

Challenging the Paradigm of Singularity Excision in Gravitational Collapse

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A paradigm deeply rooted in modern numerical relativity calculations prescribes the removal of those regions of the computational domain where a physical singularity may develop. We here challenge this paradigm by performing three-dimensional simulations of the collapse of uniformly rotating stars to black holes without excision. We show that this choice, combined with suitable gauge conditions and the use of minute numerical dissipation, improves dramatically the long-term stability of the evolutions. In turn, this allows for the calculation of the waveforms well beyond what was previously possible, providing information on the black-hole ringing and setting a new mark on the present knowledge of the gravitational-wave emission from the stellar collapse to a rotating black hole.

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Numerical relativity simulations have recently recorded important breakthroughs, which have allowed for long-term stable and accurate calculations of curved and highly dynamical spacetimes, thus increasing their potential for providing physically relevant predictions for gravitational-wave astronomy. Behind this rapid progress are various novel approaches, some of which involve the dropping of assumptions or techniques that were considered to be important or simply necessary. First, the dismissal of the $3 + 1$ ADM formulation of the field equations, which, after large-scale efforts [1], has shown not to have the stability properties necessary for long-term fully three-dimensional (3D) simulations. In lieu of the ADM equations, new formulations have been either reconsidered (as in the case of the conformal and traceless formulation of the ADM equations [2]) or investigated for the first time in 3D simulations (as in the case of the harmonic formulation of the Einstein equations [3]). Both approaches have been shown to provide long-term stability on time scales sufficiently large to evolve accurately a large class of spacetimes, including black holes [3–5] and neutron stars [6,7]. Second, the abandoning of the use of numerical grids with uniform spacing, in lieu of which several codes now use mesh-refinement techniques (either fixed [8] or with adaptivity [4]). This conceptually simple but technologically challenging improvement allows concentration of computational resources where the truncation error needs to be the smallest, while saving them where they may not be needed. In turn, nonuniform grids have allowed us to place the outer boundaries of the computational domain at very large distances, thus reducing the influence of inaccurate outer-boundary conditions and making it possible to have the wave zone within the domain and to extract there the precious gravitational-wave signal [9]. Third, for some years now, successful long-term 3D evolutions of black-hole vacuum spacetimes have been possible only thanks to the use of excision techniques (see, e.g., Ref. [10] for a first

example), that is, by ignoring the spacetime regions inside black-hole horizons. These are causally disconnected from the outside and should have no effect on the rest of the evolution as long as a suitable treatment of the equations is made at the excision surface. Furthermore, recent simulations of the collapse of rotating neutron stars to Kerr black holes [7,11] have shown the effectiveness of excision techniques also for spacetimes with matter, where they were applied separately to the field equations and to the hydrodynamical equations. In those simulations, in fact, the use of excision has extended considerably the lifetime of the simulations, allowing for an accurate investigation of the dynamics of the trapped surfaces formed during the collapse and for the extraction of the first gravitational waveforms from 3D collapse to rotating black holes [9].

Although the assumption that a region of spacetime that is causally disconnected can be ignored without this affecting the solution in the remaining portion of the spacetime is certainly true for signals and perturbations travelling at physical speeds, numerical signals, such as gauge waves or constraint violations, may travel at velocities larger than that of light and thus leave the physically disconnected region. Indeed, this is what is commonly observed when excising a topologically spherical surface in Cartesian coordinates within a conformal and traceless formulation of the Einstein equations and without the massive use of numerical dissipation as in Ref. [3].

In this Letter we challenge the paradigm of singularity excision and show that accurate numerical simulations can be carried out even in the absence of an excised region and independently of whether the spacetime is vacuum and the singularity modeled as a “puncture” [4,5]. In addition, we show that the absence of an excised region improves dramatically the long-term stability, allowing for the calculation of the gravitational waveforms well beyond what was previously possible and past the black-hole quasi-normal-mode (QNM) ringing.

