Inclusion of Matter in Inhomogeneous Loop Quantum Cosmology

Daniel Martín de Blas

daniel.martin@iem.cfmac.csic.es

Instituto de Estructura de la Materia - CSIC

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Introduction

- Loop Quantum Cosmology (LQC)
    - Quantum approach for cosmological systems inspired by the Loop Quantum Gravity (LQG)
    - Satisfactory quantization of several homogeneous cosmological models
    - New quantum phenomenology $\Rightarrow$ Resolution of initial singularity

- Hybrid quantization: Inhomogeneous models
    - Reduced model with only global constraints
    - It assumes that the most relevant effects of loop quantum geometry are in the homogeneous degrees of freedom
    - Combine LQC quantization for this homogeneous sector with a Fock quantization for inhomogeneities

- First model studied: vacuum Gowdy model with the three torus topology [Garay, Martín-Benito, Mena Marugán, '08]

Motivation

- Inclusion of a massless scalar field in the Gowdy $\mathbb{T}^3$ model
  - Minimally coupled
  - Same symmetries of the geometry

- Motivation
  - Inclusion of matter inhomogeneities in LQC.
  - Study of more realistic models, closer to the observed universe.
  - Scenario in which one can study some interesting features
    - Quantum effects of the inhomogeneities and the anisotropies on an FRW background.
    - Robustness of the Big Bounce scenario of LQC.
    - Changes in the evolution of the matter inhomogeneities due to quantum geometry effects.
    - *projection* to more symmetric quantum models.

Classical Settings

- Reduced phase space
  - Homogeneous sector: Bianchi I + homogeneous massless scalar field ϕ
  - Inhomogeneous sector: Matter inhomogeneities and gravitational waves (propagating in $\theta \in S^1$)

- Ashtekar-Barbero variables for Bianchi I
  - $su(2)$ connection: c^j;   densitized triad: p_j $i \in \{\theta, \sigma, \delta\}$

- Satisfactory Fock quantization of the inhomogeneities:
  - Unitary dynamics + Vacuum invariant under S^1 translations.
  - Parametrization of the matter φ and gravitational ξ inhomogeneities
  - Creation-annihilation variables (free m. s. f.): $\left(a_m^{(\alpha)*}, a_m^{(\alpha)}\right)$, $\alpha = \xi, \varphi$

- Two global constraints remain:
  - Diffeomorphism constraint: $C_\theta = C_\theta^\xi + C_\theta^\varphi$
  - Densitized Hamiltonian constraint: $\mathcal{C} = \mathcal{C}_{\mathsf{hom}} + \mathcal{C}_{\mathsf{inh}}$

Hybrid Quantization: Kinematics

Kinematical Hilbert space

$$\mathcal{H}_{\text{kin}} = \mathcal{H}_{\text{kin}}^{\text{hom}} \otimes \mathcal{H}_{\text{kin}}^{\text{inh}} = \mathcal{H}_{\text{kin}}^{\text{BI}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}^\xi \otimes \mathcal{F}^\varphi$$

- Fock Spaces $\mathcal{F}^\alpha$:
    - $a_m^{(\alpha)*}, a_m^{(\alpha)} \rightarrow \hat{a}_m^{(\alpha)\dagger}, \hat{a}_m^{(\alpha)}$: creation-annihilation operators.
    - n-particle states: $|\mathfrak{n}^\alpha\rangle = |\ldots, n_{-m}^\alpha, \ldots, n_m^\alpha, \ldots\rangle$, $n_m^\alpha \in \mathbb{N}$, $\sum_m n_m^\alpha < \infty$

- $\mathcal{H}_{\text{kin}}^{\text{hom-mat}} = L^2(\mathbb{R}, d\phi)$:
    - Standard Schrödinger quantization: $\hat{\phi}$, $\hat{p}_\phi = -i\hbar\partial_\phi$

