Coherent follow-up of Continuous Gravitational-Wave candidates: minimal required observation time

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Abstract. We derive two different methods to compute the minimal required integration time of a fully coherent follow-up of candidates produced in wide parameter space semi-coherent searches, such as global correlation StackSlide searches using Einstein@Home. We numerically compare these methods in terms of integration duration and computing cost. In a Monte Carlo study we confirm that we can achieve the required detection probability.

1. Introduction

Isolated neutron stars as potential sources of continuous gravitational waves are optimally studied with fully coherent matched filtering methods. These methods are not directly applicable to previously unknown objects due to the large parameter space that needs to be covered in all-sky wide parameter space searches and the related enormous computing cost [1]. Advanced semi-coherent techniques, e.g. StackSlide searches on the distributed computing environment Einstein@Home [2], produce candidates that require follow-up in greatly reduced parameter space regions. A follow-up scheme consists of two basic stages. In the first *refinement* stage, we find the maximum-likelihood estimator and associated optimal search volume V_0. In the second *zoom* stage, we zoom in on the optimal search volume by semi-coherent or fully-coherent integration. In this paper we focus on a fully-coherent zoom for which we derive and discuss two different methods to compute the minimal required coherent integration time in order to distinguish real signals from noise.

2. Properties of $\mathcal{F}$-statistic searches

The $\mathcal{F}$-statistic was first derived in [3] for the single detector case and generalized to multi-detector searches in [4]. Continuous gravitational-wave signals are monochromatic and sinusoidal in the frame of the gravitational-wave source and undergo phase- and amplitude modulation due to the rotation and orbital motion of the detector. The $\mathcal{F}$-statistic is analytically amplitude-maximized, thus the parameter space to search for signals is spanned by the remaining "Doppler parameters" λ, namely sky position (α - right ascension, δ - declination) and intrinsic frequency and frequency derivatives ($f, \dot{f}, \ddot{f}...$), further referred to as spindowns. Searching for previously unknown objects with matched filtering implies computing matched filters for different points in parameter space, also referred to as templates. As realized in [5, 6] in the context of searches for

gravitational waves from inspiraling binaries, a geometrical approach is best suited for optimal template placement and template counting. This is made possible by the introduction of a metric tensor g_{ij} on the parameter space and mismatch m

$$m = g_{ij}\Delta\lambda^i\Delta\lambda^j + \mathcal{O}(\Delta\lambda^3) \ , \tag{1}$$

where the mismatch m measures the fractional loss of (squared) signal to noise ratio (SNR) ρ^2 due to the usage of a nearby template λ_c with offset $\Delta\lambda = \lambda_c - \lambda_s$ from the true parameters of a putative signal λ_s

$$m = \frac{\rho_s^2 - \rho_c^2}{\rho_s^2} \ , \tag{2}$$

with the squared SNR ρ_s^2 and ρ_c^2 obtained at point λ_s and λ_c, respectively. Given the metric, the problem of efficient lattice and alternative random and stochastic template-bank construction is studied in [7, 8, 9].

2.1. Fully-coherent search
A fully-coherent search is the classical and most sensitive $\mathcal{F}$-statistic-based search in the case of unlimited available computing power or a sufficiently cheap computing cost requirement. The squared SNR ρ^2 scales linearly with the observation time T, according to the following formula:

$$\rho^2 = h_0^2 R N_{\mathrm{d}} T S^{-1}(f) \ , \tag{3}$$

where h_0 is the intrinsic signal amplitude, R represents the geometrical "detector response" , S is the one-sided noise spectral density, which is assumed constant in a narrow frequency band around f, and N_{d} is the number of detectors [10]. In the presence of a signal, the $\mathcal{F}$-statistic follows a non-central χ^2 distribution with four degrees of freedom and non-centrality parameter ρ^2. Thus the expectation value is

$$E[2\mathcal{F}_S] = 4 + \rho^2 \ , \tag{4}$$

with standard deviation

$$\sigma(2\mathcal{F}_S) = \sqrt{2(4 + 2\rho^2)} \ . \tag{5}$$

2.2. Semi-coherent search
At fixed and limited computing cost a more sensitive detection statistic can be constructed from the incoherent combination of results obtained by coherent integration of shorter data segments. In particular we consider a Stack-Slide search [11, 12, 13], where the statistic is the sum of the $\mathcal{F}$-statistic over the segments:

