Kinetic and mass mixing with three abelian groups

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We present the possible mixing effects associated with the low-energy limit of a Standard-Model extension by two abelian gauge groups $U(1)_1 \times U(1)_2$. We derive general formulae and approximate expressions that connect the gauge eigenstates to the mass eigenstates. Applications using the well-studied groups $U(1)_B$, $U(1)_{B-L}$, $U(1)_{L_\alpha - L_\beta}$ (L_α being lepton flavor numbers), and $U(1)_{\rm DM}$ (a symmetry acting only on the dark matter sector) are discussed briefly.

I. INTRODUCTION

Augmenting the Standard Model (SM) gauge group $G_{\rm SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ by an additional abelian group $U(1)'$ is well motivated by grand unified theories (GUTs) [1], flavor symmetries [2], and dark matter (DM) models [3]. Depending on the symmetry breaking scheme a non-diagonal mass matrix for the neutral gauge bosons is possible, so the physical mass eigenstates are linear combinations of the original gauge eigenstates (henceforth referred to as mass mixing). The precise measurements of the masses and couplings of the SM gauge bosons $Z_{\rm SM}$ and $W^\pm$ at LEP put stringent constraints on the mixing parameters and consequently on the symmetry breaking sector. An entirely different type of mixing is associated with the kinetic terms of the gauge fields: Since the field strength tensor $F^{\mu\nu}$ of an abelian gauge group is a gauge invariant object of mass dimension 2, a renormalizable Lagrangian can contain non-canonical kinetic cross-terms $\propto \sin\chi F_1^{\mu\nu} F_{2,\mu\nu}$ if the gauge group includes $U(1)_1 \times U(1)_2$. The kinetic mixing angle χ modifies the coupling of the corresponding gauge bosons and can therefore lead to observable effects [4]. The case of two abelian groups – one of them being the hypercharge gauge group $U(1)_Y$ – is well studied and widely used in model building, but the generalization to more abelian factors is seldom discussed, even though this structure naturally occurs in some string theory and GUT models [5]. Renormalizability of the theory requires the gauge group to be free of anomalies, which drastically limits the allowed additional $U(1)'$ groups, unless additional fermions are introduced. This condition is of course even more constraining in gauge extensions with several new abelian factors; even without tapping into the various GUT-inspired symmetries, there are several interesting combinations of well-studied symmetries that lead to valid models, e.g. $U(1)_L \times U(1)_B$ [6], $U(1)_B \times U(1)_{\rm DM}$, or $U(1)_{B-L} \times U(1)_{L_\mu - L_\tau}$.

We will present the generalization of the well-studied gauge group $G_{\rm SM} \times U(1)'$ to $G_{\rm SM} \times U(1)' \times U(1)''$, which introduces three kinetic mixing angles and three mass-mixing parameters. To demonstrate possible applications in model building we show that $U(1)_B \times U(1)_{\rm DM}$ generates isospin-dependent nucleon-DM scattering and that $U(1)_{B-L} \times U(1)_{L_\mu - L_\tau}$ can in principle induce non-standard neutrino interactions (NSIs).

The remaining part of this work is organized as follows. In Sec. II we will derive the connection between gauge and mass eigenstates for the neutral vector bosons and give approximate expressions for the mixing matrix and mass shifts. Specific models for dark matter model building and flavor symmetries will be presented in Sec. III. We summarize and conclude our findings in Sec. IV.

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II. KINETIC AND MASS MIXING

The most general effective Lagrange density after breaking $G_{\rm SM} \times U(1)_1 \times U(1)_2$ to $SU(3)_C \times U(1)_{\rm EM}$ can be written as $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{X_1} + \mathcal{L}_{X_2} + \mathcal{L}_{\rm mix}$, with