- Bianchi I kinematical Hilbert space:
    - LQC: Triads p_i and holonomies of connections $\mathcal{N}_{\mu_i}(c_i)$.
    - Improved dynamics: minimum length, $\bar{\mu}_j$, in the holonomies.
    - $\mathcal{H}_{\text{kin}}^{\text{BI}} = \overline{\text{span}\{|\lambda_\theta, \lambda_\sigma, v\rangle : \lambda_\theta, \lambda_\sigma, v \in \mathbb{R}\rangle\}}$ $v = 2\lambda_\theta \lambda_\sigma \lambda_\delta$
    - Discrete inner product: $\langle \lambda_\theta, \lambda_\sigma, v | \lambda_\theta', \lambda_\sigma', v \rangle = \delta_{\lambda_\theta \lambda_\theta'} \delta_{\lambda_\sigma \lambda_\sigma'} \delta_{vv'}$.
    - $\hat{\mathcal{N}}_{\pm\bar{\mu}_i}$: Scale λ_i such that shift v in ± 1; $\hat{p}_j$: $p_j \propto \text{sgn}(\lambda_j)\lambda_j^2$.

Operators on the inhomogeneous Hilbert space

- Diffeomorphism constraint operator

$$\widehat{\mathcal{C}}_\theta = \sum_{m=1}^\infty m \left(\widehat{X}_m^\xi + \widehat{X}_m^\varphi \right), \qquad \widehat{X}_m^\alpha = \hat{a}_m^{(\alpha)\dagger}\hat{a}_m^{(\alpha)} - \hat{a}_{-m}^{(\alpha)\dagger}\hat{a}_{-m}^{(\alpha)}.$$

  - $\widehat{\mathcal{C}}_\theta |\mathfrak{n}^\xi\rangle \otimes |\mathfrak{n}^\varphi\rangle \quad \Rightarrow \quad \sum_{m=1}^\infty m(X_m^\xi + X_m^\varphi) = 0, \quad X_m^\alpha = n_m^\alpha - n_{-m}^\alpha.$

  - $\mathcal{F}_{\mathsf{p}} \equiv$ proper subspace of $\mathcal{F}_\xi \otimes \mathcal{F}_\varphi$.

- Operators in $\widehat{\mathcal{C}}_{\mathsf{inh}}$

  - $\hat{H}_0 = \displaystyle\sum_{\alpha \in \{\xi,\varphi\}} \sum_{m=1}^\infty m \hat{N}_m^\alpha, \qquad \hat{N}_m^\alpha = \hat{a}_m^{(\alpha)\dagger}\hat{a}_m^{(\alpha)} + \hat{a}_{-m}^{(\alpha)\dagger}\hat{a}_{-m}^{(\alpha)}.$

  - $\hat{H}_{\mathsf{int}} = \displaystyle\sum_{\alpha \in \{\xi,\varphi\}} \sum_{m=1}^\infty \frac{1}{m} \left(\hat{N}_m^\alpha + \hat{a}_m^{(\alpha)\dagger}\hat{a}_{-m}^{(\alpha)\dagger} + \hat{a}_m^{(\alpha)}\hat{a}_{-m}^{(\alpha)} \right).$

Hamiltonian constraint operator $\widehat{\mathcal{C}} = \widehat{\mathcal{C}}_{\mathsf{hom}} + \widehat{\mathcal{C}}_{\mathsf{inh}}$

- $\widehat{\mathcal{C}}_{\mathsf{hom}} = -\sum_{i \neq j} \sum_j \frac{\widehat{\Theta}_i \widehat{\Theta}_j}{16\pi G \gamma^2} - \frac{\hbar^2}{2} \left[\frac{\partial}{\partial \phi} \right]^2, \qquad i, j \in \{\theta, \delta, \sigma\}.$

- $\widehat{\mathcal{C}}_{\mathsf{inh}} = 2\pi\hbar \widehat{|p_\theta|} \widehat{H}_0 + \hbar \left[\frac{1}{|p_\theta|^{\frac{1}{4}}} \right]^2 \frac{\left(\widehat{\Theta}_\delta + \widehat{\Theta}_\sigma \right)^2}{16\pi\gamma^2} \left[\frac{1}{|p_\theta|^{\frac{1}{4}}} \right]^2 \widehat{H}_{\mathsf{int}}.$

