Towards A Possible Fluid Flow Underlying the Kerr Spacetime

Jianwei Mei

Max Planck Institute for Gravitational Physics (Albert Einstein Institute)
Am Mühlenberg 1, 14476 Golm, Germany

ABSTRACT

Based on the idea of emergent spacetime, we consider the possibility that the material underlying our spacetime is modelled by a fluid. We are particularly interested in possible connections between the geometrical properties of the emergent spacetime and the properties of the underlying fluid. We find some partial results that support this possibility. By using the Kerr spacetime as an example, we construct from the Riemann curvature tensor a vector field, which behaves just like the speculated fluid flow.

1 Introduction

By far the geometrical properties are the only thing that we know about our spacetime. As a physical entity, however, the space must have more interesting substructures. But the substructures of the space may not become obvious until the Planck scale, so it will be extremely difficult to obtain non-geometrical information about the spacetime.

In view of this difficulty, we firstly ask a much easier question: what could be our best guess about the materials underlying the space? Although an answer to this question does not solve the real physics problem, it may invite ideas that can help solve the real problem in the end. Here we want to consider the possibility that the material underlying our space is modelled by a fluid. Hints toward such a possibility come from at least two different directions.

The first is related to a branch of study in the Gauge/Gravity duality (see, e.g. [1, 2, 3, 4, 5, 6]). As a general feature, the duality always involves a gravitational theory in the bulk and a gauge field theory on the boundary. The gauge field theory describes a matter system that can often be modelled by a fluid in some appropriate limit [7, 8, 9, 10, 11]. It is hoped that the Gauge/Gravity duality can lead to a consistent formulation of the quantum gravity theory. If such a hope does become true, it is then quite probable that the material underlying our spacetime is related to the gauge theory in the Gauge/Gravity dual, and thus may behave like a fluid in the appropriate limit.

The second is related to the existence of classical mechanical analogs of relativistic effects [12]. Here the key observation is that in some condensed matter system, certain excitations of the system only "see" a Minkowski spacetime. (A simple example can be found with a chain of identical harmonic oscillators.) If relativistic-like effects can arise from a real condensed matter system, then it is possible that Special Relativity or even General Relativity can also emerge from some more sophisticated condensed matter systems [13]. One can imagine that all currently known matter fields are excitations of the material underlying the space, and our measured spacetime is *emergent* in the sense that it is purely determined by the equations obeyed by the matter fields.[1]

Of course, the above arguments are merely speculations. So it is desirable if one can find more concrete evidence to support (or to reject) the fluid picture. In this note, we want to show that such a possible evidence dose exist. In [15], defects in solids are related to geometrical quantities such as the Riemann curvature tensor and the torsion tensor. Here

[1]Note our meaning of the *emergent spacetime* is rather different from that in [14].

we want to turn the logic around and ask if one can obtain some information about the underlying material, by using the geometrical properties of the emergent spacetime. We will show that it is possible to construct from the Riemann curvature tensor a vector field, which behaves just like the speculated fluid flow underlying our spacetime.

Lets firstly explain what we expect for the proposed fluid. We assume that the fluid is similar to the ones that we know in the emergent spacetime. Most importantly, we assume that the fluid is also characterized by a density ρ and a four velocity u^μ, satisfying $g_{\mu\nu}u^\mu u^\nu = -1$, where $g_{\mu\nu}$ is the metric of the emergent spacetime. When the emergent spacetime is flat, we expect the underlying fluid to be static (in one particular coordinate system) and everywhere uniform. The presences of matter fields in the emergent spacetime should correspond to non-trivial disturbances in the underlying fluid. Especially, when the matter fields have a net angular momentum, like in the case of a rotating star or a Kerr black hole, there might be a corresponding global current in the underlying fluid. In this case, the fluid is still static and uniform at the spatial infinity, but will have a circulation motion near the center. For all these features, it is enough to consider only stationary spacetimes, which are characterized by the presence of a time-like Killing vector.

Our main purpose of this note is to show that all the above expectations are fulfilled by the vector field that we are going to construct from the Riemann curvature tensor. We will present the vector field in the next section. Then we will discuss its main features and the possible phenomenological consequences. A short summary is at the end.

