Fluxes in M-theory on 7-manifolds:
G-structures and Superpotential

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ABSTRACT

We consider compactifications of M-theory on 7-manifolds in the presence of 4-form fluxes, which leave at least four supercharges unbroken. We focus especially on the case, where the 7-manifold supports two spinors which are SU(3) singlets and the fluxes appear as specific SU(3) structures. We derive the constraints on the fluxes imposed by supersymmetry and calculate the resulting 4-dimensional superpotential.

1 Introduction

Compactifications of string or M-theory in the presence of fluxes are expected to improve
a number of problems appearing in generic 4-dimensional field theories obtained by
Kaluza-Klein reductions. In order to get contact to the standard model, one has to solve
e.g. the moduli problem, the problem of missing chiral fermions and the gauge hierarchy
problem. We refer to non-trivial fluxes as 10- or 11-dimensional gauge field strengths
that are non-zero in the vacuum[1] and hence give a potential upon dimensional reduction,
which lift at least part of the moduli space. So far it is unclear whether all moduli can
be lifted in this way – especially not if one wants to preserve some supersymmetries.
Concerning chiral fermions, the situation was hopeless for long time and only recently
it was realized that chiral fermions can appear on the world volume of D-branes; e.g.
on D3-branes at specific singularities in the internal space or on intersecting D6-branes.
Since D-branes are sources for fluxes, (intersecting) D-brane configurations represent
one example of flux compactifications. One should of course distinguish between branes
which produce the background geometry (e.g. $AdS_5 \times S_5$) and localized probe branes
which are assumed to produce no back reaction on the geometry. Another feature
of flux compactifications is the non-trivial warping yielding a suppression of the bulk
(gravitational) scale in comparison to the world volume gauge theory scale and has
therefore been discussed with respect to the gauge hierarchy problem.

By now one can find a long list of literature about this subject. A starting point
was the work by Candelas and Raine [1] for an un-warped metric which was generalized
later in [2] (for an earlier work on warp compactification see [3]) and the first examples,
which preserve N=1 supersymmetry appeared in [4, 5]. The subject was revived around
10 years later by the work of Polchinski and Strominger [6], where flux compactifications
in type II string theory was considered. Since then many aspects have been discussed
[7]–[36]; we will come back to some highlights of these papers.

Recall, a somewhat trivial example of flux compactifications are (intersecting) brane

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[1]Note, we are using the flux/brane notation in a rather loose sense and for other applications one
might relate fluxes only to closed forms whereas fluxes coming from (magnetic) branes are related to
source terms in the Bianchi identities.

configurations where one assumes that the external space is part of the (common) world-volume directions; as e.g. the intersecting M5-brane configurations [42, 11, 37, 38]. As the branes have to follow certain intersection rules imposed by supersymmetry also the fluxes have to satisfy certain constraints, i.e. some components have to be zero and others will fix the warp factor. At the same time, fluxes induces a non-trivial back reaction onto the geometry and this back reaction can be made explicit by re-writing the flux components as specific con-torsion components, see e.g. [20, 22, 24, 23, 30, 33]. The resulting spaces are in general non-Kählerian and their moduli spaces are unfortunately only purely understood. On the other hand, the fixing of moduli can be addressed by deriving the corresponding superpotential as function of the fluxes in a way discussed in [39, 18, 25]. But also here one observes difficulties. Non-trivial Bianchi identities exclude an expansion of the fluxes in terms of harmonic forms, which in turn makes it difficult to decide to what extend the moduli space is lifted, see [22].

In this paper we will consider M-theory compactifications in the presence of 4-form fluxes, which keep the external 4-d space time maximal symmetric, i.e. either flat or anti deSitter (AdS), where in the latter case the superpotential remains non-zero in the vacuum. In many papers the superpotential is derived directly by Kaluza-Klein reduction, whereas we introduce it as a mass term for the 4-d gravitino, i.e. a (non-vanishing) covariant derivative of the 4-d Killing spinors. In general one has not only a single internal and external spinor, which gives a classification of the solutions by the number of spinors. One can have up to eight spinors, but this case is technically very involved and severely constrains the internal space, see [5]. On the other hand, one can consider first the single spinor case, followed by the two-spinor case, the three-spinor case and so on. These spinors are singlets under the structure group of the corresponding manifold and fluxes will introduce the corresponding G-structures (or torsion components). Thus, this is a classification of the vacua with respect to the G-structure group, which is a subgroup of SO(7). The single spinor case which should allow for general G_2-structures is rather trivial, see also [28]. The first non-trivial case is given by two internal spinors which give rise to SU(3)-structure and which is the main subject of present paper; see also [31] for a discussion for this case without superpotential. The reduction of the structure group is a consequence of the appearance of a vector field

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built as a fermionic bi-linear. But this is not the most general case. Since on any 7-manifold, that is spin, exist up to three independent vector fields [40], one can always define SU(2)-structures. We shall add here a warning. Any vacuum which preserves four supercharges, (i.e. $N=1$, D=4), should finally be described by one 4-d spinor and one 7-d spinor, but the 11-d spinor is in general not a direct product of these two spinors. In fact, the 7-d spinor will exhibit a γ^5-dependent rotation while moving on the 7-manifold, see also [41]. In our counting we refer to the maximal number of independent spinors to the number of singlet spinors under a given structure group and the resulting projector constraints give us a powerful tool to solve the Killing spinor equations. At the end of the day, the different 4-d spinors might be related to each other so that we still describe $N=1$ vacua.

We have organized the paper as follows. In the next section we will make the ansatz for the metric and the 4-form field strength and separate the gravitino variation into an internal and external part. In order to solve these equations, we have to decompose the 11-d spinor into an external and internal part. This is done in Section 3. Using a global vector field, which defines a foliation of the 7-manifold X_7 by a 6-manifold X_6, we define SU(3) singlet spinors on X_7. The corresponding projector equations are given in detail and can be used to re-write the flux contribution as con-torsion components, which is discussed in Section 4, where we also introduce the superpotential as mass term for the 4-d gravitino. After having introduced all relevant relations, we will derive in Section 5 the supersymmetry constraints on the fluxes and the warp factor and we distinguish especially between the single and 2-spinor case. These sections are rather technical and the reader more interested in the results might go directly to Section 6, where we give a discussion of our results after projecting them on X_6 (i.e. after making a SU(3) decomposition). The M-theory 4-form F decomposes on X_6 into a 3-form H and a 4-form G. The holomorphic (3,0)-part of H defines the 4-d superpotential, the non-primitive (2,1)-part fixes the warp factor whereas the primitive part is un-constrained. The 4-form G has to be of (2,2)-type on X_6. Finally in Section 7 we comment on special cases: (i) if the vector field is closed; (ii) if it is Killing, (iii) the vector is Killing, but with non-constant norm and finally we comment in (iv) on the MQCD-brane world model as discussed by Witten [42].

Note added. While preparing this paper, we came aware of the work done by Dall'Agata and Prezas [43], which have some overlap with our results, but discuss in more detail the reduction to type IIA string theory.

2   Warp compactification in the presence of fluxes

The compactifications of M-theory in the presence of 4-form fluxes imply in the generic situation not only a non-trivial warping, but yield a 4-d space time that is not anymore flat. This is a consequence of the fact, that for generic supersymmetric compactifications, the fluxes generate a superpotential, that is non-zero in the vacuum and the resulting (negative) cosmological constant implies a 4-d anti deSitter vacuum. Note, we are not interested in the situation where the 4-d superpotential exhibits a run-away behavior, i.e. has no fixed points. We consider therefore as ansatz for the metric and the 4-form field strength

$$
\begin{aligned}
ds^2 &= e^{2A}\Big[g^{(4)}_{\mu\nu} dx^\mu dx^\nu + h_{ab} dy^a dy^b\Big] \ , \\
F &= \tfrac{m}{4!}\,\epsilon_{\mu\nu\rho\lambda} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\lambda + \tfrac{1}{4!} F_{abcd}\, dy^a \wedge dy^b \wedge dy^c \wedge dy^d
\end{aligned}
\tag{2.1}
$$

where $A = A(y)$ is a function of the coordinates of the 7-manifold with the metric h_{ab}, m is the Freud-Rubin parameter and the 4-d metric $g^{(4)}_{\mu\nu}$ is either flat or anti deSitter.