$$
\begin{aligned}
\mathcal{L}_{\rm SM} &= -\frac{1}{4}\hat{B}_{\mu\nu}\hat{B}^{\mu\nu} - \frac{1}{4}\hat{W}^a_{\mu\nu}\hat{W}^{a\mu\nu} + \frac{1}{2}\hat{M}_Z^2 \hat{Z}_\mu \hat{Z}^\mu - \frac{\hat{e}}{\hat{c}_W}j_Y^\mu \hat{B}_\mu - \frac{\hat{e}}{\hat{s}_W}j_W^{a\mu}\hat{W}^a_\mu \,, \\
\mathcal{L}_{X_i} &= -\frac{1}{4}\hat{X}_{i\,\mu\nu}\hat{X}_i^{\mu\nu} + \frac{1}{2}\hat{M}_{X_i}^2 \hat{X}_{i\,\mu}\hat{X}_i^\mu - \hat{g}_i j_i^\mu \hat{X}_{i\,\mu}\,, \qquad i=1,2\,, \\
\mathcal{L}_{\rm mix} &= -\frac{\sin\alpha}{2}\hat{B}_{\mu\nu}\hat{X}_1^{\mu\nu} - \frac{\sin\beta}{2}\hat{B}_{\mu\nu}\hat{X}_2^{\mu\nu} - \frac{\sin\gamma}{2}\hat{X}_{1\,\mu\nu}\hat{X}_2^{\mu\nu} \\
&\quad + m_1^2\,\hat{Z}_\mu \hat{X}_1^\mu + m_2^2\,\hat{Z}_\mu \hat{X}_2^\mu + m_3^2\,\hat{X}_{1\,\mu}\hat{X}_2^\mu\,.
\end{aligned}
\tag{1}
$$

The currents are defined as

$$
\begin{aligned}
j_Y^\mu &= -\sum_{\ell=e,\mu,\tau}\left[\overline{L}_\ell \gamma^\mu L_\ell + 2\,\overline{\ell}_R \gamma^\mu \ell_R\right] + \frac{1}{3}\sum_{\rm quarks}\left[\overline{Q}_L \gamma^\mu Q_L + 4\,\overline{u}_R \gamma^\mu u_R - 2\,\overline{d}_R \gamma^\mu d_R\right]\,, \\
j_W^{a\mu} &= \sum_{\ell=e,\mu,\tau}\overline{L}_\ell \gamma^\mu \frac{\sigma^a}{2}L_\ell + \sum_{\rm quarks}\overline{Q}_L \gamma^\mu \frac{\sigma^a}{2}Q_L\,,
\end{aligned}
\tag{2}
$$

with the left-handed $SU(2)_L$ doublets Q_L and L_ℓ and the Pauli matrices σ^a. We also define the electromagnetic current $j_{\rm EM} \equiv j_W^3 + \frac{1}{2}j_Y$ and the weak neutral current $j_{\rm NC} \equiv 2j_W^3 - 2\hat{s}_W^2 j_{\rm EM}$; the currents j_1 and j_2 are left unspecified for now. We furthermore define the fields $\hat{A} \equiv \hat{c}_W \hat{B} + \hat{s}_W \hat{W}_3$ and $\hat{Z} \equiv \hat{c}_W \hat{W}_3 - \hat{s}_W \hat{B}$, corresponding to the photon and the $Z_{\rm SM}$ boson in the absence of $\mathcal{L}_{\rm mix}$. Here and in the following we will often omit the Lorentz indices on currents and gauge fields, expressions such as jA are to be read as $j^\mu A_\mu$.

Due to our parameterization of the kinetic mixing angles, the hypercharge field strength tensor $\hat{B}_{\mu\nu}$ and the field strength tensors $\hat{X}_i^{\mu\nu}$ of $U(1)_1 \times U(1)_2$ share the symmetric mixing matrix

$$
\mathcal{L} \supset -\frac{1}{4}\left(\hat{B}^{\mu\nu},\ \hat{X}_1^{\mu\nu},\ \hat{X}_2^{\mu\nu}\right)\begin{pmatrix} 1 & \sin\alpha & \sin\beta \\ \cdot & 1 & \sin\gamma \\ \cdot & \cdot & 1 \end{pmatrix}\begin{pmatrix} \hat{B}_{\mu\nu} \\ \hat{X}_{1\,\mu\nu} \\ \hat{X}_{2\,\mu\nu} \end{pmatrix}.
\tag{3}
$$

In complete analogy to Ref. [4] we can transform the gauge fields $(\hat{B}, \hat{X}_1, \hat{X}_2)$ into a basis (B, X_1, X_2) with canonical (diagonal) kinetic terms

$$
\begin{pmatrix} \hat{B} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \begin{pmatrix} 1 & -t_\alpha & (t_\alpha s_\gamma - s_\beta/c_\alpha)/D \\ 0 & 1/c_\alpha & (t_\alpha s_\beta - s_\gamma/c_\alpha)/D \\ 0 & 0 & c_\alpha/D \end{pmatrix}\begin{pmatrix} B \\ X_1 \\ X_2 \end{pmatrix},
\tag{4}
$$