$$\Sigma = \sum_{k=1}^{N} 2\mathcal{F}_k(\lambda). \tag{6}$$

This new statistic Σ follows a non-central χ^2 distribution with $4N$ degrees of freedom, thus the expectation value is

$$E[\Sigma] = 4N + \rho_\Sigma^2 \ , \tag{7}$$

where the non-centrality parameter is the sum of the squared SNRs over different segments

$$\rho_\Sigma^2 = \sum_{k=1}^{N} \rho_k^2 \ . \tag{8}$$

A trivial but useful reformulation of Eq. (7) is in terms of average $2\bar{\mathcal{F}} = \frac{1}{N}\sum_k 2\mathcal{F}_k$ and $\bar{\rho}^2 = \frac{1}{N}\sum_k \rho_k^2$, namely

$$E[2\bar{\mathcal{F}}] = 4 + \bar{\rho}^2 \ . \tag{9}$$

2.3. Template counting

The number of templates sufficient to cover the search volume V_0 is given by [7]

$$\mathcal{N}_n = \theta m^{-n/2} \mathcal{V}_n \ , \tag{10}$$

where θ is the normalized thickness characterizing the geometric structure of covering, m is the maximum allowed mismatch , n the number of dimensions and

$$\mathcal{V}_n = \int d^n \lambda \ \sqrt{\det g} \ , \tag{11}$$

is the metric template-bank volume with g_{ij} the parameter space metric. This is the general form of the template counting formula, which is valid for arbitrary lattices and curved parameter spaces. In practice, using the flat metric approximation, where the metric coefficients are constant, we can take the determinant out of the integral. Moreover, if the parameter space is a n-dimensional "box", we can replace the integral over infinitesimal displacement $d\lambda$ by a product of n "search bands" $\Delta\lambda$, namely

$$\mathcal{V}_n = \sqrt{\det g} \prod_{i=1}^{n} \Delta\lambda_i \ . \tag{12}$$

Follow-up of candidates from semi-coherent searches involves a semi-coherent metric, shown in [11, 14] to be the average of the metric computed for every segment. The semi-coherent metric allows us to estimate the search band $\Delta\lambda_i$ around the follow-up candidate using the diagonal elements of the inverse Fisher matrix [15, 16], i.e.

$$\Delta\lambda_i \equiv \kappa \sqrt{\bar{\Gamma}^{ii}} \ , \tag{13}$$

with

$$\bar{\Gamma}^{ii} = \bar{g}^{ii}/\rho^2 \ , \tag{14}$$

where κ defines the confidence level and g^{ij} is the inverse matrix to g_{ij}. In the present work we use an analytical semi-coherent metric first derived by Pletsch [14]. For coherent integration time longer than a day, but much shorter than a year, the number of sky templates at fixed frequency f converges to

$$\mathcal{N}_{sky} = \frac{2\pi^3 \tau_{\mathrm{E}}^2 f^2}{m} \ , \tag{15}$$

where $\tau_{\mathrm{E}} \approx 21 \times 10^{-3}s$ is the light travel time from the Earth's center to the detector [14]. The semi-coherent parameter space is finer than the coherent one by a refinement factor γ. Using the notion of *refinement per direction* γ_n we can also obtain the search bands from the extents of the fully coherent metric, namely

$$\Delta\lambda_i = \kappa \sqrt{\frac{g^{ii}}{\gamma_i^2 \rho^2}} \ . \tag{16}$$

For uniformly distributed segments of data without gaps, based on [14] the refinement factors can be obtained as

$$\gamma_f \;=\; 1 \ , \tag{17}$$

$$\gamma_{\dot{f}} \;=\; \sqrt{5N^2 - 4} \ , \tag{18}$$

$$\gamma_{\ddot{f}} \;=\; \sqrt{(35N^4 - 140N^2 + 108)/3} \ , \tag{19}$$

$$\gamma_{\dddot{f}} \;=\; \sqrt{(105N^8 - 1260N^6 + 5012N^4 - 6160N^2 + 2304)/(5N^2 - 4)} \ . \tag{20}$$