Unbroken supersymmetry requires the existence of (at least) one Killing spinor η yielding a vanishing gravitino variation of 11-dimensional supergravity

$$
\begin{aligned}
0 = \delta\Psi_M &= \Big[\partial_M + \tfrac{1}{4}\hat\omega_M^{RS}\Gamma_{RS} + \tfrac{1}{144}\Big(\Gamma_M^{\ NPQR} - 8\,\delta_M^N\,\Gamma^{PQR}\Big)F_{NPQR}\Big]\eta \\
&= \Big[\partial_M + \tfrac{1}{4}\hat\omega_M^{RS}\Gamma_{RS} + \tfrac{1}{144}\Big(\Gamma_M\hat F - 12\,\hat F_M\Big)\Big]\eta \ .
\end{aligned}
\tag{2.2}
$$

In the second line we used the formula

$$
\Gamma_M\Gamma^{N_1\cdots N_n} = \Gamma_M^{\ N_1\cdots N_n} + n\,\delta_M^{[N_1}\Gamma^{N_2\cdots N_n]}
\tag{2.3}
$$

and introduced the abbreviation

$$
\hat F \equiv F_{MNPQ}\Gamma^{MNPQ} \quad , \qquad \hat F_M \equiv F_{MNPQ}\Gamma^{NPQ} \ .
\tag{2.4}
$$

Using the convention $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$ with $\eta = \text{diag}(-, +, + \ldots +)$, we decompose the Γ-matrices as usual

$$\Gamma^\mu = \hat{\gamma}^\mu \otimes \mathbb{1} \qquad , \qquad \Gamma^{a+3} = \hat{\gamma}^5 \otimes \gamma^a \tag{2.5}$$

with $\mu = 0, 1, 2, 3$, $a = 1, 2, \ldots 7$ and

$$\hat{\gamma}^5 = i\hat{\gamma}^0\hat{\gamma}^1\hat{\gamma}^2\hat{\gamma}^3 \ , \quad \gamma^1\gamma^2\gamma^3\gamma^4\gamma^5\gamma^6\gamma^7 = -i \tag{2.6}$$

which implies

$$i\hat{\gamma}^5\hat{\gamma}^\mu = \frac{1}{3!}\epsilon^{\mu\nu\rho\lambda}\hat{\gamma}_{\nu\rho\lambda} \quad , \quad \frac{i}{3!}\epsilon^{abcdmnp}\gamma_{mnp} = \gamma^{abcd} \equiv \gamma^{[a}\gamma^b\gamma^c\gamma^{d]} \ . \tag{2.7}$$

The spinors in 11-d supergravity are in the Majorana representation and hence, all 4-d $\hat{\gamma}^\mu$-matrices are real and $\hat{\gamma}^5$ as well as the 7-d γ^a-matrices are purely imaginary and antisymmetric.

With this notation, we can now split the gravitino variation into an internal and external part. First, for the field strength we find

$$\hat{F} = -i\,m\,\hat{\gamma}^5 \otimes \mathbb{1} + \mathbb{1} \otimes F \ ,$$

$$\hat{F}_\mu = \tfrac{1}{4}\,i\,m\hat{\gamma}^5\hat{\gamma}_\mu \otimes \mathbb{1} \quad , \qquad \hat{F}_a = \hat{\gamma}^5 \otimes F_a \tag{2.8}$$

where F and F_a are defined as in (2.4), but using the 7-d γ^a-matrices instead of the 11-d matrices. In order to deal with the warp factor, we use

$$ds^2 = e^{2A}\widetilde{ds}^2 \quad \rightarrow \quad D_M = \tilde{D}_M + \frac{1}{2}\Gamma_M^N\partial_N A \tag{2.9}$$

and find for the external components of the gravitino variation

$$0 = \left[\nabla_\mu \otimes \mathbb{1} + \hat{\gamma}_\mu\hat{\gamma}^5 \otimes \left(\frac{1}{2}\partial A + \frac{i\,m}{36}\right) + \frac{1}{144}e^{-3A}\,\hat{\gamma}_\mu \otimes F\right]\eta \tag{2.10}$$

where $\partial A \equiv \gamma^a\partial_a A$ and ∇_μ is the 4-d covariant derivative in the metric $g_{\mu\nu}^{(4)}$. In the same way, we get for the internal gravitino variation

$$0 = \left[\mathbb{1} \otimes \left(\nabla_a^{(h)} + \frac{1}{2}\gamma_a{}^b\,\partial_b A + \frac{i\,m}{144}\,\gamma_a\right) + \frac{1}{144}e^{-3A}\,\hat{\gamma}^5 \otimes \left(\gamma_a F - 12F_a\right)\right]\eta \ . \tag{2.11}$$

Note, from this equation we can eliminate the term $\sim \gamma_a F\eta$ by multiplying eq. (2.10) with $(\frac{1}{4}\hat{\gamma}^5\hat{\gamma}^\mu \otimes \gamma^a)$ and subtracting both expression. As result we obtain

$$0 = \left[\mathbb{1} \otimes \left(\nabla_a^{(h)} - \frac{1}{2}\partial_a A + \frac{im}{48}\gamma_a\right) - \frac{1}{4}\hat{\gamma}^5\hat{\gamma}^\mu\nabla_\mu \otimes \gamma^a - \frac{1}{12}e^{-3A}\,\hat{\gamma}^5 \otimes F_a\right]\eta \ . \tag{2.12}$$

Before we can continue in solving this equation, we shall decompose the spinor and introduce the superpotential.

3   Decomposition of the Killing spinor

The 11-d Majorana spinor can be expanded in all independent spinors

$$\eta = \sum_{i=1}^{N} (\epsilon^i \otimes \theta^i + cc) \ .$$

where ϵ^i and θ^i denote the 4- and 7-d spinors, respectively. If there are no fluxes, all of these spinors are covariantly constant and N gives the number of extended supersymmetries in 4 dimensions, where $N = 8$ is the maximal possible. In presence of fluxes, the spinors are not independent anymore and hence N does not refer to the number of unbroken supersymmetries, but nevertheless, this number gives a classification of the supersymmetric vacua. Basically, this is a classification with respect to the subgroup of SO(7) under which the spinors are invariant and the embedding of this subgroup is parameterized by globally well-defined vector fields. If there is e.g., only a single spinor on the 7-manifold, it can be chosen as a real G_2 singlet spinor; if there are two spinors, one can combine them into a complex $SU(3)$ singlet; three spinors can be written as $Sp(2) \simeq SO(5)$ singlets and four spinors as $SU(2)$ singlets. Of course, all eight spinors cannot be a singlet of a non-trivial subgroup of $SO(7)$. Apart from the latter case, the internal spinors satisfy certain projectors, which annihilate all components that would transform under the corresponding subgroup and, at the same time, these projectors can be used to derive simple differential equations for the spinors (as we will discuss in more detail in the next section). For vanishing superpotential a discussion of the single spinor case is given in [28] whereas the two spinor case has been discussed before in [31].

These spinors should be globally well-defined on the 7-manifold, and hence, they can be used to build differential forms as bi-linears

$$\theta^i \gamma_{a_1 \cdots a_n} \theta^j \ .$$

The 7-dimensional γ-matrices are in the Majorana representation and satisfy the relation: $(\gamma_{a_1 \cdots a_n})^T = (-)^{\frac{n^2+n}{2}} \gamma_{a_1 \cdots a_n}$, implying that the differential form is antisymmetric in $[i, j]$ if $n = 1, 2, 5, 6$ and otherwise symmetric [we assumed here of course that θ^i are commuting spinors and the external spinors are anti-commuting]. This gives the well-known statement that a single spinor does not give rise to a vector or 2-form, but only a
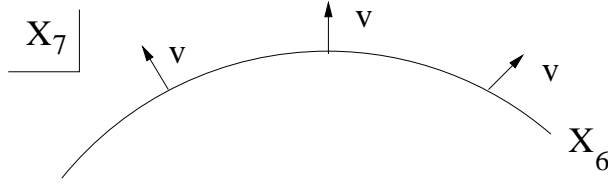
Figure 1: We have shown here a simple case, where the vector v built out of two 7-d spinors gives a foliation of the 7-manifold X_7 by a 6-manifold X_6. This foliation is not unique since on any 7-manifold that is spin exists at least three globally well-defined vectors.

3-form and its dual 4-form [the 0- and 7-form exist trivially on any spin manifold]. If we have two 7-spinors $\theta^{\{1/2\}}$, we can build one vector v and one 2-form (and of course its dual 5- and 6-form). Since the spinor is globally well-defined also the vector field is well defined on X_7 and it can be used to obtain a foliation of the 7-d space by a 6-manifold X_6, see figure for a simple example. Similarly, if there are three 7-spinors we can build three vector fields as well as three 2-forms and having four spinors the counting yields six vectors combined with six 2-forms. In addition to these vector fields and 2-forms, one obtains further 3-forms the symmetrized combination of the fermionic bi-linears. We have however to keep in mind, that all these forms are not independent, since Fierz re-arrangements yield relations between the different forms, see [20, 24, 33] for more details.

In the simplest case with a single spinor, this G_2 singlet can be written as

$$\theta = e^Z \theta_0 \tag{3.1}$$

where θ_0^T is a normalized real spinor obeying $\theta_0^T \theta_0 = 1$, e.g. $\theta_0^T = (1, 0, \ldots, 0)$. In this case, the 11-d spinor is written as

$$\eta = \epsilon \otimes \theta \tag{3.2}$$

and since the 11- and 7-d spinor are Majorana also the 4-d spinor ϵ has to be Majorana. As we will see below, the differential equation for this G_2 singlet spinor becomes $\nabla_a \theta \sim \gamma_a \theta$, which implies that X_7 is a space of weak G_2 holonomy. This in turn implies, that the 8-d space built as a cone over this 7-manifold has $Spin(7)$ holonomy. As we will see below, the internal fluxes in this case have to be trivial.