The simplest and most impressive way to prove the improvements resulting from not excising the region inside the apparent horizon (AH) is to reconsider the same initial data of Ref. [9], which lead to gravitational collapse to rotating black holes. More specifically, we consider rotating relativistic stars calculated as equilibrium solutions of the Einstein equations, with a polytropic equation of state $p = K\rho^\Gamma$, with $\Gamma = 2$ and with the polytropic constant initially set to $K_{\text{ID}} = 100$ [12]. To illustrate the collapse for the range of neutron stars rotating either very slowly or near the mass-shedding limit, we consider two representative models, indicated as D1 and D4 in Ref. [11], with different initial angular momenta. The first one is a slowly rotating star of mass $M = 1.67M_\odot$, circumferential equatorial radius $R_e = 11.43$ km and $a \equiv J/M^2 = 0.21$, where J is the angular momentum; the second one has $M = 1.86M_\odot$, $R_e = 14.25$ km and is rotating close to the mass-shedding limit with $a = 0.54$.

The numerical methods used for these evolutions are the same as discussed in Ref. [9]. In particular, we use a conformal and traceless formulation of the ADM equations [13], which are solved with fourth-order finite-difference operators, while we evolve the hydrodynamical equations with the WHISKY code [14], implementing high-resolution shock-capturing (HRSC) techniques with a variety of approximate Riemann solvers and reconstruction algorithms and an overall second-order truncation error [11]. The numerical grid setup makes use of the same mesh refinement implemented in the numerical infrastructure described in Ref. [8]. Besides a fixed number of refinement levels which are present already on the initial slice, new refined levels are added at predefined positions during the evolution. More specifically, as the star collapses, the innermost (most refined) grid box is progressively further refined with box-in-box grids, the final one having a resolution of $\Delta x \approx 0.02M$; the outermost grid, instead, which is not further refined, has a resolution of $\Delta x \approx 1.5M$, sufficient to capture the details of the gravitational radiation. In this way, our outer boundaries are placed at $\approx 165M$, so as to minimize the influence of our imperfect outer-boundary conditions on the very small gravitational-wave signal. We note that the “switching-on” of different levels of resolution does have a small but appreciable effect on its dynamics, essentially due to the necessary interpolation to the new refined grids, which also changes the growth time of the most unstable, exponentially growing mode. This effect is negligible in the evolution of the matter but can influence the waveforms. To provide a direct comparison with Ref. [9], we have here followed the same approach.

Figure 1 summarizes the dynamics of the matter for model D4 by showing the time evolution of the maximum of the rest-mass density ρ_{max} (continuous lines) and of the total rest mass M_* (dashed lines) when normalized to their initial values (a similar behavior is observed also for the slowly rotating model D1). The evolution obtained without excision is to be contrasted with the one in which the excision is made (the used excision technique is described

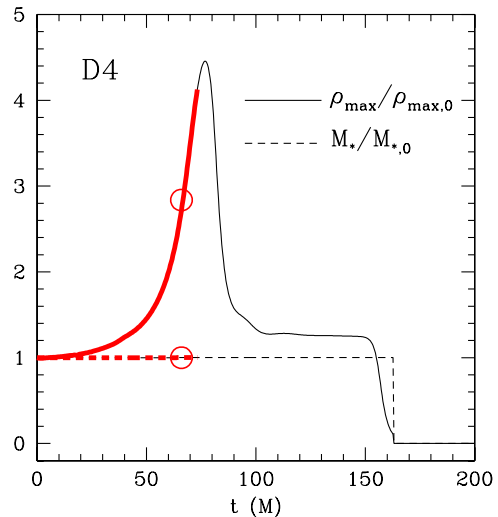


FIG. 1 (color online). Evolution of the normalized maximum of the rest-mass density ρ_{max} (solid line) and of the total rest-mass M_* (dashed line) for model D4. Circles indicate the times at which the AH is first found and thick lines the results obtained with excision.

in detail in Ref. [15]) and which is indicated with thick lines. In that case, the excision was started soon after an AH [16] was found (this is marked with circles on the two curves) and the corresponding evolution terminated at $t \geq 77M$, when the code crashed with large violations in the Hamiltonian constraint. Note that while the maximum values for the rest-mass density attained during the evolution are comparable in the two cases, the evolution without excision can be carried out to much later times (i.e. $t \geq 300M$) without appreciable loss of accuracy or sign of instability. Indeed, as the matter collapses and concentrates over a very few grid points (and ultimately on only one), the high accuracy of the HRSC methods is able to conserve the rest mass to very high precision with a loss of less than 0.2% up to when the rest-mass density distribution is diffused as a result of the poorly resolved gradients.

