Schwarzschild radius, i.e. it would immediately collapse into a single massive BH. The other grey region correspond to clusters in which the two-body relaxation time would be shorter than 10^4 yr. Such a cluster would probably not last long enough for stars to form and turn into COs. The dashed diagonal lines in grey indicate an estimate of the number density in the cluster, from the relation $n \approx N/R^3$.

would become BHs. This number fraction f depends on the initial mass function (IMF) of high mass stars. Some galactic nuclei seem to be top-heavy (Maness et al. 2007), so that in principle f ranges between 10^{-3} and 10^{-2}, yielding, at most, $N = 10^5$ BHs in very large globular cluster.

A galaxy contains a much larger number of COs but most of them inhabit regions of very low density. BHs born in the central regions are likely to accumulate at the centre, in the galactic nucleus, through the process of mass segregation, which is basically an effect of dynamical friction. But, in the case of our Galaxy, dynamical friction is only effective, over a Hubble time, within a few parsecs of the centre. Hence, at most of order 10^4 BHs may have gathered within the innermost 0.3 pc (Freitag, Amaro-Seoane & Kalogera 2006; Amaro-Seoane, Freitag & Preto, in preparation). A dark mass concentration weighing $4 \times 10^6\,\mathrm{M}_\odot$ has been detected at the Galactic centre through the analysis of the orbits of bright IR stars, the so-called S- or SO-stars, around the weak radio source Sgr A* (Eisenhauer et al. 2005; Ghez et al. 2005, 2008; Gillessen et al. 2009; Genzel, Eisenhauer & Gillessen 2010). It is generally assumed that it is a massive BH although the only strict constraint on its size is that it has to fit within the periapse of the IR stars' orbits, the tightest of which is that of S-2 (SO-2), imposing $R \lesssim 5 \times 10^{-4}$ pc. If we assume that this object is actually a cluster of $10\,\mathrm{M}_\odot$ BHs, with $N \approx 4 \times 10^5$, it would need to be so compact in order to host triple relativistic encounters that its relaxation time would be extremely short, making its existence at the present time an extraordinary coincidence (See also Maoz 1998; Miller 2006). Furthermore, no mechanism to form such a dense cluster is known.

While we have limited the analysis to a system of three BHs, it is nevertheless obvious that for more BHs the event rates are much more unlikely. We therefore conclude that the waveforms to develop need only to include two BHs for the searches of GWs in the data streams.

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