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Next, allowing for two spinors on X_7, one can build a vector field v, which can be used to combine these two spinor into one complex spinor defined as

$$\theta = \frac{1}{\sqrt{2}} e^Z \left(\mathbb{1} + v_a \gamma^a\right)\theta_0 \quad , \qquad v_a v^a = 1 \tag{3.3}$$

where the constant spinor θ_0 is again the G_2 singlet and Z is a complex function. Both spinors, θ and its complex conjugate $\theta^\star$, are chiral spinors on X_6 (see figure) and moreover, both spinors are $SU(3)$ singlets, which will become clear from the properties that we will discuss below. In this case, the 11-d Majorana spinor is decomposed as

$$\eta = \epsilon \otimes \theta + \epsilon^\star \otimes \theta^\star \ . \tag{3.4}$$

Of course, now ϵ is not a 4-d Majorana spinor, which would bring us back to the single spinor case discussed before $(\theta + \theta^\star \sim \theta_0)$. Instead ϵ and $\epsilon^\star$ are two 4-d Weyl spinors of opposite chirality. Now, the differential equation for this 7-d spinor of the form $\nabla_a \theta \sim \gamma_a \theta$ identifies X_7 as a Sasaki-Einstein space, which implies that the cone over it is a 8-d space of SU(4)-holonomy.

The other cases become now technically more and more involved. If there are three different spinors on X_7, we can build three vectors as well as three 2-forms and the three spinors are now $Sp(2)$ singlets. One might think that the existence of three well-defined vector fields on X_7 would give restrictions on the manifold, but this is not the case. On any 7-manifold that is spin, exists at least three no-where vanishing vector fields, [40]. The manifold can again be identified from the equation satisfied by the spinors. The equation $\nabla_a \theta^i \sim \gamma_a \theta^i$ implies e.g., that X_7 a 3-Sasaki-space (i.e. the cone yields an 8-d Hyperkähler space with $Sp(2)$ holonomy), with the Aloff-Walach space $N^{1,1}$ as the only regular examples [44] (apart from S^7); non-regular examples are in [45].

The four spinors case yield up to six vector fields (recall, due to Fierz re-arrangements they are not necessarily independent), which would however severely constrain the manifold (e.g. that X_7 factorizes into $R_3 \otimes X_4$).

Let us now return to the spinor θ_0, which is fixed by the requirement to be a G_2 singlet. Note, the Lie algebra $\mathfrak{so}(7)$ is isomorphic to Λ^2 and a reduction of the structure group on a general X_7 from $SO(7)$ to the subgroup G_2 implies the following splitting:

$$\mathfrak{so}(7) = \mathfrak{g}_2 \oplus \mathfrak{g}_2^\perp \ . \tag{3.5}$$

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This induces a decomposition of the space of 2-forms in the following irreducible G_2-modules,

$$\Lambda^2 = \Lambda_7^2 \oplus \Lambda_{14}^2 \,, \tag{3.6}$$

where

$$\Lambda_7^2 = \{u \lrcorner \varphi | u \in TX_7\} = \{\alpha \in \Lambda^2 \,|\, *(\varphi \wedge \alpha) - 2\alpha = 0\}$$

$$\Lambda_{14}^2 = \{\alpha \in \Lambda^2 \,|\, *(\varphi \wedge \alpha) + \alpha = 0\} \cong \mathfrak{g}_2$$

and we introduced the abbreviation $u \lrcorner \varphi \equiv u^m \varphi_{mnp}$ and φ denotes the G_2-invariant 3-index tensor, which is expressed as fermionic bi-linear in (3.11). The operator $*(\varphi \wedge \alpha)$ splits therefore the 2-forms correspondingly to the eigenvalues 2 and -1. These relations serve us to define the orthogonal projections $\mathcal{P}_k$ onto the k-dimensional spaces:

$$\mathcal{P}_7(\alpha) = \frac{1}{3}(\alpha + *(\varphi \wedge \alpha)) = \frac{1}{3}\left(\alpha + \frac{1}{2}\alpha \lrcorner \psi\right) \,, \tag{3.7}$$

$$\mathcal{P}_{14}(\alpha) = \frac{1}{3}(2\alpha - *(\varphi \wedge \alpha)) = \frac{2}{3}\left(\alpha - \frac{1}{4}\alpha \lrcorner \psi\right) \tag{3.8}$$

where $\psi = *\varphi$. To be concrete, the G_2-singlet spinor satisfies the condition

$$(\mathcal{P}_{14})^{cd}_{ab}\, \gamma_{cd}\,\theta_0 = \frac{2}{3}\left(\mathbb{1}^{cd}_{\ ab} - \frac{1}{4}\psi^{cd}_{\ ab}\right)\gamma_{cd}\,\theta_0 = 0 \,.$$

This constraint is equivalent to the condition

$$\gamma_{ab}\theta_0 = i\varphi_{abc}\gamma^c\theta_0 \tag{3.9}$$

which gives after multiplication with γ-matrices

$$\begin{aligned}
\gamma_{abc}\theta_0 &= \left(i\varphi_{abc} + \psi_{abcd}\gamma^d\right)\theta_0 \,, \\
\gamma_{abcd}\theta_0 &= \left(-\psi_{abcd} - 4i\varphi_{[abc}\gamma_{d]}\right)\theta_0 \,.
\end{aligned} \tag{3.10}$$

Since it is a normalized spinor and due to the properties of the 7-d γ-matrices (yielding especially $\theta_0^T\gamma_a\theta_0 = 0$), we get from these relations the following bi-linears

$$\begin{aligned}
1 &= \theta_0^T\theta_0 \,, \\
i\,\varphi_{abc} &= \theta_0^T\gamma_{abc}\theta_0 \,, \\
-\psi_{abcd} &= \theta_0^T\gamma_{abcd}\theta_0 \,, \\
i\,\epsilon_{abcdmnp} &= \theta_0^T\gamma_{abcdmnp}\theta_0 \,.
\end{aligned} \tag{3.11}$$

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All other bi-linears vanish. These identities can now be used to derive projectors for the complex 7-spinor in (3.3)

$$
\begin{aligned}
\gamma_a \theta &= \tfrac{e^Z}{\sqrt{2}}(\gamma_a + v_a + i\varphi_{abc}v^b\gamma^c)\theta_0 \ , \\[4pt]
\gamma_{ab}\theta &= \tfrac{e^Z}{\sqrt{2}}(i\varphi_{abc}\gamma^c + i\varphi_{abc}v^c + \psi_{abcd}v^c\gamma^d - 2v_{[a}\gamma_{b]})\theta_0 \ , \\[4pt]
\gamma_{abc}\theta &= \tfrac{e^Z}{\sqrt{2}}(i\varphi_{abc} + \psi_{abcd}\gamma^d + 3iv_{[a}\varphi_{bc]d}\gamma^d - \psi_{abcd}v^d - 4i\varphi_{[abc}\gamma_{d]}v^d)\theta_0 \ , \\[4pt]
\gamma_{abcd}\theta &= \tfrac{e^Z}{\sqrt{2}}(-\psi_{abcd} - 4i\varphi_{[abc}\gamma_{d]} - 5\psi_{[abcd}\gamma_{e]}v^e - 4iv_{[a}\varphi_{bcd]} - 4v_{[a}\psi_{bcd]e}\gamma^e)\theta_0 \ , \\[4pt]
\gamma_{abcde}\theta &= \tfrac{e^Z}{\sqrt{2}}(-5\psi_{[abcd}\gamma_{e]} - i\varepsilon_{abcdefg}\gamma^g v^f - 5v_{[a}\psi_{bcde]} - 20iv_{[a}\varphi_{bcd}\gamma_{e]})\theta_0 \ , \\[4pt]
\gamma_{abcdef}\theta &= \tfrac{e^Z}{\sqrt{2}}(-i\varepsilon_{abcdefg}\gamma^g + \varepsilon_{abcdefg}v_h\gamma_j\varphi^{ghj} - i\varepsilon_{abcdefg}v^g)\,\theta_0 \ .
\end{aligned}
$$

Using complex notation, we can introduce the following two sets of bi-linears $[\hat\theta^\dagger = (\hat\theta^*)^T]$:

$$
\Omega_{a_1\cdots a_k} \equiv \theta^\dagger \gamma_{a_1\cdots a_k}\theta \qquad \text{and} \qquad \tilde\Omega_{a_1\cdots a_k} \equiv \theta^T \gamma_{a_1\cdots a_k}\theta
$$

and we define the associated k-forms by

$$
\Omega^k \equiv \frac{1}{k!}\Omega_{a_1\cdots a_k}e^{a_1\cdots a_k} \qquad \text{and} \qquad \tilde\Omega^k \equiv \frac{1}{k!}\tilde\Omega_{a_1\cdots a_k}e^{a_1\cdots a_k} \ .
$$