Finally, for simplicity of the template-bank construction, we use a hyper-cubic lattice to place templates, though hyper-cubic lattices are in general suboptimal, compared to better solutions, e.g. A_n^* lattice. The normalized thickness for an n-dimensional hyper-cubic grid is [7]

$$\theta_n = n^{n/2}\, 2^{-n} \ . \tag{21}$$

The proper choice of the number of dimensions that maximizes the number of templates [1, 11, 12, 13] $\mathcal{N}$ is:

$$\mathcal{N} = \max_n \mathcal{N}_n \ . \tag{22}$$

2.4. Computing cost
In the follow-up of real candidates, especially weak signal candidates, along with the constraint of the total amount of available data, the computing cost constraint may limit significantly the feasibility of the search. Thus the computing-cost requirement is of particular interest. There are currently two different strategies to implement an $\mathcal{F}$-statistic search code in LIGO's reference software suite `lalsuite`[17], namely the SFT-method based on short Fourier transforms of the data with duration T_{SFT} [10] and the FFT-method based on barycentric resampling [18]. Regarding the computational cost, the FFT method is preferable, as the computational requirement to calculate the $\mathcal{F}$-statistic, for a single point in the parameter space, scales only with $\log T$, while the cost of the SFT algorithm scales with T. However, for historical reasons the SFT method is currently still more often used by LIGO/LSC [19, 20, 21], is well tested and we can use recent timing information. The computing cost of a SFT-based $\mathcal{F}$-statistic search is

$$C = \mathcal{N} c_0 N_{\mathrm{SFT}} \ , \tag{23}$$

where N_{SFT} is the number of used SFTs, namely

$$N_{\mathrm{SFT}} = N_{\mathrm{d}} T / T_{\mathrm{SFT}} \tag{24}$$

and c_0 is the fundamental implementation- and hardware-specific computing constant per SFT and template.

3. Minimal required observation time
The main scope of the present work is to find the minimal required observation time that guarantees a certain detection probability of a putative signal buried deep in the detector noise at a certain confidence level by using the fully-coherent $\mathcal{F}$-statistic search technique. We consider two different methods to compute the required integration duration. In method 1, which is closely related to hypothesis testing, we use the concept of false-alarm and false-dismissal probability to achieve certain detection probability. This is the natural way to compute the required integration time. In method 2 we alternatively use the more intuitive notion of expectation value to find the observation duration that guarantees the required detection probability.

3.1. Method 1
In absence of a signal, the probability density function of the $\mathcal{F}$-statistic reduces to a central χ^2-distribution, and the false-alarm probability is given by

$$p_{\mathrm{fA}}^1 = \int_{2\mathcal{F}_{\mathrm{th}}}^{\infty} d(2\mathcal{F}) \chi_4^2(2\mathcal{F}; 0) \ , \tag{25}$$

where p_{fA}^1 denotes single trial false-alarm probability and $\chi_4^2(2\mathcal{F};0)$ is the central χ^2-distribution with 4 degrees of freedom. The integration of $\chi_4^2(2\mathcal{F},0) = \frac{1}{2}\mathcal{F}e^{-\mathcal{F}}$ yields

$$p_{\mathrm{fA}}^1 = (1 + \mathcal{F}_{\mathrm{th}})e^{-\mathcal{F}_{\mathrm{th}}} \ . \tag{26}$$

The overall false-alarm probability of crossing the threshold $2\mathcal{F}_{\mathrm{th}}$ in $\mathcal{N}$ trials is

$$p_{\mathrm{fA}} = 1 - (1 - p_{\mathrm{fA}}^1)^{\mathcal{N}} \approx p_{\mathrm{fA}}^1 \mathcal{N}, \tag{27}$$

when $p_{\mathrm{fA}}^1 \mathcal{N} \ll 1$ [3, 22], thus

$$p_{\mathrm{fA}}^1 = p_{\mathrm{fA}}/\mathcal{N} \ . \tag{28}$$

We cannot solve Eq. (26) analytically, but numerical solution gives a threshold $2\mathcal{F}_{\mathrm{th}}$ value. This allows us to numerically integrate the false-dismissal probability