2 Calculating the Fluid Flow

To get started, it is natural to firstly seek a possible connection between two rather different but also similar quantities — the vorticity of the fluid flow and the Riemann curvature tensor. They are similar in the sense that both are related to some non-trivial integrals along a closed path. The vorticity is related to the exterior derivative of the fluid flow. So to make the connection, we can also try to construct a closed two-form from the Riemann curvature tensor. It turns out that the time-like Killing vector also becomes a helpful ingredient in the construction. We find that the following relation can give a physically sensible result:

$$d\tilde{u} \equiv d(\rho u) = i\bar{\psi}\gamma^{\mu\nu}\psi R_{\mu\nu\rho\sigma} dx^\rho \wedge dx^\sigma \,, \tag{1}$$

where $\gamma^{\mu\nu} = -\frac{i}{4}\left[\gamma^\mu \,, \gamma^\nu\right]$. Note (1) only determines $\tilde{u}$ up to an exact form. This ambiguity can be fixed by imposing appropriate boundary conditions on $\tilde{u}$. The spinor field ψ can be viewed as the "square root" of the time-like Killing vector ξ at the spatial infinity,

$$\bar{\psi}\gamma^\mu\psi \to -c_\psi \xi^\mu \quad \text{as} \quad r \to +\infty\,, \tag{2}$$

where $c_\psi > 0$ is a normalization constant, and r is the radial direction in a usual spherical coordinate system. For (1) to be true, one must also impose the following constraint on the spinor field,

$$R_{\mu\nu\rho\sigma}\partial_\alpha(\bar{\psi}\gamma^{\mu\nu}\psi)dx^\alpha \wedge dx^\rho \wedge dx^\sigma = 0\,. \tag{3}$$

In four dimensions, (2) and (3) contain eight equations in total. A general spinor field with eight degrees of freedom is just enough to satisfy all the constraints.

Note relation (1) is motivated by the fact that both $d\tilde{u}$ and $i\bar{\psi}\gamma^{\mu\nu}\psi$ represent some kind of angular momentum. The presence of the Riemann curvature tensor can guarantee that we have a trivial result for the flat spacetime.

In the following, we will use the Kerr black hole to illustrate what may be obtained from (1). To be explicit, we write the metric of the Kerr black hole as

$$\begin{aligned} ds^2 = \eta_{ab}e^a{}_\mu e^b{}_\nu dx^\mu dx^\nu = \eta_{ab}e^a e^b\,, \quad \eta = diag\{+++-\}\,,\\ e^1 = f_x dr\,, \quad e^2 = f_y d\theta\,, \quad e^3 = f_p(d\phi - f_a dt)\,, \quad e^4 = f_t(dt - f_b d\phi)\,,\\ f_x = \frac{f_y}{\sqrt{X}}\,, \quad f_y = \sqrt{r^2 + a^2\cos^2\theta}\,, \quad f_p = \frac{(r^2+a^2)\sin\theta}{f_y}\,, \quad f_t = \frac{\sqrt{X}}{f_y}\,,\\ f_a = \frac{a}{r^2+a^2}\,, \quad f_b = a\sin^2\theta\,, \quad X = r^2 + a^2 - 2mr\,. \end{aligned} \tag{4}$$

The gamma matrices are taken to be

$$\gamma^a = i\begin{pmatrix} & \sigma^a \\ -\sigma^a & \end{pmatrix}\,,\ a = 1,2,3\,, \quad \gamma^4 = i\begin{pmatrix} \mathbf{1}_2 & \\ & -\mathbf{1}_2 \end{pmatrix}\,, \tag{5}$$

where $\sigma^{1,2,3}$ are the usual Pauli matrices and $\mathbf{1}_2$ is the two-dimensional unit matrix. Note the gamma matrices in (5) are in the vierbein bases. Gamma matrices with real coordinate indices can be obtained by using $\gamma^\mu = e_a{}^\mu\gamma^a$ with $a = 1,2,3,4$ and $\mu = r, \theta, \phi, t$. We write the spinor field as