where $D \equiv \sqrt{1 - s_\alpha^2 - s_\beta^2 - s_\gamma^2 + 2s_\alpha s_\beta s_\gamma}$, $s_x \equiv \sin x$, $c_x \equiv \cos x$, and $t_x \equiv \tan x$. The transformation (4) diagonalizes the kinetic terms and yields the massless photon A and the mass matrix for the massive neutral fields in the basis (Z, X_1, X_2)

$$
\mathcal{M}^2 = \begin{pmatrix} \hat{M}_Z^2 & m_1^2/c_\alpha + \hat{M}_Z^2 \hat{s}_W t_\alpha & M_{13}^2 \\ \cdot & \hat{M}_{X_1}^2/c_\alpha^2 + \hat{s}_W t_\alpha(2m_1^2 + \hat{M}_Z^2 \hat{s}_W s_\alpha)/c_\alpha & M_{23}^2 \\ \cdot & \cdot & M_{33}^2 \end{pmatrix},
\tag{5}
$$

with the three extra long expressions

$$
\begin{aligned}
M_{13}^2 \cdot c_\alpha D &\equiv (\hat{M}_Z^2 \hat{s}_W(s_\beta - s_\alpha s_\gamma) + m_1^2(s_\alpha s_\beta - s_\gamma) + m_2^2 c_\alpha^2)\,, \\
M_{23}^2 \cdot c_\alpha^2 D &\equiv \hat{M}_{X_1}^2(s_\alpha s_\beta - s_\gamma) + \hat{M}_Z^2 \hat{s}_W^2 s_\alpha(s_\beta - s_\alpha s_\gamma) + m_1^2 \hat{s}_W(s_\beta - 2s_\alpha s_\gamma + s_\beta s_\alpha^2) \\
&\quad + m_2^2 \hat{s}_W s_\alpha c_\alpha^2 + m_3^2 c_\alpha^2\,, \\
M_{33}^2 \cdot c_\alpha^2 D^2 &\equiv \hat{M}_{X_2}^2 c_\alpha^4 + \hat{M}_{X_1}^2(s_\gamma - s_\alpha s_\beta)^2 + \hat{M}_Z^2 \hat{s}_W^2(s_\beta - s_\alpha s_\gamma)^2 \\
&\quad - 2m_1^2 \hat{s}_W(s_\alpha s_\beta - s_\gamma)(s_\alpha s_\gamma - s_\beta) + 2m_2^2 c_\alpha^2 \hat{s}_W(s_\beta - s_\alpha s_\gamma) \\
&\quad + 2m_3^2 c_\alpha^2(s_\alpha s_\beta - s_\gamma)\,.
\end{aligned}
\tag{6}
$$

$\mathcal{M}^2$ is a real symmetric matrix and can therefore be diagonalized by an orthogonal matrix U: $U^T \mathcal{M}^2 U = \text{diag}(M_1^2, M_2^2, M_3^2)$, with M_i^2 being the physical fields. This diagonalization introduces in general three more mixing angles ξ_i that are connected to the entries in $\mathcal{M}^2$. The gauge eigenstates $\hat{A}$, $\hat{Z}$, $\hat{X}_1$, and $\hat{X}_2$ couple to the currents $\hat{e}j_{\text{EM}}$, $\hat{g}_Z j_{\text{NC}}$,[1] $\hat{g}_1 j_1$, and $\hat{g}_2 j_2$, respectively, and are connected to the physical mass eigenstates A, Z_1, Z_2, and Z_3 via

$$
\begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\hat{c}_W t_\alpha & \hat{c}_W(s_\alpha s_\gamma - s_\beta)/c_\alpha D \\ 0 & 1 & \hat{s}_W t_\alpha & \hat{s}_W(s_\beta - s_\alpha s_\gamma)/c_\alpha D \\ 0 & 0 & 1/c_\alpha & (s_\alpha s_\beta - s_\gamma)/c_\alpha D \\ 0 & 0 & 0 & c_\alpha/D \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & U & \\ 0 & & & \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} , \tag{7}
$$

or, inverted:

$$
\begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & U^T & \\ 0 & & & \end{pmatrix} \begin{pmatrix} 1 & 0 & \hat{c}_W s_\alpha & \hat{c}_W s_\beta \\ 0 & 1 & -\hat{s}_W s_\alpha & -\hat{s}_W s_\beta \\ 0 & 0 & c_\alpha & (s_\gamma - s_\alpha s_\beta)/c_\alpha \\ 0 & 0 & 0 & D/c_\alpha \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} . \tag{8}
$$

Due to our parameterization, we can identify $\hat{e} = e = \sqrt{4\pi\alpha_{\text{EM}}}$ with the usual electric charge. The physical Weinberg angle is defined via

$$
s_W^2 c_W^2 = \frac{\pi \alpha_{\text{EM}}(M_1)}{\sqrt{2} G_F M_1^2} , \tag{9}
$$

which leads to the identity $s_W c_W M_1 = \hat{s}_W \hat{c}_W \hat{M}_Z$ [4].