Thus, we obtain the following forms (cp. also [31])

$$
\begin{aligned}
\Omega^0 &= e^{2\,\mathrm{Re}(Z)} \ , \\[4pt]
\Omega^1 &= e^{2\,\mathrm{Re}(Z)}\,v \ , \\[4pt]
\Omega^2 &= i\,e^{2\,\mathrm{Re}(Z)}\,v \lrcorner \varphi = i\,e^{2\,\mathrm{Re}(Z)}\,\omega \ , \\[4pt]
\Omega^3 &= i\,e^{2\,\mathrm{Re}(Z)}\Big[v \wedge (v \lrcorner \varphi)\Big] = i\,e^{2\,\mathrm{Re}(Z)}\,v \wedge \omega \ , \\[4pt]
\tilde\Omega^3 &= i\,e^{2\,\mathrm{Re}(Z)}\Big[e^{2i\,\mathrm{Im}(Z)}\big(\varphi - v\wedge\omega - i\,v\lrcorner\psi\big)\Big] = i\,e^{2\,\mathrm{Re}(Z)}\,\Omega^{(3,0)} \ , \\[4pt]
\tilde\Omega^4 &= e^{2\mathrm{Re}(Z)}\Big[v \wedge (v \lrcorner \psi) - i\,v\wedge\varphi\Big] = -i\,e^{2\,\mathrm{Re}(Z)}v \wedge \bar\Omega^{(0,3)} \ , \\[4pt]
\Omega^4 &= -e^{2\mathrm{Re}(Z)}\Big[\psi - v\wedge(v\lrcorner\psi)\Big] = -\tfrac{1}{2}e^{2\,\mathrm{Re}(Z)}\,\omega\wedge\omega \ .
\end{aligned}
\tag{3.12}
$$

The associated 2-form to the almost complex structure on X_6 is ω and with the projectors $\tfrac{1}{2}(\mathbb{1}\pm i\omega)$ we can introduce (anti) holomorphic indices so that $\Omega^{(3,0)}$ can be identified as the holomorphic $(3,0)$-form on X_6. There exists a topological reduction from a G_2-structure to a $SU(3)$-structure (even to a $SU(2)$-structure). The difficulties arise by

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formulating the geometrical reduction. Using the vector v one is able to formulate an explicit embedding of the given $SU(3)$-structure in the G_2-structure to obtain such a geometrical reduction:

$$
\begin{aligned}
\varphi &= \mathrm{Re}(e^{-2i\,\mathrm{Im}(Z)}\,\Omega^{(3,0)}) + v \wedge \omega = \chi_+ + v \wedge \omega \,, \\
\psi &= \mathrm{Im}(e^{-2i\,\mathrm{Im}(Z)}\,\Omega^{(3,0)}) \wedge v + \tfrac{1}{2}\omega^2 = \chi_- \wedge v + \tfrac{1}{2}\omega^2
\end{aligned}
\tag{3.13}
$$

with the compatibility relations

$$
e^{-2i\,\mathrm{Im}(Z)}\,\Omega^{(3,0)} \wedge \omega = (\chi_+ + i\,\chi_-) \wedge \omega = 0 \,, \tag{3.14}
$$

$$
\chi_+ \wedge \chi_- = \frac{2}{3}\,\omega^3 \,. \tag{3.15}
$$

Since the phase factor, coming form $\mathrm{Im}(Z)$, does not play any role in the following and we will set it to zero. Before we will use these relations for the discussion of the Killing spinor equations, we have first to discuss the deformation of the geometry due to the fluxes.

4   Back reaction onto the geometry

Due to gravitational back reaction the fluxes deform the geometry, not only of the internal but also of the external space. This is related to the fact that the 4- as well as the 7-d spinor is not anymore covariantly constant. There is also an ongoing discussion on this subject in the mathematical literature, see e.g. [46, 47, 48, 34, 49].

Consider first the 7-d internal space. The spinor is singlet under a subgroup of the structure group and hence satisfies certain projector relations; see previous section. This can be used to re-write all flux terms in the Killing spinor equation as con-torsion terms entering the covariant derivative as follows

$$
\tilde{\nabla}_a \theta \equiv (\nabla_a - \frac{1}{4}\tau_a^{\ bc}\gamma_{bc})\theta = 0 \,.
$$

From the symmetry it follows that it has $7 \times 21 = 7 \times (7+14)$ components, but if θ is a G_2-singlet the $\mathbf{14}$ drops out and hence $\tau \in \Lambda_1 \otimes \mathfrak{g}_2^\perp$, where Λ_1 is the space of 1-forms and $\mathfrak{g}_2^\perp$ is defined by the Lie-algebra relation $\mathfrak{g}_2^\perp \oplus \mathfrak{g}_2 = \mathfrak{so}(7)$. These components decompose as

$$
\mathbf{49} = \mathbf{1} + \mathbf{7} + \mathbf{14} + \mathbf{27} = \tau_1 + \tau_7 + \tau_{14} + \tau_{27}
$$

where τ_i are called G_2-structures. They are related to differential forms that can be obtained from $d\varphi$ and $d\psi$ as follows

$$
\begin{aligned}
d\varphi &\in \Lambda^4 = \Lambda_1^4 \oplus \Lambda_7^4 \oplus \Lambda_{27}^4 ~, \\
d\psi &\in \Lambda^5 = \Lambda_7^5 \oplus \Lambda_{14}^5 ~,
\end{aligned}
\tag{4.1}
$$

where the $\mathbf{7}$ in Λ_7^4 is the same as in Λ_7^5 up to a multiple. Note, since the spinor is not covariantly constant, neither can be the differential form φ and ψ. The different components in the differentials $d\varphi$ can be obtained by using the following projectors

$$
\begin{aligned}
\mathcal{P}_1(\beta) &= \tfrac{1}{4!}\psi \lrcorner\, \beta ~~, \\
\mathcal{P}_7^4(\beta) &= -\tfrac{1}{3!}\varphi \lrcorner\, \beta, \\
\mathcal{P}_{27}(\beta)_{ab} &= \tfrac{1}{3!}(\beta_{cde\{a}\psi_{b\}}{}^{cde})_0
\end{aligned}
\tag{4.2}
$$

where β denotes a 4-form and in $(\cdot)_0$ we removed the trace. Let us summarize:

$$
\begin{aligned}
\tau^{(1)} &\longleftrightarrow \psi \lrcorner\, d\varphi ~~, \\
\tau^{(7)} &\longleftrightarrow \varphi \lrcorner\, d\varphi ~, \\
\tau^{(14)} &\longleftrightarrow *d\psi - \tfrac{1}{4}(*d\psi) \lrcorner\, \psi ~, \\
\tau^{(27)} &\longleftrightarrow (d\varphi_{cde\{a}\psi_{b\}}{}^{cde})_0 ~.
\end{aligned}
\tag{4.3}
$$

With these definitions τ_{14} and τ_{27} have to satisfy: $\varphi_3 \wedge \Lambda_{27}^3 = \varphi_3 \wedge \tau_{14} = 0$. In the case of two 7-spinors, which combine to an SU(3)-singlet spinor, these G_2-structures decompose into SU(3)-structures, which consist of five components again related to the differential forms: $d\omega$ and $d\Omega$, where the 2-form ω and 3-form Ω were introduced in (3.12). We refer here to literature for more details [50]. Note, the Killing spinor is invariant under the G-structures group and therefore, the more spinors we have the less is the G-structure group.

Let us end this section with the back reaction onto the external space. As we said at the beginning, we are interested only in the case where this gives at most a cosmological constant yielding a 4-d anti deSitter vacuum. As special case we recover of course the flat space vacuum. In supergravity, the superpotential appears as mass term for the

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gravitino and this means, that the corresponding Killing spinor cannot be covariantly constant. Therefore, we introduce the superpotential by assuming that our 4-d spinors solve the equation

$$\nabla_\mu \epsilon^i \sim \hat{\gamma}_\mu \, e^{K/2} \, (W_1^{ij} + i\hat{\gamma}^5 \, W_2^{ij}) \, \epsilon_j \tag{4.4}$$

where we took also into account a non-trivial Kähler potential K. If there is only a single spinor as in eq. (3.2), this equation simplifies to

$$\nabla_\mu \epsilon \sim \hat{\gamma}_\mu e^{K/2} \, (W_1 + i \, \hat{\gamma}^5 \, W_2) \, \epsilon \; .$$

If ϵ is a Weyl spinor, as in eq. (3.4), this equation becomes $\nabla_\mu \epsilon = \hat{\gamma}_\mu \bar{W} \epsilon^\star$ with the complex superpotential

$$W = W_1 + i \, W_2 \; . \tag{4.5}$$

Therefore, in the 11-d spinor equations, we introduce the superpotential by assuming that 11-d spinor satisfies the equation

$$\left[\nabla_\mu \otimes \mathbb{1}\right]\eta = (\hat{\gamma}_\mu \otimes \mathbb{1})\tilde{\eta} \qquad \text{with}: \qquad \tilde{\eta} = \begin{cases} e^{K/2} \left[(W_1 + i\,\hat{\gamma}^5 W_2) \otimes \mathbb{1}\right] \eta & \text{M} \\ e^{K/2} \, W\epsilon \otimes \theta^\star + cc & \text{W} \end{cases} \tag{4.6}$$

where M/W refers to a 4-d Majorana or Weyl spinor ϵ. We do not consider here the case where the superpotential is matrix valued.