$$p_{\mathrm{fD}}(2\mathcal{F}_{\mathrm{th}}, \rho^2) = \int_{-\infty}^{2\mathcal{F}_{\mathrm{th}}} (d2\mathcal{F})\chi_4^2(2\mathcal{F}, \rho^2) \ , \tag{29}$$

where $p_{\mathrm{fD}}(2\mathcal{F}_{\mathrm{th}}) = 1 - p_{\mathrm{det}}$, with the desired detection probability p_{det} and $\chi_4^2(2\mathcal{F}, \rho^2)$ is the non-central χ^2-distribution with 4 degrees of freedom and non-centrality parameter ρ^2. At fixed p_{fA}^* and p_{fD}^*, using the above equation, we can compute a threshold SNR $\rho_{\mathrm{th}}(p_{\mathrm{fA}}^*, p_{\mathrm{fD}}^*)$. The required T is such that the inequality

$$\rho_{\mathrm{ac}}^2(T) \geq \rho_{\mathrm{th}}^2(p_{\mathrm{fA}}^*, p_{\mathrm{fD}}^*) \tag{30}$$

holds, where $\rho_{\mathrm{ac}}^2(T)$ is the accumulated SNR due to the presence of signal in the analyzed data. Assuming that the follow-up search will use data of similar constant noise floor, we can rewrite Eq. (3) as

$$\rho_{\mathrm{ac}}^2(T) = \rho_{\mathrm{c}}^2 \frac{N_{\mathrm{d}} T}{N_{\mathrm{d}}^c \Delta T} \ , \tag{31}$$

where ΔT is the length of one segment in the semi-coherent search using data from N_{d}^c number of detectors. With the average $2\bar{\mathcal{F}}_c$ value of the candidate, we can compute its SNR ρ_{c} from Eq. (9), namely

$$\rho_{\mathrm{c}}^2 = E[2\bar{\mathcal{F}}_{\mathrm{c}}] - 4 \ . \tag{32}$$

Substitution in the equations above yields the accumulated SNR in presence of signal

$$\rho_{\mathrm{ac}}^2 = \left(E[2\bar{\mathcal{F}}_{\mathrm{c}}] - 4\right) \frac{N_{\mathrm{d}} T}{N_{\mathrm{d}}^c \Delta T} \ , \tag{33}$$

which gives the required minimal T.

3.2. Method 2
Computation of the $\mathcal{F}$-statistic on data with no signal, has a certain expectation value, therefore we ask what is the expected maximal $2\mathcal{F}$ value $E[2\mathcal{F}_{\mathcal{N}}]$ in $\mathcal{N}$ trials in Gaussian noise, where $\mathcal{F}_{\mathcal{N}} \equiv \max\{\mathcal{F}\}_{i=1}^{\mathcal{N}}$. The probability to get $(\mathcal{N}-1)$ values of $2\mathcal{F}$ less than $2\mathcal{F}_{\mathcal{N}}$ follows a binomial distribution, namely

$$p_{\mathcal{N}}(2\mathcal{F}_{\mathcal{N}}) = \binom{\mathcal{N}}{1}\chi_4^2(2\mathcal{F}, 0)(1 - \alpha_1)^{\mathcal{N}-1} \tag{34}$$

$$= \frac{1}{2}\mathcal{N}\mathcal{F}_{\mathcal{N}}e^{-\mathcal{F}_{\mathcal{N}}}\left(1 - (1 + \mathcal{F}_{\mathcal{N}})e^{-\mathcal{F}_{\mathcal{N}}}\right)^{\mathcal{N}-1} \ . \tag{35}$$

With this we can numerically integrate the expectation value

$$E[2\mathcal{F}_\mathcal{N}] = \int_0^\infty d(2\mathcal{F}_\mathcal{N})\, 2\mathcal{F}_\mathcal{N}\, p_\mathcal{N}(2\mathcal{F}_\mathcal{N}) \ , \tag{36}$$

and standard deviation

$$\sigma_\mathcal{N}(2\mathcal{F}_\mathcal{N}) = \left(\int_0^\infty d(2\mathcal{F}_\mathcal{N})\, (2\mathcal{F}_\mathcal{N} - E[2\mathcal{F}_\mathcal{N}])^2\, p_\mathcal{N}(2\mathcal{F}_\mathcal{N}) \right)^{1/2} . \tag{37}$$