$$\psi = \begin{pmatrix} \psi_{1a} + i\psi_{1b} \\ \psi_{2a} + i\psi_{2b} \\ \psi_{3a} + i\psi_{3b} \\ \psi_{4a} + i\psi_{4b} \end{pmatrix}\,, \tag{6}$$

where all the functions $\psi_{Ia} = \psi_{Ia}(r,\theta)$ and $\psi_{Ib} = \psi_{Ib}(r,\theta)$ $(I = 1, \cdots, 4)$ are real. For the metric (4), we find

$$\bar{\psi}\gamma^r\psi \;=\; -\frac{2z_1}{f_x}\,, \quad \bar{\psi}\gamma^\phi\psi = -\frac{f_a}{1 - f_a f_b}\Big[\frac{2z_3}{f_a f_p} + \frac{z_4}{f_t}\Big]\,,$$

$$\bar{\psi}\gamma^\theta\psi = -\frac{2z_2}{f_y}\,, \quad \bar{\psi}\gamma^t\psi = -\frac{1}{1-f_a f_b}\Big[\frac{2f_b z_3}{f_p} + \frac{z_4}{f_t}\Big]\,, \tag{7}$$

with

$$\begin{aligned}
z_1 &= (\phi_{2a}\phi_{3a} + \phi_{2b}\phi_{3b} + \phi_{1a}\phi_{4a} + \phi_{1b}\phi_{4b})\,,\\
z_2 &= (\phi_{2b}\phi_{3a} - \phi_{2a}\phi_{3b} - \phi_{1b}\phi_{4a} + \phi_{1a}\phi_{4b})\,,\\
z_3 &= (\phi_{1a}\phi_{3a} + \phi_{1b}\phi_{3b} - \phi_{2a}\phi_{4a} - \phi_{2b}\phi_{4b})\,,\\
z_4 &= (\phi_{1a}^2 + \phi_{1b}^2 + \phi_{2a}^2 + \phi_{2b}^2 + \phi_{3a}^2 + \phi_{3b}^2 + \phi_{4a}^2 + \phi_{4b}^2)\,.
\end{aligned} \tag{8}$$

For the Kerr metric (4), the time-like Killing vector is $\xi = \partial_t$. Now (2) can be satisfied with

$$z_1 = z_2 = z_3 = 0\,, \quad z_4 = c_\psi(1 - f_a f_b) f_t\,. \tag{9}$$

The first three equations are easily solved by

$$\phi_{3b} = -\frac{\phi_{3a}}{\phi_{1b}}\phi_{1a}\,, \quad \phi_{4a} = \frac{\phi_{3a}}{\phi_{1b}}\phi_{2b}\,, \quad \phi_{4b} = -\frac{\phi_{3a}}{\phi_{1b}}\phi_{2a}\,. \tag{10}$$

From (1), we find that

$$d\tilde{u} = f_{rh} dr\wedge d\theta + f_{pt} d\phi\wedge dt + f_{hp} d\theta\wedge d\phi + f_{rp} dr\wedge d\phi + f_{ht} d\theta\wedge dt + f_{rt} dr\wedge dt\,, \tag{11}$$