5   BPS constraints

With the superpotential as introduced before, equation (2.10) becomes

$$0 = \tilde{\eta} + \left[\hat{\gamma}^5 \otimes \left(\frac{1}{2}\partial A + \frac{im}{36}\right) + \frac{1}{144}e^{-3A} \left(\mathbb{1} \otimes F\right)\right]\eta \; . \tag{5.1}$$

and if

$$\hat{\eta} = e^{-\frac{A}{2}}\eta \tag{5.2}$$

equation (2.12) yields

$$0 = \mathbb{1} \otimes \left(\nabla_a^{(h)} + \frac{i\,m}{48}\right)\hat{\eta} - i\,\hat{\gamma}^5\hat{\gamma}_a \, e^{-\frac{A}{2}}\tilde{\eta} - \frac{1}{12}\, e^{-3A}\, \hat{\gamma}^5 \otimes F_a\hat{\eta} \; . \tag{5.3}$$

We will now derive the constraints imposed by these two equations for the two cases: (i) that the 4-d spinor is Majorana or (ii) that we have two Weyl spinors of opposite

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chirality. This in turn implies that we consider the cases that on X_7 is a single or two spinors.

5.1 Manifolds with a single 7-spinor

In this case, which has been discussed also in [28], we can use the relations (3.10) and find

$$F\theta_0 \equiv F_{abcd}\gamma^{abcd}\theta_0 \;\; = \;\; \left(- F_{abcd}\psi^{abcd} - 4i F_{abcd}\varphi^{abc}\gamma^d \right)\theta_0 \; ,$$

$$F_a\theta_0 \equiv F_{abcd}\gamma^{bcd}\theta_0 \;\; = \;\; \left(i\, F_{abcd}\varphi^{bcd} + F_{abcd}\psi^{bcde}\gamma_e \right)\theta_0 \; .$$

Next, the internal part of the 4-form field strength has 35 components, that decompose under G_2 as $\mathbf{35} \to \mathbf{1} + \mathbf{7} + \mathbf{27}$ with

$$F_{abcd} = \frac{1}{7}\,\mathcal{F}^{(1)}\,\psi_{abcd} + \mathcal{F}^{(7)}_{[a}\,\varphi_{bcd]} - 2\,\mathcal{F}^{(27)}_{e[a}\psi^e{}_{bcd]} \tag{5.4}$$

or

$$\mathcal{F}^{(1)} \;\; = \;\; \tfrac{1}{4!}\,F_{abcd}\psi^{abcd} \; ,$$

$$\mathcal{F}^{(7)}_a \;\; = \;\; \tfrac{1}{3!}\,F_{abcd}\varphi^{bcd} \; , \tag{5.5}$$

$$\mathcal{F}^{(27)}_{ab} - \tfrac{4}{7}\mathcal{F}^{(1)}\,\delta_{ab} \;\; = \;\; \tfrac{1}{3!}\,F_{cde\{a}\psi_{b\}}{}^{cde} \; .$$

Therefore, we can also write

$$F\theta_0 \;\; = \;\; -4!\left[\mathcal{F}^{(1)} + i\mathcal{F}^{(7)}_a\gamma^a\right]\theta_0 \; ,$$

$$F_a\theta_0 \;\; = \;\; \left[3!\, i\,\mathcal{F}^{(7)}_a + \left(F_{cde[a}\psi_{b]}{}^{cde} + F_{cde\{a}\psi_{b\}}{}^{cde}\right)\gamma^b\right]\theta_0 \tag{5.6}$$

$$\;\; = \;\; 3!\left[i\,\mathcal{F}^{(7)}_a + \left(\varphi_{ab}{}^c\mathcal{F}^{(7)}_c + \mathcal{F}^{(27)}_{ab} - \tfrac{4}{7}\mathcal{F}^{(1)}\delta_{ab}\right)\gamma^b\right]\theta_0 \; .$$

Note, by contracting with φ, one can verify that: $F_{cde[a}\psi_{b]}{}^{cde} \sim \varphi_{ab}{}^e\mathcal{F}^{(7)}{}_e$. With these relations it is now straightforward to solve the Killing spinor equations. Because the 11-d spinor η is Majorana, also the 7-spinor θ as well as the 4-spinor ϵ have to be Majorana and as consequence, terms $\mathcal{O}(\epsilon)$ and $\mathcal{O}(\hat{\gamma}^5\epsilon)$ have to vanish separately. We find the equations

$$0 = \left(ie^{K/2}\,W_2 + \tfrac{1}{2}\partial A + \tfrac{im}{36}\right)\theta_0 = \left(ie^{K/2}\,W_2 + \tfrac{1}{2}\gamma^a\partial_a A + \tfrac{im}{36}\right)\theta_0 \; ,$$

$$0 = \left(e^{K/2}\,W_1 + \tfrac{1}{144}e^{-3A}F\right)\theta_0 = \left(e^{K/2}\,W_1 - \tfrac{1}{6}e^{-3A}[\mathcal{F}^{(1)} + i\mathcal{F}^{(7)}_a\gamma^a]\right)\theta_0 \; .$$

Since terms $\mathcal{O}(\theta_0)$ and $\mathcal{O}(\gamma^a \theta_0)$ cannot cancel, we get finally as solution

$$A = const. , \qquad m = -36\, e^{K/2}\, W_2 \qquad \mathcal{F}_a^{(7)} = 0 , \qquad \frac{e^{-3A}}{6} \mathcal{F}^{(1)} = e^{K/2}\, W_1 . \qquad (5.7)$$

The remaining differential equation (5.3) can be solved in the same way. With eq. (5.6) and $\mathcal{F}_a^{(7)} = 0$ we get the equations

$$\begin{aligned}
0 &= \left(\nabla_a - i\left[e^{K/2}\, W_2 - \tfrac{m}{48}\right]\gamma_a\right) e^Z \theta_0 = \left(\nabla_a + i\, \tfrac{7m}{144}\, \gamma_a\right) e^Z \theta_0 , \\
0 &= \left(e^{K/2}\, W_1 \gamma_a + \tfrac{1}{12}\, e^{-3A}\, F_a\right)\theta_0 = \left(e^{K/2}\, W_1 \delta_{ab} + \tfrac{1}{2}\, e^{-3A}\left[\mathcal{F}_{ab}^{(27)} - \tfrac{4}{7}\mathcal{F}^{(1)}\delta_{ab}\right]\right)\gamma^b \theta_0 .
\end{aligned}$$

Using (5.7) we find from the second equation

$$W_1 = \mathcal{F}^{(1)} = \mathcal{F}_{ab}^{(27)} = 0 \qquad (5.8)$$

and thus all internal 4-form components have to vanish

$$F_{abcd} = 0 . \qquad (5.9)$$

The other equation gives now a differential equations for the spinor $e^Z \theta_0$. The factor e^Z does not contain any γ-matrices[2] and if we contract this equation with $\theta_0^T e^{-Z}$ and use the fact that $\theta_0^T \theta_0 = 1$ (and therefore $\theta_0^T \nabla_a \theta_0 = 0$) we find

$$\partial_a Z = 0$$

and this constant factor can be dropped from our analysis. The differential equations for θ_0 fixes the 7-manifold to have a weak G_2 holonomy. In fact, after taking into account the vielbeine, this gives the known set of first order differential equations for the spin connection 1-form ω^{ab}

$$\omega^{ab} \varphi_{abc} = \frac{7}{36}\, m\, e_b \qquad (5.10)$$

where m was the Freud-Rubin parameter [note ω is here the spin connection and should not be confused with the associated 2-form introduced before].

Using the differential equation for the 7-spinor, it is straightforward to verify that

$$d\varphi = -\frac{7\,m}{18}\,\psi \quad , \qquad d\psi = 0$$

---

[2]Recall, as a G_2 singlet θ has only one component, which we normalized by allowing a non-trivial factor e^Z.

and therefore only $\tau^{(1)}$ is non-zero.