The general case is complicated to discuss and hardly illuminating, which is why we will work with several approximations from here on out. In the limit $m_i^2 \ll \hat{M}_Z^2, \hat{M}_{X_j}^2$, $\alpha, \beta, \gamma \ll 1$ the mass matrix (5) simplifies to

$$
\mathcal{M}^2 \simeq \begin{pmatrix} \hat{M}_Z^2 & \hat{M}_Z^2 \hat{s}_W \alpha + m_1^2 & \hat{M}_Z^2 \hat{s}_W \beta + m_2^2 \\ \cdot & \hat{M}_{X_1}^2 & -\hat{M}_{X_1}^2 \gamma + m_3^2 \\ \cdot & \cdot & \hat{M}_{X_2}^2 \end{pmatrix} . \tag{10}
$$

Diagonalization leads to the resulting connection between gauge and mass eigenstates

$$
\begin{pmatrix} \hat{A} \\ \hat{Z} \\ \hat{X}_1 \\ \hat{X}_2 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & -\hat{c}_W \alpha & -\hat{c}_W \beta \\ 0 & 1 & \frac{\hat{s}_W \alpha \hat{M}_{X_1}^2 + m_1^2}{\hat{M}_{X_1}^2 - \hat{M}_Z^2} & \frac{\hat{s}_W \beta \hat{M}_{X_2}^2 + m_2^2}{\hat{M}_{X_2}^2 - \hat{M}_Z^2} \\ 0 & -\frac{\hat{s}_W \alpha \hat{M}_Z^2 + m_1^2}{\hat{M}_{X_1}^2 - \hat{M}_Z^2} & 1 & -\frac{\gamma \hat{M}_{X_2}^2 - m_3^2}{\hat{M}_{X_2}^2 - \hat{M}_{X_1}^2} \\ 0 & -\frac{\hat{s}_W \beta \hat{M}_Z^2 + m_2^2}{\hat{M}_{X_2}^2 - \hat{M}_Z^2} & \frac{\gamma \hat{M}_{X_1}^2 - m_3^2}{\hat{M}_{X_2}^2 - \hat{M}_{X_1}^2} & 1 \end{pmatrix} \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} , \tag{11}
$$

and one can calculate the mass shift of the Z boson

$$
M_1^2/\hat{M}_Z^2 \simeq 1 + \frac{\left(\hat{s}_W \alpha + m_1^2/\hat{M}_Z^2\right)^2}{1 - \hat{M}_{X_1}^2/\hat{M}_Z^2} + \frac{\left(\hat{s}_W \beta + m_2^2/\hat{M}_Z^2\right)^2}{1 - M_{X_2}^2/\hat{M}_Z^2} . \tag{12}
$$

With this formula we can express $\hat{M}_Z^2$ in terms of measurable masses:

$$
\frac{\hat{M}_Z^2}{M_1^2} = \frac{s_W^2 c_W^2}{\hat{s}_W^2 \hat{c}_W^2} \simeq 1 - \frac{\left(s_W \alpha + m_1^2/M_1^2\right)^2}{1 - M_2^2/M_1^2} - \frac{\left(s_W \beta + m_2^2/M_1^2\right)^2}{1 - M_3^2/M_1^2} . \tag{13}
$$

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[1] Here we defined the coupling strength of the $\hat{Z}$ boson $\hat{g}_Z \equiv \hat{e}/2\hat{c}_W \hat{s}_W$.

The direction of the shift depends on the hierarchy of $\hat{M}_Z^2$ and $\hat{M}_{X_i}^2$; a cancellation is possible for $\hat{M}_{X_1}^2 < \hat{M}_Z^2 < \hat{M}_{X_2}^2$, which would reduce stringent constraints from the ρ parameter (hiding one Z' with another). A different way of relaxing the limits on a Z' model by adding additional heavy bosons with specific charges was recently discussed in Ref. [7]. For completeness we show the effects of heavy Z' bosons in terms of the oblique parameters S and T, which can be read off the modified Z_1 couplings to j_W^3 and j_{EM} in the limit $\hat{g}_{1,2} \equiv 0$ [4]:

$$
\begin{aligned}
\alpha_{\mathrm{EM}} T &\simeq \frac{s_W^2 \alpha^2 - m_1^4/M_1^4}{1 - M_2^2/M_1^2} + \frac{s_W^2 \beta^2 - m_2^4/M_1^4}{1 - M_3^2/M_1^2}\,, \\
\alpha_{\mathrm{EM}} S &\simeq 4 s_W c_W^2 \alpha\, \frac{s_W \alpha + m_1^2/M_1^2}{1 - M_2^2/M_1^2} + 4 s_W c_W^2 \beta\, \frac{s_W \beta + m_2^2/M_1^2}{1 - M_3^2/M_1^2}\,.
\end{aligned}
\tag{14}
$$