To safely distinguish a real signal from pure noise, we can require the following inequality to hold:

$$E[2\mathcal{F}_S] - h\sigma_S(2\mathcal{F}_S) > E[2\mathcal{F}_\mathcal{N}] + h\sigma_\mathcal{N}(2\mathcal{F}_\mathcal{N}) \ , \tag{38}$$

where the expectation value $E[2\mathcal{F}_S]$ of a real signal and its standard deviation $\sigma_S(2\mathcal{F}_S)$ are computed using Eqs. (4) and (5). As all terms in inequality (38) are function of the observation time, this gives an alternative method to compute the minimal required integration time. Fine-tuning of Eq. (38) is possible through the safety parameter h, which we quantify by using Chebyshev's inequality. For a random variable X, with expected value $E[X]$ and standard deviation σ,

$$P(|X - E[X]| \geq h\sigma) \leq 1/h^2 \ , \tag{39}$$

which means that at least a fraction

$$p = 1 - 1/h^2 \tag{40}$$

of the data is within h standard deviations on either side of the mean [23]. Rearranging the above equation yields

$$h = 1/\sqrt{1-p} \ . \tag{41}$$

Having two independent random variables, $2\mathcal{F}_S$ and $2\mathcal{F}_\mathcal{N}$, we can label the fraction of data around each mean as p_S and $p_\mathcal{N}$ and introduce the joint probability $p_J = p_S p_\mathcal{N}$. We see, that the same joint probability can be achieved for different combinations of p_S and $p_\mathcal{N}$. However, a natural choice is $p_S = p_\mathcal{N}$, thus

$$h = 1/\sqrt{1 - \sqrt{p_J}}. \tag{42}$$

We give a set of p_J values and related h in Table 1.

p_J	0.75	0.90	0.95	0.99
h	2.73	4.41	6.28	14.12

Table 1. Joint probability p_J and corresponding required h standard deviations.

Fixing p_J to some value and with this h in inequality (38), we can compute the minimal required coherent observation time T, such that (38) holds. For this integration time, the joint probability p_J becomes the separation probability $p_{\mathrm{sep}} = p_J$. This is the probability, that a candidate due to the presence of a signal is consistent with the signal hypothesis and a candidate due to the noise is consistent with the noise hypothesis. Taking into account that $p_S = 1 - p_{\mathrm{fD}}$ and $p_\mathcal{N} = 1 - p_{\mathrm{fA}}$, we find the relation of the separation probability to the detection probability, namely $p_{\mathrm{sep}} = p_{\mathrm{det}}(1 - p_{\mathrm{fA}})$, or for negligible false-alarm $p_{\mathrm{det}} \approx p_{\mathrm{sep}}$.

4. Method comparison

4.1. Numerical predictions

In the following we compare the two methods to find the minimal required integration time described in the previous section in terms of observation duration and computing cost. We consider a StackSlide search with $N = 205$ segments of duration $\Delta T = 25$ hours, each using data from $N_{\mathrm{d}}^{\mathrm{c}} = 2$ detectors. For a hypothetical candidate with fixed Doppler parameters $\alpha = 1.45$ rad, $\delta = 0$ rad $f = 185$ Hz, $\dot{f} = -1 \times 10^{-9}$ Hz/s, we pick an average strength in the range $2\bar{\mathcal{F}}_c \in [5, 13]$. Then using Eq. (13) with $\kappa = 1$ and the semi-coherent metric we compute the search bands associated with such a candidate. Having that, for mismatch $m = 0.01$ and a hyper-cubic lattice, we can compute the number of templates using Eq. (10) and the fully-coherent metric. Using method 1, requiring detection probability $p_{\mathrm{det}}^* = 0.9$ at overall false-alarm probability $p_{\mathrm{fA}}^* = 0.01$ using Eq. (29) we compute $\rho_{\mathrm{th}}^2(p_{\mathrm{fA}}^*, p_{\mathrm{fD}}^*)$ and the minimal required observation time T_1, which substituted in Eq. (33) with $N_{\mathrm{d}} = N_{\mathrm{d}}^{\mathrm{c}}$ satisfies Eq. (30). For method 2 a separation probability equal to p_{det}^* yields safety factor $h = 4.41$, see Table 1. We label the integration time that satisfies Eq. (38) as T_2 and plot both integration times $T_1(2\bar{\mathcal{F}}_c)$ and $T_2(2\bar{\mathcal{F}}_c)$ in Figure 1 (a) as function of $2\bar{\mathcal{F}}_c$. With the number of templates for T_1 and T_2 we estimate the computing cost C_1 and C_2 using the fundamental computing cost constant $c_0 = 7 \times 10^{-8}s$ in Eq. (24) and assuming SFTs of duration $T_{\mathrm{SFT}} = 1800$ s in Eq. (25). $C_1(2\bar{\mathcal{F}}_c)$ and $C_2(2\bar{\mathcal{F}}_c)$ are plotted in Figure 1 (b). In Figure 1 (c) we plot how the expectation value from a real signal grows with increasing T compared to loudest candidate from Gaussian noise. In this plot the candidate strength is fixed to $2\bar{\mathcal{F}}_c = 8.5$.