To conclude this case, we have verified a 7-manifold of weak G_2 holonomy, which is Einstein and the cosmological constant is given by the Freud-Rubin parameter [17, 28]. Note also, that the theory of weak holonomy, G_2-structures forces m to be a real constant. The holonomy group with respect to the Levi-Civita connection reduces in this case *not* to the group G_2 and is therefore of generic type. Only in case of $m = 0$ we get back to the special manifolds having precisely holonomy group G_2. We found that in this case all internal flux components have to vanish. The 4-d superpotential is only given by the Freud-Rubin parameter, ie.

$$W \sim i \int_{X_7} {}^\star F \tag{5.11}$$

which fixes the overall size of the 7-manifold. In the limit of flat 4-d Minkowski vacuum, the Freud-Rubin parameter has to vanish and we get back to the Ricci-flat G_2-holonomy manifold. In order to allow for non-trivial fluxes we have therefore to consider the case with two spinors on X_7 or equivalently to use 4-d Weyl spinors.

5.2   Manifolds with two 7-spinors

The case before was rather trivial, as next step we consider the more general spinor as given in (3.4), which comprises of two 4- and 7-spinors and we shall solve again the equations (5.1) and (5.3). But now, the complex 4-spinor is chiral and we choose

$$\gamma^5 \epsilon = \epsilon \quad , \qquad \gamma^5 \epsilon^\star = -\epsilon^\star \ .$$

Due to the opposite chirality, terms with ϵ and $\epsilon^\star$ are independent and the term $\mathcal{O}(\epsilon)$ in eq. (5.1) becomes

$$0 = e^{K/2} W \theta^\star + \left(\frac{im}{36} + \frac{1}{2} \partial A + \frac{1}{144} e^{-3A} F \right) \theta \ . \tag{5.12}$$

The equation coming from the terms $\mathcal{O}(\theta^\star)$ do not contain new information, but gives just the complex conjugate of this equation. Now, with the relations from section 3 we can collect all terms $\mathcal{O}(\theta_0)$ and $\mathcal{O}(\gamma^a \theta_0)$ which have to vanish separately and find

$$0 = e^{K/2} W + \frac{im}{36} + \frac{1}{2} v^a \partial_a A - \frac{1}{6} e^{-3A} \left[\mathcal{F}^{(1)} + i v^a \mathcal{F}_a^{(7)} \right] \tag{5.13}$$

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$$0 = \left[-e^{K/2} W + \frac{im}{36} \right] v_a + \frac{1}{2} \left[\delta_{ab} + i\varphi_{abc} v^c \right] \partial^b A \tag{5.14}$$
$$- \frac{1}{6} e^{-3A} \left[\mathcal{F}^{(1)} v_a - i\mathcal{F}_a^{(7)} + \frac{1}{3} F_{bcd\{a} \psi_{e\}}{}^{bcd} v_e \right] .$$

And separating the real and imaginary part, we get four equations

$$0 = e^{K/2} W_1 - \frac{1}{6} e^{-3A} \mathcal{F}^{(1)} + \frac{1}{2} v^a \partial_a A , \tag{5.15}$$

$$0 = e^{K/2} W_2 + \frac{m}{36} - \frac{1}{6} e^{-3A} \mathcal{F}_k^{(7)} v^k , \tag{5.16}$$

$$0 = \left[-e^{K/2} W_1 + \frac{1}{42} e^{-3A} \mathcal{F}^{(1)} \right] v_a + \frac{1}{2} \partial_a A - \frac{1}{3} e^{-3A} \mathcal{F}_{ak}^{(27)} v^k , \tag{5.17}$$

$$0 = \left[-e^{K/2} W_2 + \frac{m}{36} \right] v_a + \frac{1}{6} e^{-3A} \mathcal{F}_a^{(7)} - \frac{1}{2} \varphi_{abc} v^b \partial^c A . \tag{5.18}$$

If we contract the last two equations with the vector v^a and add/subtract the results to the first two equations we infer

$$W = W_1 + i W_2 = \frac{1}{6} \left[\frac{4}{7} \mathcal{F}^{(1)} - v^a \mathcal{F}_{ab}^{(27)} v^b + i v^a \mathcal{F}_a^{(7)} \right]$$
$$v^a \partial_a e^{3A} = \frac{3}{7} \mathcal{F}^{(1)} + v^a \mathcal{F}_{ab}^{(27)} v^b \tag{5.19}$$
$$m = 0$$

We set $K = -6A$, i.e. identified the Kähler potential[3] with the warp factor, which yields a holomorphic superpotential, see eq. (6.2) below. In the discussion we will re-write these expressions in terms of quantities on the 6-manifold X_6. For the other flux components we find

$$(\delta_a{}^b - v_a v^b) \mathcal{F}_b^{(7)} = \varphi_{abc} v^b \partial^c e^{3A} = 2 \varphi_a{}^{bc} v_b \mathcal{F}_{cd}^{(27)} v^d , \tag{5.20}$$
$$2 \mathcal{F}_{ab}^{(27)} v^b = \left[-\frac{3}{7} \mathcal{F}^{(1)} + v^c \mathcal{F}_{cb}^{(27)} v^b \right] v_a + \partial_a e^{3A} \tag{5.21}$$

[the flux components were introduced in (5.4)].

Finally, we have to investigate the internal part (5.3) of the variation giving

$$\nabla_a^{(h)} \hat{\theta} = e^{K/2} W \gamma_a \theta^\star + \frac{1}{12} e^{-3A} F_a \hat{\theta} \tag{5.22}$$

---

[3]At least for specific Kähler gauge this is possible and appeared also in the supergravity solutions [58].

with $\hat{\theta} = \frac{e^{-\frac{A}{2}+Z}}{\sqrt{2}}(\mathbb{1}+v)\theta_0$. On the rhs of this equation we can use our previous formulae and write

$$F_a\hat{\theta} = \frac{e^{Z-\frac{A}{2}}}{\sqrt{2}}(F_a{}^1 + F_a{}^7 + F_a{}^{27})\theta_0$$

where

$$\gamma_a\hat{\theta}^\star = \frac{1}{\sqrt{2}}e^{Z^\star}\left(\gamma_a - v_a - i\,\varphi_{abc}v^b\gamma^c\right)\theta_0\ ,$$

$$F_a{}^1\theta_0 = \frac{24}{7}\mathcal{F}^{(1)}(v_a - \gamma_a)\theta_0\ ,$$

$$F_a{}^7\theta_0 = 3\left[2i\mathcal{F}_a^{(7)} - \varphi_{ab}{}^c(v^b - \gamma^b)\mathcal{F}_c^{(7)} - i\left(v_a\gamma^b + \gamma_a v^b\right)\mathcal{F}_b^{(7)}\right]\theta_0\ ,$$

$$F_a{}^{27}\theta_0 = 6\left[-\mathcal{F}_{ab}^{(27)}(v^b - \gamma^b) + i\varphi_{ad}{}^b\left(\gamma^c v^d + \gamma^d v^c\right)\mathcal{F}_{bc}^{(27)}\right]\theta_0\ .$$

With these expressions we can now calculate the G-structures as introduced in (4.1) and find

$$\begin{aligned} \tau^{(1)} &\longleftrightarrow W_2\ , \\ \tau_a^{(7)} &\longleftrightarrow 48\,W_1\,v_a - \frac{24}{7}\,\mathcal{F}^{(1)}\,v_a + \frac{3}{2}\,\varphi_a{}^{bc}v_b\mathcal{F}_c^{(7)} + 27\mathcal{F}_{ab}^{(27)}v^b\ . \end{aligned} \tag{5.23}$$

We used here the constraint

$$0 = d[-A + 2\mathrm{Re}(Z)] \tag{5.24}$$

which one can derive from

$$\partial_a\|\hat{\theta}\| \equiv \partial_a[\hat{\theta}^\dagger\hat{\theta}] = e^{-A+2\mathrm{Re}(Z)}\,\partial_a[-A + 2\,\mathrm{Re}(Z)] = (\nabla_a\hat{\theta})^T\hat{\theta}^* + \hat{\theta}^T\nabla_a(\hat{\theta}^*) = 0$$

To make the set of equations complete, we have to derive the differential equations obeyed by the vector field v, which is straightforward if we use again the differential equation for the spinor. To simplify the notation and for later convenience we use here the 2-form ω which was introduced in (3.13) and find for the covariant derivative

$$\begin{aligned} \nabla_m v_n &= -\tfrac{1}{12}e^{-3A-2\mathrm{Re}(Z)}\,\theta^+[F_m, \gamma_n]\theta \\ &= \tfrac{1}{12}e^{-3A}F_{mbcd}\omega^{bc}\omega^d{}_n \end{aligned} \tag{5.25}$$

recall $\omega_{ab} = \varphi_{abc}v^c$. Note, $v^n\nabla_m v_n = 0$, which is consistent with $|v|^2 = 1$. Using the decomposition (5.4) one finds for the (anti)symmetrized components

$$\nabla_{[m}v_{n]} = \left(\delta_{[m}^a\delta_{n]}^b + \frac{1}{2}\psi_{mn}{}^{ab}\right)\mathcal{F}_{ac}^{(27)}v^c v_b + \frac{1}{4}\varphi_{mn}{}^a(\delta_a{}^b - v_a v^b)\mathcal{F}_b^{(7)}\ , \tag{5.26}$$

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$$\nabla_{\{m}v_{n\}} = -\frac{2}{7}\left(\delta_{mn} - v_{\{m}v_{n\}}\right)\mathcal{F}^{(1)} - \frac{1}{2}v_{\{m}\varphi_{n\}}{}^{ab}v_a\mathcal{F}^{(7)}_b$$
$$+\frac{1}{2}\left(\delta_m{}^a\delta_n{}^b + \omega_m{}^a\omega_n{}^b\right)\mathcal{F}^{(27)}_{ab} - \frac{1}{2}\delta_{mn}\mathcal{F}^{(27)}_{ab}v^av^b \tag{5.27}$$

The first term in the anti-symmetric part is the projector onto the **7**, see (3.7), and by contracting with φ and employing eqs. (5.20) and (5.21), one can verify that

$$d(e^{3A}v) = 0 \ .$$

Note, in comparison with [31] our internal metric is conformally rescaled.