While very little extra is needed for the evolution of the hydrodynamical quantities in the absence of an excision algorithm, a stable evolution of the Einstein equations requires at least two important ingredients. The first one is represented by gauge conditions for the lapse function α with suitable singularity-avoidance properties. Our experience has shown that hyperbolic K -driver slicing conditions of the form $\partial_t \alpha = -f(\alpha)\alpha^2(K - K_0)$, with $f > 0$ and K_0 being the trace of the extrinsic curvature at $t = 0$, are essential to “freeze” the evolution in those regions of the computational domain inside the AH, where the metric functions experience the growth of very large gradients. In practice, we confirm that using the generalized “1 + log” slicing condition [17] as obtained by setting $f = 2/\alpha$ provides the desired singularity avoidance and is computationally efficient. With a good choice for the slicing condition, the results do not depend sensitively on the gauge condition for the shift. We have found that the

use of the ‘‘Gamma-driver’’ shift conditions discussed in Ref. [11] is sufficient to compensate for the ‘‘slice stretching’’ induced by the singularity-avoiding slicing.

The second important ingredient is the introduction of an artificial dissipation of the Kreiss-Oliger type [18] on the right-hand sides of the evolution equations for the spacetime variables and the gauge quantities (no dissipation is introduced for the hydrodynamical variables). This is needed mostly because all the field variables develop very steep gradients in the region inside the AH. Under these conditions, small high-frequency oscillations (either produced by finite-differencing errors or by small reflections across the refinement boundaries) can easily be amplified, leave the region inside the AH and rapidly destroy the solution. In practice, for any time-evolved quantity u , the right-hand side of the corresponding evolution equation is modified with the introduction of a term of the type $\mathcal{L}_{\text{diss}}(u) = -\varepsilon \Delta x_i^3 \partial_{x_i}^4 u$, where ε is the dissipation coefficient, which is allowed to vary in space. We have experimented with configurations in which the coefficient was either constant over the whole domain or larger for the grid points inside the AH. No significant difference has been noticed between the two cases. Much more sensitive is, instead, the choice of the value of ε . In the simulations reported here we have set $\varepsilon = 0.01$ for model D1 and $\varepsilon = 0.0075$ for D4, respectively. However, smaller values (e.g. $\varepsilon = 0.005$) are not sufficient to yield the long-term stability discussed here, while larger values (e.g. $\varepsilon = 0.05$ and $\varepsilon = 0.01$, respectively) alter significantly the waveforms, which would not match the ones obtained without dissipation until the latter can be computed.

In the region within the AH, as a consequence of large gradients in the spacetime variables, which cannot be resolved sufficiently despite the use of several mesh-refinement levels, the solution of the Einstein equations becomes increasingly inaccurate as the collapse proceeds. Figure 2 offers a measure of this loss of accuracy by showing the time evolution of the $L2$ norm of the Hamiltonian-constraint violation for model D1. To distinguish the different amplitudes of the errors, the violation has been computed separately for the domain inside the AH and for the rest of the grid.

First, the two errors are considerably different in amplitude, the one inside the AH being much larger. Second, the sudden decrease in the long-dashed and continuous lines at the time of AH formation at $t \approx 60M$ is due to the fact that some grid points (those with the largest violations) are no longer included in the calculation of the $L2$ norm. Third, the violation outside the AH grows only at a slow pace over the remaining evolution. Finally, shown with circles is the violation for a high-resolution simulation. The values are rescaled to second order before AH formation (when the dominant truncation error comes from hydrodynamics and the HRSC methods used are only second-order) and to third order afterwards (when the smooth accretion flow of the atmosphere boosts the accuracy of the hydrodynam-

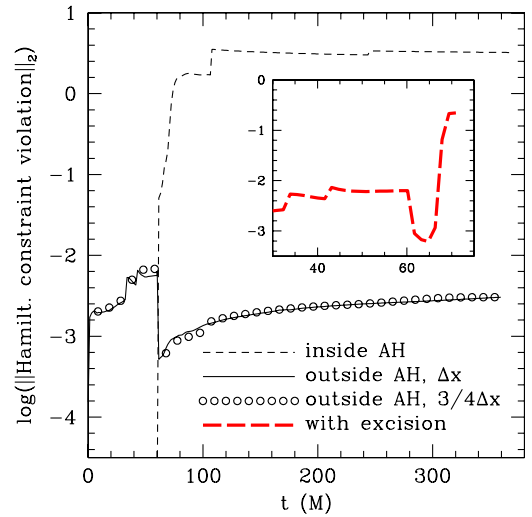


FIG. 2 (color online). Evolution of the $L2$ norm of the Hamiltonian-constraint violation for model D1. Different curves refer to the violation computed only inside the AH (short-dashed line), only outside it (continuous line) or when the excision is made (long-dashed line in the inset). The circles show the rescaled violation for a higher resolution.