We see that method 2 yields much longer observation time, at same candidate strength compared to method 1. Due to the resulting much larger number of templates, the computing cost, especially for weak candidates, is much higher. The inferiority of method 2 compared to method 1 in terms of required integration duration and computing power can be explained by the *ad hoc* construction of method 2 and the use of Chebyshev's inequality, which is only a lower bound. In this sense method 2 is a more conservative approach, though the important information about false-alarm and false-dismissal probability gets lost in this framework. The computing cost of method 1 looks very promising even for weak candidates, however we should keep in mind that this is lower limit and the cost of a search with real data would most likely be much higher. The reason for this is that gaps in the data are direct penalty for the growth of ρ_{ac}^2, while ρ_{th}^2 remains unaffected. Furthermore, for very weak signals, the required integration duration may violate the assumption of constant sky resolution, thus we would underestimate the number of templates, resulting in a higher false-dismissal.

4.2. Monte Carlo results

To confirm the numerical predictions of method 1 we perform the following Monte Carlo studies. We create a set of 205 segments with duration 25 hours of Gaussian noise and draw a set of pulsar parameters $\alpha \in (0, 2\pi)$, $\delta \in (-\pi/2, \pi/2)$, $\cos \iota \in (-1, 1)$, $\psi \in (0, 2\pi)$, $\phi_0 \in (0, 2\pi)$ at fixed frequency of $f = 185$ Hz and spindown value in the range $\dot{f} \in (-f/\tau, 0)$, where $\tau = 2220$ yr is the minimal spindown age of the source [1]. We inject a signal with the above parameters and intrinsic signal amplitude h_0 high enough to produce a candidate with expected average strength $E[2\bar{\mathcal{F}}_S] \in [12, 13]$. To find the actual injected value we first do a targeted StackSlide search at the point of the injection. With this measured injected $2\bar{\mathcal{F}}_S$ value, using Eq. (13) we compute Fisher extents, from which we draw a random parameter point λ_c satisfying