6 Discussion

Let us now summarize and discuss our results. We considered flux compactifications of M-theory on a 7-manifold X_7, where we distinguished between the number of (real) spinors. The case with a single 7-d spinor was rather trivial: all (internal) fluxes had to vanish and the Freud-Rubin parameter deformed X_7 to a manifold with weak G_2-holonomy. Let us concentrate here on the situation with two 7-d spinors with the constraints derived in the previous subsection. In contrast to the single spinor case, the Freud-Rubin parameter has to vanish in this case. The no-where vanishing vector field v, which was built as fermionic bi-linear, can be used to foliate X_7 by a 6-d manifold X_6 and for the discussion it is useful to project the fluxes onto X_6, which in turn is equivalent to the decomposition under SU(3). So we define

$$H_{abc} = v^m F_{mabc} \quad (H \equiv v \lrcorner F) \quad , \qquad G_{abcd} = F_{abcd} \tag{6.1}$$

where the indices $a, b, \ldots = 1, \ldots, 6$ are related here to coordinates on X_6 and we denote the 4-form on X_6 by G. Strictly speaking, this reduction is valid only as long as v is not fibered over X_6, which would result in a non-trivial connection A and hence would result in a Chern-Simons term in G of the form $A \wedge H$. This is well known from Kaluza-Klein reduction, but since this is not important for the discussion here, we will define G without this Chern-Simons term. Actually we are not doing the dimensional reduction (v is in general not Killing), we only project the equations onto X_6 and distinguish between different components. Now, counting the number of components and decomposing them

under SU(3) we find

$$[H] \;=\; \mathbf{20} = \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \bar{\mathbf{1}}$$

$$[G] \;=\; \mathbf{15} = \mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$$

where we used the projector $\frac{1}{2}(\mathbb{1} \pm i\,\omega)$ to introduce (anti) holomorphic indices on X_6.

Our main results are in eqs. (5.19) – (5.21) combined with eqs. (5.25) and let us now discuss the BPS constraints on the different (projected) components.

The $\mathbf{1} + \bar{\mathbf{1}}$ of H are the (3,0) and (0,3) part, which fixes the superpotential as written in the form of

$$W = \frac{i}{36}\bar{\Omega}^{(0,3)} \lrcorner\, H \;\sim\; \frac{1}{36}\int_{X_7} F \wedge \Omega^{(3,0)} \;. \tag{6.2}$$

This is obviously a holomorphic superpotential, which can be verified from (5.19) by contracting the last two equations in (5.5) with v^a. It can also be obtained by contracting the spinor equation (5.12) by θ^T which gives the holomorphic expression

$$W \;\sim\; \theta^T F \theta \;.$$

Note, the differential forms $\tilde{\Omega}^k$ as given in (3.12) are only non-zero for the 3- or its dual 4-form.

Similarly, one can verify that the $\mathbf{3} + \bar{\mathbf{3}}$ components of G have to vanish. They are given by the (3,1)-part of G and we can project onto them by contracting (5.12) with $\theta^T \gamma_a$ which gives $0 = \theta^T \gamma_a F \theta \sim G_a{}^{bcd}\Omega_{bcd}$ on X_6, see again (3.12). But we can also see this by using (3.13) to write

$$\mathcal{F}^{(7)}|_{X_6} \equiv -\frac{1}{3!}\varphi \lrcorner\, F|_{X_6} \;=\; -\frac{1}{6}(\chi_+)\lrcorner\, G - \frac{1}{2}\omega \lrcorner\, H = d(e^{3A})\lrcorner\,\omega$$

where we used eqs. (5.20) and (5.21). Similarly, we get

$$2v \lrcorner\, \mathcal{F}^{(27)}|_{X_6} \;=\; -\frac{1}{6}(\chi_-)\lrcorner\, G + \frac{1}{2}H \lrcorner\,\omega^2 = d(e^{3A})|_{X_6} \;.$$

These two equations imply now that

$$\Omega \lrcorner\, G = 0 \tag{6.3}$$

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and

$$de^{3A} \lrcorner \omega = \frac{1}{2}\omega \lrcorner H \ . \tag{6.4}$$

Therefore, the $\mathbf{3} + \bar{\mathbf{3}}$ of G has to vanish defining G as a (2,2)-form and the warp factor is fixed by the non-primitive (2,1)-part of H (i.e. its $\mathbf{3} + \bar{\mathbf{3}}$).

Thus, the only non-zero components are so far

$$[H] = \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \bar{\mathbf{1}} \quad , \qquad [G] = \mathbf{8} + \mathbf{1} \ . \tag{6.5}$$

The $\mathbf{1} + \mathbf{8}$ of G are the components of a (2,2)-form, which is equivalent to the (1,1)-form obtained by $\omega \lrcorner G$. The $\mathbf{1}$ is the component that fixes the gradient of the warp factor along the vector v

$$\frac{1}{2\,4!}\, G^{(1)} \equiv \frac{1}{144}\omega^2 \lrcorner G = \left(\frac{3}{7}\mathcal{F}^{(1)} + v^a \mathcal{F}^{(27)}_{ab} v^b\right) = v^a \partial_a e^{3A} \tag{6.6}$$

which is obtained from (5.21). The $\mathbf{8}$ is the (1,1) part in $\mathcal{F}^{(27)}_{ab}$ and hence we can project onto it by[4]

$$\left[\delta_c{}^a \delta_d{}^b + \omega_c{}^a \omega_d{}^b\right] \mathcal{F}^{(27)}_{ab}$$

and it appears in (5.25) as the traceless symmetric component.

The remaining components of $\mathcal{F}^{(27)}_{ab}$, comprise a symmetric (2,0) and (0,2) tensor and give the $\mathbf{6} + \bar{\mathbf{6}}$ that enter H. These 12 components are related to the primitive (2,1)-part of H, i.e. the components that satisfy: $H^{(2,1)} \wedge \omega = 0$. They drop out in all our expressions and hence these components are unconstrained.

We might also ask whether X_6 is a complex manifold, i.e. whether $d\omega \in \Lambda^{(2,1)} \oplus \Lambda^{(1,2)}$ and $d\Omega \in \Lambda^{(3,1)}$ when projected on X_6 [51]. These requirements mean, that $d\omega$ has no (3,0) or (0,3) part and similarly $d\Omega$ has no (2,2) part (see also [22, 23]). Using the differential equations for the spinor (5.22), we find however that

$$\Omega^{(3,0)} \wedge d\omega \ \sim \ W\, i\,(\Omega^{(3,0)} \wedge \bar{\Omega}^{(0,3)})$$

and hence X_6 can only be complex if the superpotential vanishes. In fact, it is straight-forward to show that in this case also $(d\Omega)^{(2,2)} = 0$.

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[4]Note the contraction of (anti) holomorphic indices with ω yield "$\pm i$" so that this combination projects onto the (1,1)-part.

But this expression implies also that we can express the superpotential purely geo-metrically (without fluxes) by

$$W \sim \int_{X_7} iv \wedge d\omega \wedge \Omega^{(3,0)}$$

which has to be contrasted with the form derived in (6.2). Since our setup is only sensible to value of the superpotential in the vacuum it is difficult to distinguish between both expressions. For the case that v is Killing, we can however compare our results with the ones derived in [22, 27, 32] and this suggest that we have to add both expressions yielding

$$W \sim \int_{X_7} (F + iv \wedge d\omega) \wedge \Omega^{(3,0)} . \tag{6.7}$$

Upon reduction to 10 dimensions, this superpotential contains only fields that are com-mon in all string theories and this proposal based on a number of consistency checks (that it is U-dual to the type IIB superpotential and anomaly-free on the heterotic side).

7   Examples

So far, our discussion was general and let us now consider specific cases in more detail.