III. APPLICATIONS

We will now show some applications of the framework laid out above. It is not our intention to examine the models in complete detail, but only to consider a few interesting effects. In most cases it suffices to work with the approximation in Eq. (11), which is used to read off the couplings of the mass eigenstates to the different currents/particles. Once a proper model is defined by additional scalars and fermions, one can perform more sophisticated analyses which make use of numerical diagonalization of the neutral boson mass matrix in Eq. (5). In particular, in specific models the loop-induced kinetic mixing angles can be calculated.

A. Crossing the streams

Model building with mixing between $U(1)_1$ and $U(1)_2$ often makes use of the induced coupling of currents, i.e. $\mathcal{L}_{\mathrm{mix}} \sim \varepsilon\, j_1 j_2$, which connects the two gauge sectors even if no particle is charged under both groups. We will now derive a necessary condition for such a non-diagonal term at tree level. Taking all of the mixing parameters in Eq. (1) to be zero except for m_3 and γ, we obtain the coupling of the mass eigenstates Z_2 and Z_3 to the currents

$$
\mathcal{L} \supset -\left(\hat{g}_1 j_1\,,\ \hat{g}_2 j_2\right) \begin{pmatrix} 1 & -t_\gamma \\ 0 & 1/c_\gamma \end{pmatrix} \begin{pmatrix} c_\xi & -s_\xi \\ s_\xi & c_\xi \end{pmatrix} \begin{pmatrix} Z_2 \\ Z_3 \end{pmatrix} \equiv -\left(\hat{g}_1 j_1\,,\ \hat{g}_2 j_2\right) V_\gamma U_\xi \begin{pmatrix} Z_2 \\ Z_3 \end{pmatrix}, \tag{15}
$$

where U_ξ diagonalizes the mass matrix. Integrating out the heavy mass eigenstates yields an effective four-fermion interaction of the form

$$
\begin{aligned}
\mathcal{L}_{\mathrm{eff}} &= -\frac{1}{2} \left(\hat{g}_1 j_1\,,\ \hat{g}_2 j_2\right) V_\gamma U_\xi \begin{pmatrix} 1/M_2^2 & 0 \\ 0 & 1/M_3^2 \end{pmatrix} U_\xi^T V_\gamma^T \begin{pmatrix} \hat{g}_1 j_1 \\ \hat{g}_2 j_2 \end{pmatrix} \\
&= -\frac{1}{2} \left(\hat{g}_1 j_1\,,\ \hat{g}_2 j_2\right) \begin{pmatrix} \hat{M}_{X_1}^2 & m_3^2 \\ m_3^2 & \hat{M}_{X_2}^2 \end{pmatrix}^{-1} \begin{pmatrix} \hat{g}_1 j_1 \\ \hat{g}_2 j_2 \end{pmatrix}.
\end{aligned}
\tag{16}
$$

It is obvious that the coupling matrix is diagonal if $m_3 = 0$, independent of γ. An analogous calculation can be performed for the coupling of j_i to j_{NC} via m_i and α, β, respectively, although it is a bit more tedious because of the additional Weinberg rotation. Nevertheless, the result is the same: an off-diagonal effective coupling $j_i j_{\mathrm{NC}}$ only arises for $m_i \neq 0$, i.e. $\mathcal{L}_{\mathrm{eff}} \propto m_{1,2}^2\, j_{1,2}\, j_{\mathrm{NC}}$. Since the Weinberg rotation induces a coupling of j_i to the electromagnetic current (first row in Eq. (11)), interesting couplings can arise even for $m_{1,2} = 0$.