$$\bar{\Gamma}_{ij}\Delta\lambda^i\Delta\lambda^j < 1 \ . \tag{43}$$

The point λ_c is within the 1-σ Fisher ellipsoid of the true signal location and becomes the candidate to follow up. Following the scheme for method 1 as described above, we compute
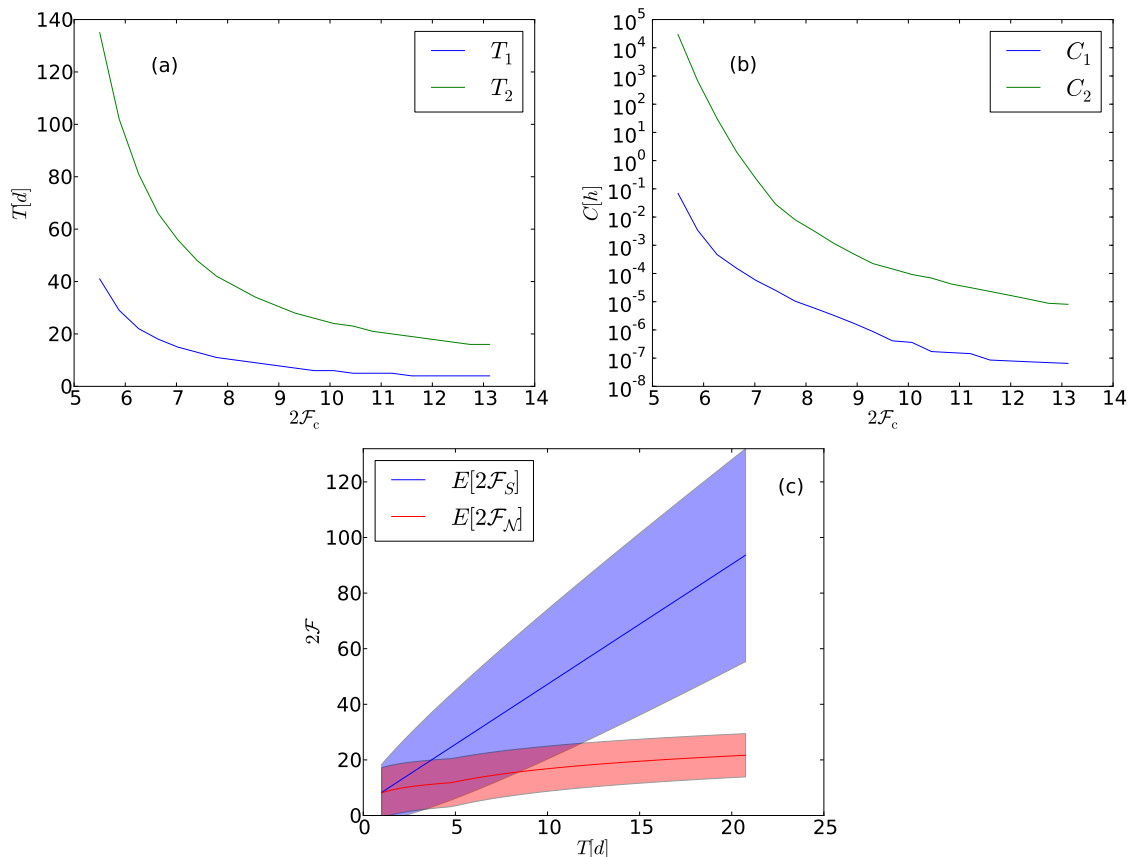
Figure 1. Numerical comparison between method 1 and method 2 (quantities labeled with 1 and 2, respectively). Figure (a) shows the required coherent integration time as function of the strength of the candidate, (b) shows the computing cost depending on the strength of the candidate, (c) shows expected value of signal, noise and related $h = 4.41$ standard deviations for detection probability $p_{\text{det}} = 0.9$ of a candidate with $2\bar{\mathcal{F}}_c = 8.5$.

the minimal required coherent observation time targeting detection probability $p^*_{\text{det}} = 0.9$ and search for the signal. After computation of $2\mathcal{F}_S$ using the data with the injected signal, we compute $2\mathcal{F}_{\mathcal{N}}$ with the same grid and integration duration using the noise only data. We claim "detection" whenever the loudest measured $2\mathcal{F}_S$ value in the data with injected signal is higher than the loudest measured $2\mathcal{F}_{\mathcal{N}}$ of the noise. The result of the Monte Carlo simulations is as follows: in 897, out of 1000 trials, the measured $2\mathcal{F}_S$ value in the data containing injected signal exceeds the measured $2\mathcal{F}_{\mathcal{N}}$ value of the noise only data. With this the achieved detection probability $p_{\text{det}} = 0.897 \pm 0.023$ is in accordance with the targeted detection probability p^*_{det}.

5. Discussion

We derived two different methods to compute the minimal required coherent integration time in a fully-coherent $\mathcal{F}$-statistic search in the *zoom* stage of follow-up of candidates from a semi-coherent StackSlide search. By numerical comparison we showed that method 1 is superior to method 2 in terms of required integration duration and computing cost. We confirmed in a Monte Carlo study that the predicted coherent integration time is sufficient to achieve the desired detection probability. The results of this paper have been derived for Gaussian data without gaps and two detectors of equal noise floor. Further extension of this work is closely

related to the data selection problem.

Acknowledgments

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