Case (i): The vector v is closed ($dv = 0$)

If v is closed, we can (at least locally) write $v = dz$, where the coordinate z can be regarded as the 11th or 5th coordinate. From (5.25) and the BPS constraints (5.20) we obtain now

$$v \wedge dA = 0 \quad \rightarrow \quad \mathcal{F}_a^{(7)} = \mathcal{F}_{am}^{(27)} v^m = 0 .$$

Therefore, only the non-vector components of $\mathcal{F}^{(27)}$ and $\mathcal{F}^{(1)}$ are non-zero and in the notation of (6.5), the $\mathbf{3} + \bar{\mathbf{3}}$ have to vanish. It is natural to consider a reduction over X_6 to obtain an effective 5-dimensional description. Note, X_6 does not need to be simple connected and one may also consider the case where X_6 factorizes into different components with different z-dependent warping. In any case, the resulting 5-d solution is not flat, but our 4-d external space represents a domain wall with z as the transversal coordinate. Since the fluxes on X_6 are non-trivial we get a potential for the 5-d theory,

which agrees with the real superpotential in [39, 38]

$$W_{5d} \sim \frac{1}{144} G^{(1)} = \frac{1}{144} \int_{X_6} G \wedge \omega \ .$$

Let us mention here a subtle issue. For the case discussed in this paper (i.e. SU(3) structures), the 4-d superpotential W does not come from the 5-d potential, but represents in 5 dimensions a kinetic term of two (axionic) scalars of a hyper multiplet[5], which is non-zero in the vacuum and curves the domain wall. That kinetic terms of (axionic) scalars act effectively as a potential can be understood from massive T-duality [52, 53] (if one allows for a linear dependence of the 5th direction) and we expect that these flux compactifications are dual to the curved domain walls discussed in [54, 55, 56]. The 5-d superpotential W_{5d} is instead compensated by the warp factor as in eq. (6.6), which is the known (5-d) BPS equation (in proper coordinates)

$$\partial_z A(z) = -3\, W_{5d} \ .$$

It is also instructive to discuss the fixing of the moduli. If we start with the scalars entering W_{5d} and expand the 4-form G as well as ω in a complete basis of harmonic 4- and 2-forms, resp. As result, the superpotential becomes $W_{5d} = p_I t^I = e^{2\varphi} p_I X^I$, where p_I are the coefficients from the expansion of the 4-form G, t^I are the Kähler class moduli, which are rescaled by the volume scalar $e^{2\varphi}$. The resulting fields $X^I = X^I(\phi^A)$ parameterize the vector multiplet moduli space $\mathcal{M}_V$ of 5-d (gauged) supergravity and *all* of them are fixed in the vacuum. In fact their value can be determined explicitly using the so-called attractor mechanism[6] [59]. The (5-d) vacuum is namely defined by the equations: $\frac{\partial}{\partial \phi^A} W_{5d} = p_I \frac{\partial X^I}{\partial \phi^A} = 0$ and since $\frac{\partial X^I}{\partial \phi^A}$ is a tangent vector on the moduli space $\mathcal{M}_V$, the vacuum is given by the point where the flux vector p_I is normal on the manifold $\mathcal{M}_V$, see [38] for more details. In a generic situation this is a point on $\mathcal{M}_V$ and thus *all* vector moduli are fixed by these fluxes. Note, the volume scalar gives a run-away behavior of the potential, which results into a power-like behavior of the warp factor in the coordinate z (especially these models do not give rise to an exponential warping) and consequently the supergravity solution exhibits a naked singularity.

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[5]These are the scalars coming from the (3,0) and (0,3) part of the 11-dimensional 3-form potential.

[6]It was originally introduced to calculate the black hole entropy, but can also be used in gauged supergravity to determine the fixed point values of vector multiplet moduli. A number of explicit solutions of these algebraic equations are known in 5 but also 4 dimensions [37, 38, 57, 58].

How about the complex structure moduli? They are related to scalars in hyper multiplet, which parameterize a quaternionic manifold and they enter the 5-d superpotential only if it is SU(2)-valued (coming from the momentum maps), see e.g. [60, 61]. This is beyond our spinor ansatz, but we do have 3-form fluxes on X_6, which enter the 4-d superpotential and might lift the complex structure moduli space nevertheless. In fact, for the 4-d superpotential as given in the form (6.2) we could repeat the procedure as done for W_{5d} and the corresponding 4-d attractor formalism will yield fixed point values. But since X_6 is not a complex manifold for $W \neq 0$, the expansion in harmonic forms becomes subtle and we do not know whether this is a reliable procedure. On the other hand, the 4-d superpotential corresponds in 5 dimensions to kinetic terms of scalar fields in hypermultiplets. Because these kinetic terms do not vanish in the vacuum and the quaternionic metric is not invariant under generic deformations of the hyperscalar field, the complex structure moduli space seemed to be lifted for all deformation that do not represent isometries of the quaternionic space.

Setting $H = W = 0$ our setup describes the model discussed in [37, 38]. In this case X_6 is Kähler and it is straightforward to introduce a brane which is a source for the flux and corresponds to an M5-brane wrapping a holomorphic curve in X_6. The worldvolume of the M5-brane can be identified by the 6-form: $*(dA \wedge F)$. Since $dA \sim dz \sim v$, the 5-brane does not wrap the v-direction, but the 2-cycle corresponding to the (1,1)-form $*_6 G$ [note G is (2,2) form containing the SU(3) representations $\mathbf{1} + \mathbf{8}$].

Case (ii): v is Killing, i.e. $\nabla_{\{m} v_{n\}} = 0$

If both indices are on X_6, eq. (5.25) implies that $\nabla_{\{a} v_{b\}}$ is fixed by the components $\mathbf{1} + \mathbf{8}$ of G and if one index is along v, i.e. $v^m \nabla_{\{m} v_{b\}}$, which is $\omega_{bc} \partial^c A$. Since the $G^{(3,1)} = 0$ we conclude that $\nabla_{\{m} v_{n\}} = 0$ implies

$$G_{abcd} = 0 \qquad , \qquad dA|_{X_6} = 0 \ .$$

Note, if $G = 0$ also $v^m \partial_m A = 0$ and thus the warp factor has to be constant, see (6.6). Therefore, the only non-zero flux components are the $\mathbf{1} + \bar{\mathbf{1}} + \mathbf{6} + \bar{\mathbf{6}}$ of H. If in addition, the superpotential vanishes, only the primitive $H^{(2,1)}$ is non-zero and since $d\omega|_{X_6} \in \Lambda_0^{(2,1)}$ and $d\Omega|_{X_6} = 0$, X_6 is an Iwasawa manifold. Because v is a Killing vector, we can make

a dimensional reduction and these manifolds have been discussed in string theory in more detail in [23]. Note, since $|v|^2 = 1$, the 10-d dilaton as well as the warp factor is constant for these string theory vacua.

Case (iii): $\tilde{v} \equiv e^{\phi} v$ is Killing, i.e. $\nabla_{\{m} v_{n\}} = -v_{\{n} \partial_{m\}} \phi$

This case is closely related to the last case. Again we get

$$G_{abcd} = 0 \ ,$$

but now, all components of the H-field can be non-zero and the $\mathbf{3} + \bar{\mathbf{3}}$ gives $d\phi$. We find

$$\phi = -3A$$

and note ϕ as well as A do not depend on the coordinate along the vector v, because from (6.6) follows for $G = 0$ that $v^m \partial_m A = 0$. If we now use this Killing vector $\tilde{v}$ for dimensional reduction, the scalar field ϕ becomes up to a rescaling the dilaton ($|\tilde{v}|^2 = e^{2\phi}$). Since all components of the 4-form G are zero, this case describes the common sector of all string models and if the superpotential vanishes, we obtain the vacuum already discussed in heterotic string models [7, 23] (with trivial gauge fields; see [36] for a recent review).

Case (iv): realization of MQCD

Finally, we want to discuss the relation to the setup in [42, 11, 29], where a 4-dimensional N=1 field theory was realized on a M5-brane that wrap a 2-cycle in the internal space. In 10 dimensions this configuration corresponds to an intersection of two NS5-branes, which are rotated by an SU(2) angle and D4-branes end on both NS5-branes. In 11 dimension the M5-branes wrap a 2-cycle in a 6-dimensional space, which we identify with X_6 in our setup and which means that the 11th coordinate is *not* along v, but inside of X_6. Of course, our notation might now be misleading, since the H field in our setup is not related to the H appearing after dimensional reduction to 10 dimensions along v. Recall the 2-cycle wrapped by the M5-branes is identified by the 2-form $*(dA \wedge F)$ and in the simplest case as assumed in [42], the warp factor does not depend on the coordinate along v (i.e. $v^m \partial_m A = 0$) and therefore dA lies always inside X_6. As consequence, the

4-form F has always a v component and therefore G is zero and only the H field is non-trivial. This is exactly the situation that we described in the previous case and in fact both 10-d configurations have the same 11-d origin, they differ only in the choice of the 11th coordinate.

Acknowledgments

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