Up until now we discussed only one non-zero m_i and kinetic mixing angle at a time, corresponding to the well-known case of Z–Z' mixing. A more general analysis including all our mixing parameters from Eq. (1) yields the effective four-fermion interactions

$$
\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2} \begin{pmatrix} \hat{g}_Z j_{\mathrm{NC}} \\ \hat{g}_1 j_1 - e\hat{c}_W s_\alpha j_{\mathrm{EM}} \\ \hat{g}_2 j_2 - e\hat{c}_W s_\beta j_{\mathrm{EM}} \end{pmatrix}^T \begin{pmatrix} \hat{M}_Z^2 & m_1^2 & m_2^2 \\ \cdot & \hat{M}_{X_1}^2 & m_3^2 \\ \cdot & \cdot & \hat{M}_{X_2}^2 \end{pmatrix}^{-1} \begin{pmatrix} \hat{g}_Z j_{\mathrm{NC}} \\ \hat{g}_1 j_1 - e\hat{c}_W s_\alpha j_{\mathrm{EM}} \\ \hat{g}_2 j_2 - e\hat{c}_W s_\beta j_{\mathrm{EM}} \end{pmatrix}. \tag{17}
$$

Because the 3×3 coupling matrix takes the explicit form

$$\begin{pmatrix} \hat{M}_Z^2 & m_1^2 & m_2^2 \\ \cdot & \hat{M}_{X_1}^2 & m_3^2 \\ \cdot & \cdot & \hat{M}_{X_2}^2 \end{pmatrix}^{-1} = \frac{1}{\Delta^6} \begin{pmatrix} \hat{M}_{X_1}^2 \hat{M}_{X_2}^2 - m_3^4 & -m_1^2 \hat{M}_{X_2}^2 + m_2^2 m_3^2 & -\hat{M}_{X_1}^2 m_2^2 + m_1^2 m_3^2 \\ \cdot & \hat{M}_Z^2 \hat{M}_{X_2}^2 - m_2^4 & -\hat{M}_Z^2 m_3^2 + m_1^2 m_2^2 \\ \cdot & \cdot & \hat{M}_Z^2 \hat{M}_{X_1}^2 - m_1^4 \end{pmatrix}, \quad (18)$$

with $\Delta^6 \equiv \hat{M}_Z^2 \hat{M}_{X_1}^2 \hat{M}_{X_2}^2 - \hat{M}_Z^2 m_3^4 - \hat{M}_{X_2}^2 m_2^4 - \hat{M}_{X_2}^2 m_1^4 + 2m_1^2 m_2^2 m_3^2$, we end up with new off-diagonal couplings like $m_2^2 m_3^2 j_1 j_{\mathrm{NC}}$, even if there is no direct coupling $m_1^2 j_1 j_{\mathrm{NC}}$.

B. $U(1)_B \times U(1)_{\mathrm{DM}}$

It was recently shown that the seemingly incompatible results of the dark matter (DM) direct detection experiments DAMA/CoGeNT and XENON can be alleviated with the introduction of isospin-dependent couplings of nucleons to dark matter [8]. One of the models used in Ref. [9] to explain this coupling is based on gauged baryon number $U(1)_1 \equiv U(1)_B$.[2] With dark matter charged under this gauge group, the resulting cross section turns out to be too small to explain the observed events, unless the coupling of Z' to dark matter is significantly stronger than to quarks (i.e. DM carries a large baryon number). However, in a model with another gauge group $U(1)_2 \equiv U(1)_{\mathrm{DM}}$ – acting only on the DM sector [3] – the dark matter coupling constant g_{DM} can be naturally large compared to g_B, which allows for a sizable cross section as long as the mass mixing between the groups is not too small.[3] We only introduce one DM Dirac fermion χ, so the $U(1)_2$ current takes the form $j_2^\mu = j_{\mathrm{DM}}^\mu = \overline{\chi}\gamma^\mu\chi$. For clarity we take all mixing parameters in Eq. (1) to be zero – except for m_3 and β – and assume $Z_{2,3}$ to be light ($M_{2,3}^2 \ll M_1^2$) to generate a large cross section. Eq. (11) then gives the approximate couplings

$$\mathcal{L} \supset - \left(\frac{e}{2c_W s_W} j_{\mathrm{NC}} + \beta s_W g_{\mathrm{DM}} j_{\mathrm{DM}} \right) Z_1 - \left(g_B j_B - g_{\mathrm{DM}} \frac{m_3^2}{M_3^2 - M_2^2} j_{\mathrm{DM}} \right) Z_2$$
$$- \left(g_{\mathrm{DM}} j_{\mathrm{DM}} - \beta c_W e j_{\mathrm{EM}} + g_B \frac{m_3^2}{M_3^2 - M_2^2} j_B \right) Z_3 . \quad (19)$$