ics to third order). No convergence of the violation is seen inside the AH. Overall, Fig. 2 shows that the solution of the Einstein equations inside the AH is by and large incorrect, but these errors remain confined within the AH and do not contaminate the overall accuracy of the simulation. Shown for comparison in the inset is the violation computed when the singularity is excised, showing an exponential growth soon after the AH is found.

Note that the ability to perform these long-term evolutions cannot be related in any manner to the use of a puncture prescription for the physical singularity, as recently done in simulations of binary black holes [4,5]. However, it is possible that the stability provided here by the singularity-avoiding gauge and made numerically more robust by the use of dissipation is closely related to the one discussed for punctures [19].

Besides a long-term stability and the possibility of following the collapse well beyond what was possible with the use of excision techniques, the most dramatic advantage produced by the approach suggested here can be appreciated in the study of the gravitational radiation produced during the collapse. As in Ref. [9], we have extracted the gravitational-wave information through an approach in which the spacetime is matched with the non-spherical perturbations of a Schwarzschild black hole described in terms of gauge-invariant odd $Q_{\ell m}^{(o)}$ and even-parity $\Psi_{\ell m}^{(e)}$ metric perturbations, where ℓ, m are the indices of the angular decomposition. In Fig. 3 we report the lowest-order multipoles $Q_{20}^+ \equiv \lambda \Psi_{20}^{(e)}$, where $\lambda \equiv \sqrt{2(\ell+2)!/(\ell-2)!}$. The left panel, in particular, shows the signal detected by an observer at a distance of $42M$ for the slowly rotating model D1; the right panel, instead,

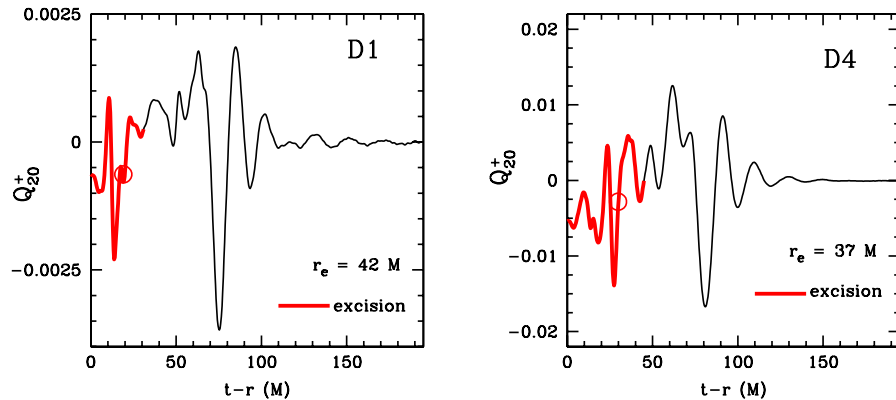


FIG. 3 (color online). Lowest gauge-invariant multipole for a slowly rotating star (left panel) and a rapidly rotating one (right panel). Thick lines refer to evolutions carried out with excision and terminate at a code crash, while the circles indicate the AH formation time.

shows the same multipole extracted at $37M$ for the rapidly rotating model D4. In both cases the thick lines refer to the evolutions carried out with the excision technique and terminate when the code crashed (cf. left panel of Fig. 1 in Ref. [9]). This comparison shows that it is now possible to detect the complete gravitational-wave signal produced by the collapse of a relativistic star to a rotating black hole well beyond what was previously possible in either 2D [20] or 3D simulations [9]. The estimated error on the phase is $\lesssim 1\%$ and $\lesssim 5\%$ on the $L2$ norm of the amplitude.

