

Initial data for black hole collisions

Sergio Dain¹

¹*Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut*

I describe the construction of initial data for the Einstein vacuum equations that can represent a collision of two black holes. I stress in the main physical ideas.

The physical system we want to describe is a binary system of two black holes. It is expected that lot of such systems exist in the universe. Moreover, these systems are expected to have the following two properties. First, gravitational radiation will be emitted by them and the new detectors will be able to measure it. Second, these systems are completely described by the Einstein field equations. By this I mean that there exist solutions of the Einstein equations that can reproduce the gravitational wave forms emitted by them. The ultimate goal is to find these solutions and compare the waves forms with the result of the measurements.

One method for constructing an appropriate class of solutions of the Einstein equation is through an initial value formulation: we give appropriate data and then we evolved them numerically. It is not obvious that an initial value formulation is the most appropriate way to construct a solution for an astrophysical problem because we can not prepare the initial conditions of an astrophysical system in the laboratory. It is, in principle, impossible to find the exact initial data for a real astrophysical system. The whole program will succeed if we are able to find some properties of the binary system that do not depend very much on fine details of the initial data. In this sense black holes are perhaps better candidates than ordinary stars, since they are much simpler, they do not involve the matter equations. For example, stationary black holes are expected to be characterized completely only by two parameters: mass and spin; on the other hand stationary stars can have a complicated multipolar structure.

A black hole is a region of no escape which does not extend out to infinity. In the standard text books [6] [9] the definition of a black hole is made using the conformal compactification of the space time. If the space time admit a suitable conformal boundary at null infinity and the causal past of this boundary is globally hyperbolic, then the black hole region is defined as the difference between the space time and the causal past of null infinity. The boundary of the black hole region is a null surface called the event horizon.

I want to discuss the problem of finding initial data for black holes. An *initial data set* for the Einstein vacuum equations is given by a triple (S, h_{ab}, K_{ab}) where S

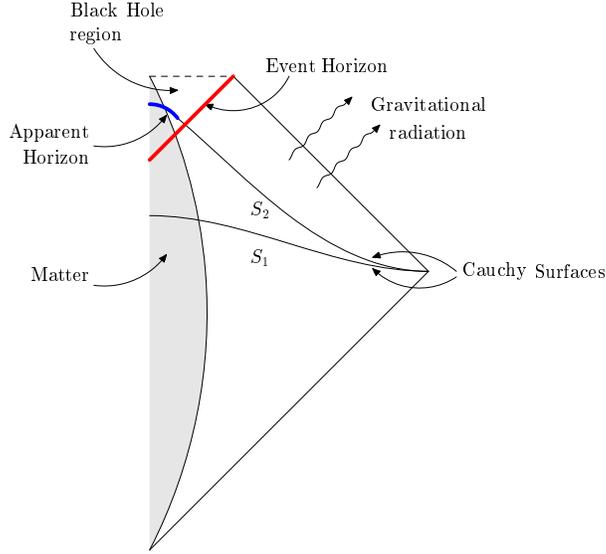


Figure 1: Penrose diagram of a space time with a black hole.

is a connected 3-dimensional manifold, h_{ab} a (positive definite) Riemannian metric, and K_{ab} a symmetric tensor field on S . They satisfy the vacuum constraint equations

$$D^b K_{ab} - D_a K = 0, \quad (1)$$

$$R + K^2 - K_{ab} K^{ab} = 0, \quad (2)$$

on S , where D_a is the covariant derivative with respect to h_{ab} , R is the trace of the corresponding Ricci tensor, and $K = h^{ab} K_{ab}$. Examples of initial data are S_1 and S_2 in Fig. 1.

We want to construct initial data set that evolve in a space time with a black hole region. In order to do this, we must recognize the black hole on the initial data. The event horizon is not very useful because it is a global property of the space time, we need to know the whole space time in order to calculate it. However, there is a remarkable consequence of the black hole theory, it is related with the concept of the apparent horizon. An *apparent horizon* can be defined as follows. Given a slice S , we say that a two surface Σ contained in S is an apparent horizon if it satisfies the following equation

$$D_a n^a - K + K_{ab} n^a n^b = 0, \quad (3)$$

where n^a is the unit normal vector to Σ . Equation (3) involves only quantities that are present on the initial data. No evolution is needed. By the singularity theorems

(see [6]) we know that a data with an apparent horizon will evolve in a geodesically incomplete space time. That is, the space time will have a singularity. If the cosmic censorship conjecture is true, this singularity will be inside the black hole region. If there is a black hole, then the apparent horizon must be inside the black region. That is, the presence of an apparent horizon indicate the presence of a black hole.

In general, a space time with a black hole will have slices with an apparent horizon and slices without it. In Fig. 1 the slice S_1 have no apparent horizon, and the slice S_2 contains one. Both data describe the same space time, but looking S_1 it will be very difficult to decide that the space time contain a black hole. The presence of apparent horizon gives as a practical criteria to distinguish a black hole data. However, it is important to recall that in principle it is possible to have a black hole without any apparent horizon.