These terms couple dark matter to nucleons via m_3, and because of β, proton and neutron couple differently, i.e. the interaction is isospin-dependent. Integrating out all the gauge bosons gives the effective vector-vector interactions in the usual parameterization

$$\mathcal{L}_{\mathrm{eff}} \supset f_p \, \overline{\chi}\gamma_\mu\chi \, \overline{p}\gamma^\mu p + f_n \, \overline{\chi}\gamma_\mu\chi \, \overline{n}\gamma^\mu n \,, \quad (20)$$

with the ratio of the neutron and proton couplings

$$f_n/f_p = \frac{1}{1+r} \,, \qquad\qquad r \simeq e c_W \frac{\beta}{g_B} \frac{M_2^2}{m_3^2} \,. \quad (21)$$

We can easily find parameters to generate $f_n/f_p \simeq -0.7$ ($r \simeq -2.4$). The overall DM-neutron cross section can be calculated to be [13]

$$\sigma_n = \frac{1}{64\pi} \left(\frac{m_\chi m_n}{m_\chi + m_n} \right)^2 f_n^2 \simeq \frac{m_n^2}{64\pi} \left(g_B g_{\mathrm{DM}} \frac{m_3^2}{M_2^2 M_3^2} \right)^2 \simeq 2\,\alpha_{\mathrm{DM}} \beta^2 \left(\frac{1\,\mathrm{GeV}}{M_3} \right)^4 10^{-31}\,\mathrm{cm}^2 \,, \quad (22)$$

where we defined $\alpha_{\mathrm{DM}} \equiv g_{\mathrm{DM}}^2/4\pi$ and assumed $m_\chi \gg m_n$. To obtain the last equation we replaced $g_B\,m_3^2$ with the demanded value for r from Eq. (21). For $\beta \sim 10^{-3}$ it is possible to generate the required DAMA/CoGeNT cross section $\sigma_n \sim 10^{-38}$–$10^{-37}\,\mathrm{cm}^2$ [8] without being in

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[2] It was pointed out in Ref. [10] that a gauge boson coupled to the baryon number B can be light. The drawback of such a symmetry is the unavoidable introduction of new chiral fermions to cancel occurring triangle anomalies. An anomaly-free symmetry (SM + right-handed neutrinos) with similarly weak constraints is $U(1)_{B-3L_\tau}$ [11].

[3] A similar model was proposed very recently in the same context, see Ref. [12].

conflict with other constraints [10, 14]. We note that the dark matter fine-structure constant α_{DM} is not restricted to be small.

Due to the required non-zero m_3^2 we will have a non-trivial scalar sector that also serves as a mediator between the SM and the dark sector. We assume these scalars to be heavy enough to not alter our foregoing discussion.

Aside from the group $U(1)_B \times U(1)_{\mathrm{DM}}$ discussed above, further interesting models using this framework in the dark matter sector could be build using leptophilic groups like $U(1)_{L_\mu - L_\tau} \times U(1)_{\mathrm{DM}}$, with the possibility to resolve the PAMELA positron excess via the small leptophilic admixture [15].

C. $U(1)_{B-L} \times U(1)_{L_\mu - L_\tau}$

A family non-universal model can be build using $U(1)_1 \equiv U(1)_{B-L}$ and $U(1)_2 \equiv U(1)_{L_\mu - L_\tau}$ without introducing anomalies. Each group is anomaly-free if the Standard Model is extended with 3 right-handed neutrinos $N_{i,R}$ carrying appropriate lepton numbers, so the only potential triangle anomalies involve both gauge groups:

$$
\begin{aligned}
U(1)_{L_\mu - L_\tau} &- U(1)_{B-L} - U(1)_{L_\mu - L_\tau} : \\
&\sum_{\mu,\tau} Y_{B-L} = 2 \left[Y_{B-L}(\mu_L) + Y_{B-L}(\nu_\mu) + Y_{B-L}(\mu_R^c) + Y_{B-L}(N_{2,R}^c) \right] = 0 \,, \\
U(1)_{L_\mu - L_\tau} &- U(1)_{B-L} - U(1)_{B-L} : \\
&\sum_{\mu,\tau} Y_{L_\mu - L_\tau} Y_{B-L}^2 = \sum_\mu Y_{B-L}^2 - \sum_\tau Y_{B-L}^2 = 0 \,, \\
U(1)_{L_\mu - L_\tau} &- U(1)_{B-L} - U(1)_Y : \\
&\sum_{\mu,\tau} Y_{L_\mu - L_\tau} Y_{B-L} Y = \sum_\mu Y_{B-L} Y - \sum_\tau Y_{B-L} Y = 0 \,,
\end{aligned}
\tag{23}
$$

where the last two relations follow from the universality of $U(1)_Y$ and $U(1)_{B-L}$. The anomalies from $SU(2) - U(1)_1 - U(1)_2$ and $SU(3) - U(1)_1 - U(1)_2$ vanish trivially in any model due to the tracelessness of the non-abelian generators. The same conclusion can, of course, be reached for any of the anomaly-free $L_\alpha - L_\beta$ symmetries. However, $L_\mu - L_\tau$ is favored over $L_e - L_\mu$ and $L_e - L_\tau$ because of a more reasonable flavor structure of the neutrino mass matrix [16].