- $\widehat{\Theta}_i = \widehat{c_i p_i} = i\pi\gamma G\hbar \sqrt{|v|} \left[\left(\hat{\mathcal{N}}_{-2\bar{\mu}_i} - \hat{\mathcal{N}}_{2\bar{\mu}_i} \right) \widehat{\mathsf{sgn}(p_i)} + \widehat{\mathsf{sgn}(p_i)} \left(\hat{\mathcal{N}}_{-2\bar{\mu}_i} - \hat{\mathcal{N}}_{2\bar{\mu}_i} \right) \right] \sqrt{|v|}$

- Symmetric factor ordering:
    - Triad operators: $v = 0$ states decouple (kin. singularity resolution)
    - $\widehat{\Theta}_j$ operators do not mix states with different sign of $\lambda_\theta, \lambda_\sigma, v$.

- $\widetilde{\mathcal{H}}_{\mathsf{kin}}^{\mathsf{BI}}$: states such that $\lambda_\theta, \lambda_\sigma, v > 0 \Rightarrow \Lambda_\theta = \log \lambda_\theta, \Lambda_\sigma = \log \lambda_\sigma$.

- Superselection sectors:
    - in v: $v \in \mathcal{L}_\epsilon = \{\epsilon + 4k; \ k \in \mathbb{N}\}, \qquad \epsilon \in (0, 4]$
    - in Λ_a: Given an initial $\Lambda_a^\star \Rightarrow \Lambda_a = \Lambda_a^\star + z_\epsilon, \ z_\epsilon \in \mathcal{Z}_\epsilon$

Physical Hilbert space

- Action of the Hamiltonian constraint
    - The coefficients do not depend on Λ_σ
    - It is a difference equation in $v \Rightarrow$ evolution equation in v
    - The solutions can be determined by a set of initial data on the section of minimum homogeneous volume

- Physical Hilbert space $\mathcal{H}_{\text{phy}} \Leftrightarrow$ Hilbert space of initial data

$$\mathcal{H}_{\text{p}} = \mathcal{H}_{\text{phys}}^{\text{BI}} \otimes L^2(\mathbb{R}, d\phi) \otimes \mathcal{F}_{\text{p}}$$

  - $\mathcal{H}_{\text{phys}}^{\text{BI}} \equiv$ Physical Hilbert space of Bianchi I

Projection to LRS-Gowdy

- The model is symmetric under the interchange of σ and δ directions.
- Classical solutions with local rotational symmetry (LRS)
- General state: $|\Psi\rangle = \sum_{\Lambda_\theta, \Lambda_\sigma, v} |\Psi(\Lambda_\theta, \Lambda_\sigma, v)\rangle \otimes |\Lambda_\theta, \Lambda_\sigma, v\rangle$

- *Projection map:*

$$|\Psi(\Lambda_\theta, \Lambda_\sigma, v)\rangle \qquad \longrightarrow \qquad \sum_{\Lambda_\sigma} |\Psi(\Lambda_\theta, \Lambda_\sigma, v)\rangle \equiv |\psi(\Lambda_\theta, v)\rangle$$

Quantum Gowdy Model $\xrightarrow{\;projection\;}$ Quantum LRS-Gowdy Model

- *projection* over Λ_θ to get the isotropic Gowdy model fails
  - There is no classical inhomogeneous and isotropic solutions.

Conclusions

- Satisfactory quantization of the Gowdy T^3 model with linearly polarized gravitational waves and a massless scalar field.
- Hybrid quantization applied as in the vacuum model.
- Inclusion of the matter field:
    - Classical isotropic solutions of the homogeneous sector.
    - Two "copies" of inhomogeneities (mathematically speaking).
    - Matter inhomogeneities in LQC.
- Same results as in the vacuum model.
    - Standard Fock quantization of the inhomogeneities is recovered.
    - Classical singularity resolved at the kinematical level.
- Study of the *projection* to more symmetric systems.
- Possibility of analyzing the effects of the anisotropies and the inhomogeneities on a flat FRW model. (Work in progress)

Thanks for your attention!