A space time contains two black black hole if the event horizon has two disconnected components. Again, is very difficult to recognize this situation on the initial data. But, we can recognize when the apparent horizon has two disconnected components. It seems to be reasonable to assume that when these two components of the apparent horizon are very separated from each other the event horizon will have also two disconnected components.

Summarizing: a simply way of constructing a two black holes initial data is by imposing that the data contain an apparent horizon with two disconnected components.

The question is now how to construct initial data with apparent horizons. In order to force the data to have an apparent horizon we have to impose to the constraint equations (1), (2) appropriate boundary conditions. Black hole data will be just a particular choice of boundary conditions.

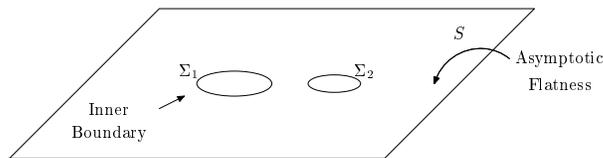


Figure 2: A black hole initial data with an apparent horizon with two disconnected components.

The boundary condition are divided naturally in two: an outer boundary condition at infinity and an inner boundary condition. The outer boundary condition is asymptotically flatness, it means that we are dealing with an isolated system. It is a fall off condition for the metric h_{ab} and the extrinsic curvature K_{ab} . This boundary condition is well understood. The inner boundary condition is the condition that we have to impose on the inner boundary in order to force it to be an apparent

horizon. The inner boundary condition is not so well understood as the outer one. The standard way to produce an apparent horizon involves the choice of a non trivial topology for the data [2] [7] [1]. These methods introduce the apparent horizon in an indirect manner. They do not really deal with an inner boundary. I think it would be very interesting to know how to prescribe directly this inner boundary condition in a consistent way. The only article I known in this direction is a numerical study by J. Thornburg [8].

There exist an infinite number of data which contain apparent horizon with two disconnected components. Which is, among them, the data that will correspond to the real astrophysical process? If the black hole are close to each other to answer this question is probably as difficult as to calculate the whole space time. But if the black hole are initially very well separated from each other then one expect some simplifications. In particular, the first requirement we should impose is that the data near each of the black hole should be, in some appropriate sense, close to the data of one isolated black hole. We call this the *far limit* of the data. The important point is that we know exactly the data for one black hole: the Kerr initial data. This suggest that one way to produce an initial data with the right far limit is by “summing” two Kerr initial data.

If the black holes do not have spins, then they should have a far limit to the Schwarzschild one. The Schwarzschild initial data can be chosen to be time symmetric, that is $K_{ab} = 0$. With this assumption the constraint equations can be reduced to a linear equation. In this case is simpler to make sense of “summing” two Schwarzschild black holes, this has been done in [2] [7].

In the case of Kerr, the constraint equation are non linear. It is possible to sum part of the initial data (the “free data”), for the rest of the data one should solve the constraint equations. One can prove that, under certain assumptions, these equations have solutions. Then, one obtain an initial data which have a far limit to the Kerr initial data. This construction has been made in [5] [4]

It important to note that in all these data the linear momentum of the individual black holes is zero in the far limit approximation. There exist data with arbitrary linear momentum, like [1], but they do not have a fat limit to Kerr. They are also numerical studies of data which have arbitrary linear momentum and also a far limit to Kerr (see the review [3] and reference therein), but so far no analytic existence proof of these type of data is available. The main difficulty is that most of the knowledge we have about the constraint equations is based on the assumption that the data is maximal (i.e; $K = 0$). The Kerr data in standard Boyer-Lindquist coordinates is maximal. However, remarkably enough, is it not known any maximal, boosted (i.e; with no trivial linear momentum), initial data for Kerr or even for Schwarzschild. These boosted maximal slices can in principle be used to construct a data for two black holes in which each of them has arbitrary linear momentum in the far limit.

References

- [1] J. M. Bowen and J. W. York, Jr. Time-asymmetric initial data for black holes and black-hole collisions. *Phys. Rev. D*, 21(8):2047–2055, 1980.
- [2] D. Brill and R. W. Lindquist. Interaction energy in geometrostatics. *Phys. Rev.*, 131:471–476, 1963.
- [3] G. B. Cook. Initial data for numerical Relativity. *Living Rev. Relativity*, 3(5), 2001. <http://www.livingreviews.org/Articles/Volume3/2000-5cook/>.
- [4] S. Dain. Initial data for a head on collision of two Kerr-like black holes with close limit. *Phys. Rev. D*, 64(15):124002, 2001. gr-qc/0103030.
- [5] S. Dain. Initial data for two Kerr-like black holes. *Phys. Rev. Lett.*, 87(12):121102, 2001. gr-qc/0012023.
- [6] S. W. Hawking and G. F. R. Ellis. *The large scale structure of space-time*. Cambridge University Press, Cambridge, 1973.
- [7] C. W. Misner. The method of images in geometrostatics. *Ann. Phys. (N.Y.)*, 24:102–117, 1963.
- [8] J. Thornburg. Coordinates and boundary conditions for the general relativistic initial data problem. *Classical and Quantum Gravity*, 4(5):1119–1131, 1987.
- [9] R. M. Wald. *General Relativity*. The University of Chicago Press, Chicago, 1984.