The gauge boson $Z_2 = Z_{B-L}$ is highly constrained by collider experiments ($M_{B-L}/g_{B-L} \gtrsim$ 6–7 TeV at 95% C.L. [17]),[4] but $Z_3 = Z_{L_\mu - L_\tau}$ can have a mass around the electroweak scale and there is actually a preferred region around $M_{L_\mu - L_\tau}/g_{L_\mu - L_\tau} \simeq 200 \,\mathrm{GeV}$ that ameliorates the tension between the theoretical and experimental values for the anomalous magnetic moment of the muon [18] (see [19] for earlier works).

In $U(1)_{B-L} \times U(1)_{L_\mu - L_\tau}$ models with non-vanishing mass mixing the parameter m_3 induces an effective coupling of the currents $j_{L_\mu - L_\tau}$ and j_{B-L} (see Sec. III A), which leads for example to non-standard neutrino interactions, usually parameterized by the non-renormalizable effective Lagrangian

$$
\mathcal{L}_{\mathrm{eff}}^{\mathrm{NSI}} = -2\sqrt{2} G_F \varepsilon_{\alpha\beta}^{fP} \left[\bar{f} \gamma^\mu P f \right] \left[\bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta \right].
\tag{24}
$$

The model at hand induces $\varepsilon_{\mu\mu}^{fP} = -\varepsilon_{\tau\tau}^{fP}$, easily read off from Eq. (16):

$$
\begin{aligned}
\varepsilon_{\mu\mu}^{eV} &\simeq -\frac{1}{2\sqrt{2} G_F} g_1 g_2 \frac{m_3^2}{M_2^2 M_3^2} \simeq -2 \times 10^{-6} \frac{1}{g_1 g_2} \left(\frac{m_3}{10 \,\mathrm{GeV}} \right)^2 \left(\frac{6 \,\mathrm{TeV}}{M_2/g_1} \right)^2 \left(\frac{200 \,\mathrm{GeV}}{M_3/g_2} \right)^2 , \\
\varepsilon_{\mu\mu}^{uV} &= \varepsilon_{\mu\mu}^{dV} = -\varepsilon_{\mu\mu}^{eV}/3 \,,
\end{aligned}
\tag{25}
$$

---

[4] The limits from LEP 2 and Tevatron have been derived under the assumption of just one additional gauge boson, but still hold approximately when additional bosons are included [7].

which are in general too small to be observable in current experiments [20]. Larger NSIs can be generated at the price of introducing mass mixing of $Z_{L_\mu - L_\tau}$ with Z_{SM} via m_2 (using the more general Eq. (17)). Even though this kind of mixing is highly constrained by collider experiments, the arising NSIs are testable in future facilities for $M_2 < M_1$ [18]. Substituting $U(1)_{B-L}$ in Eq. (25) with less constrained symmetries like $U(1)_B$ or $U(1)_{B-3L_\tau}$ (including fermions to cancel arising anomalies) allows for lighter gauge bosons and therefore also larger NSIs; a recent discussion of additional constraints on Z' bosons with non-universal couplings to charged leptons can be found in Ref. [21]. Since our framework does not involve mixing with the SM gauge bosons – at least at tree level – the bounds on the mixing parameters are less stringent.

IV. CONCLUSION

The extension of the Standard Model by an additional abelian factor $U(1)'$ is a well motivated and frequently discussed area in model building. It is not far fetched to extend this even further to $G_{SM} \times [U(1)']^n$, provided the full gauge group stays free of anomalies. We discussed the most general low-energy Lagrangian for the case $n = 2$, including kinetic mixing among the abelian groups. We showed how the mixing among several gauge groups – such as $U(1)_{B-L}$, $U(1)_{L_\mu - L_\tau}$, and $U(1)_{DM}$ – can lead to interesting effects like non-standard neutrino interactions and isospin-dependent dark matter scattering. This opens up new and interesting possibilities in model building.

Acknowledgments

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