Figure 3 also highlights that the signal can be rather different both in amplitude and form in the two cases with the exception of the final parts, when the signal is dominated by the black-hole QNM ringing. This happens between $\approx 80M$ and $\approx 120M$, where the signal we extract matches very well with a perturbative one computed using the frequencies given in Ref. [21] for the angular momenta and masses of our models. The very good agreement with the perturbative results is an important confirmation of the accuracy of our results, which are intrinsically plagued by the extreme weakness of the emitted gravitational radiation. Furthermore, the richness of details in the two waveforms opens the prospects that a careful characterization of the waveforms will provide important information on the properties of the black hole as well as on those of the collapsing matter [22].

A straightforward analysis of the *now-complete* gravitational-wave signal computed in these simulations allows us to improve the estimates provided in Ref. [9] for the detectability of the gravitational collapse of a uniformly rotating polytropic star at a distance of 10 kpc. More specifically, the energy efficiency in the emission of gravitational radiation is $E_{D1}/M = 3.3 \times 10^{-7}$ and $E_{D4}/M = 3.7 \times 10^{-6}$, with an overall accuracy of $\sim 10\%$. The resulting signal-to-noise ratios are then: $(S/N)_{D1-D4}^{\text{Virgo}} \approx 0.27 - 2.1$, $(S/N)_{D1-D4}^{\text{advLIGO}} \approx 1.2 - 11$, and $(S/N)_{D1-D4}^{\text{Dual}} \approx 3.3 - 28$ for detectors such as Virgo/LIGO, advanced LIGO, or Dual [23].

In conclusion, we have presented the first 3D calculations of the gravitational collapse of uniformly rotating stars to black holes without excision. This choice, together

with suitable gauge conditions and the use of minute numerical dissipation, improves dramatically the long-term stability of the evolutions, providing the most accurate waveforms of this process to date. While our approach represents a challenge to the paradigm of singularity excision, it does not necessarily imply that all excision techniques should be expected to lead to instabilities. Rather, it highlights that, for a conformal traceless formulation of the Einstein equations and in highly dynamical spacetimes, the excision of a spherical surface in Cartesian coordinates may be more of a problem than a solution.

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- [1] G. B. Cook *et al.*, Phys. Rev. Lett. **80**, 2512 (1998).
 - [2] T. Nakamura *et al.*, Prog. Theor. Phys. Suppl. **90**, 1 (1987).
 - [3] F. Pretorius, Phys. Rev. Lett. **95**, 121101 (2005).
 - [4] J. G. Baker *et al.*, Phys. Rev. Lett. **96**, 111102 (2006).
 - [5] M. Campanelli *et al.*, Phys. Rev. Lett. **96**, 111101 (2006).
 - [6] M. Shibata, Astrophys. J. **595**, 992 (2003).
 - [7] M. D. Duez *et al.*, Phys. Rev. D **67**, 024004 (2003).
 - [8] E. Schnetter *et al.*, Classical Quantum Gravity **21**, 1465 (2004).
 - [9] L. Baiotti *et al.*, Phys. Rev. Lett. **94**, 131101 (2005).
 - [10] E. Seidel and W.-M. Suen, Phys. Rev. Lett. **69**, 1845 (1992).
 - [11] L. Baiotti *et al.*, Phys. Rev. D **71**, 024035 (2005).
 - [12] This is subsequently reduced by 2% to trigger the collapse.
 - [13] M. Alcubierre *et al.*, Phys. Rev. D **62**, 044034 (2000).
 - [14] L. Baiotti *et al.*, Mem. Soc. Astron. It. Suppl. **1**, 210 (2003).
 - [15] M. Alcubierre *et al.*, gr-qc/0411137.
 - [16] J. Thornburg, Classical Quantum Gravity **21**, 743 (2004).
 - [17] C. Bona *et al.*, Phys. Rev. Lett. **75**, 600 (1995).
 - [18] H.-O. Kreiss and J. Olinger, GARP Publ. Ser. **10** (1973).
 - [19] M. Hannam *et al.*, gr-qc/0606099.
 - [20] R. F. Stark and T. Piran, Phys. Rev. Lett. **55**, 891 (1985).
 - [21] E. Berti *et al.*, Phys. Rev. D **73**, 064030 (2006).
 - [22] L. Baiotti *et al.* (to be published).
 - [23] M. Bonaldi *et al.*, Phys. Rev. D **68**, 102004 (2003).