with

$$\begin{aligned}
f_{rh} &= \frac{12amr_1\cos\theta}{f_y^4\sqrt{X}}k_1 + \frac{2mrr_3}{f_y^4\sqrt{X}}k_2\,,\\
f_{pt} &= \frac{4mrr_3\sin\theta\sqrt{X}}{f_y^6}k_1 - \frac{6amr_1\cos\theta\sin\theta\sqrt{X}}{f_y^6}k_2\,,\\
f_{hp} &= \frac{4amrr_3\sin^2\theta\sqrt{X}}{f_y^6}k_3 - \frac{8mr(r^2+a^2)r_3\sin\theta}{f_y^6}k_4\\
&\quad -\frac{24am(r^2+a^2)r_1\cos\theta\sin\theta}{f_y^6}k_5 - \frac{12a^2mr_1\cos\theta\sin^2\theta\sqrt{X}}{f_y^6}k_6\,,\\
f_{rp} &= \frac{12am(r^2+a^2)r_1\cos\theta\sin\theta}{f_y^6\sqrt{X}}k_3 - \frac{24a^2mr_1\cos\theta\sin^2\theta}{f_y^6}k_4\\
&\quad +\frac{8amrr_3\sin^2\theta}{f_y^6}k_5 + \frac{4mr(r^2+a^2)r_3\sin\theta}{f_y^6\sqrt{X}}k_6\,,\\
f_{ht} &= -\frac{4mrr_3\sqrt{X}}{f_y^6}k_3 + \frac{8amrr_3\sin\theta}{f_y^6}k_4 + \frac{24a^2mr_1\cos\theta\sin\theta}{f_y^6}k_5\\
&\quad +\frac{12amr_1\cos\theta\sqrt{X}}{f_y^6}k_6\,,\\
f_{rt} &= -\frac{12a^2mr_1\cos\theta\sin\theta}{f_y^6\sqrt{X}}k_3 + \frac{24amr_1\cos\theta}{f_y^6}k_4 - \frac{8mrr_3}{f_y^6}k_5 - \frac{4amrr_3\sin\theta}{f_y^6\sqrt{X}}k_6\,,
\end{aligned} \tag{12}$$

where $r_1 = r^2 - \frac{1}{3}a^2\cos^2\theta$, $r_3 = r^2 - 3a^2\cos^2\theta$ and

$$\begin{aligned}
k_1 &= \phi_{1b}\phi_{3a} - \phi_{1a}\phi_{3b} - \phi_{2b}\phi_{4a} + \phi_{2a}\phi_{4b}\,, \\
k_2 &= \phi_{1a}^2 + \phi_{1b}^2 - \phi_{2a}^2 - \phi_{2b}^2 - \phi_{3a}^2 - \phi_{3b}^2 + \phi_{4a}^2 + \phi_{4b}^2\,, \\
k_3 &= \phi_{2a}\phi_{3a} + \phi_{2b}\phi_{3b} - \phi_{1a}\phi_{4a} - \phi_{1b}\phi_{4b}\,, \\
k_4 &= \phi_{1a}\phi_{2a} + \phi_{1b}\phi_{2b} - \phi_{3a}\phi_{4a} - \phi_{3b}\phi_{4b}\,, \\
k_5 &= \phi_{2b}\phi_{3a} - \phi_{2a}\phi_{3b} + \phi_{1b}\phi_{4a} - \phi_{1a}\phi_{4b}\,, \\
k_6 &= \phi_{1b}\phi_{2a} - \phi_{1a}\phi_{2b} - \phi_{3b}\phi_{4a} + \phi_{3a}\phi_{4b}\,.
\end{aligned} \tag{13}$$

The condition $d(d\tilde{u}) = 0$ and (11) lead to

$$\partial_r f_{pt} = \partial_\theta f_{pt} = 0\,, \quad \partial_r f_{hp} = \partial_\theta f_{rp}\,, \quad \partial_r f_{ht} = \partial_\theta f_{rt}\,. \tag{14}$$

For $\tilde{u}$ to be physically meaningful, we should in fact take $f_{pt} = 0$ in (11), giving

$$\phi_{1a}^2 + \phi_{1b}^2 = \phi_{2a}^2 + \phi_{2b}^2 = \phi_v^2\,, \quad \phi_v > 0\,. \tag{15}$$

The $dr \wedge d\theta$ term in (11) also vanishes due to these relations. (9) and (15) can be solved by

$$\phi_{1a} = \sqrt{\phi_v^2 - \phi_{1b}^2}\,, \quad \phi_{2a} = \sqrt{\phi_v^2 - \phi_{2b}^2}\,, \quad \phi_{3a} = -\phi_{1b}\sqrt{\frac{c_\psi f_t}{2\phi_v^2}(1 - f_a f_b) - 1}\,. \tag{16}$$

Now we are left with three undetermined functions ϕ_{1b}, ϕ_{2b}, ϕ_v and two unsolved equations in (14). We have achieved an extra degrees of freedom because two equations in $d(d\tilde{u}) = 0$ are solved with the same condition (15).

It is difficult to find exact solutions to the last two equations in (14). It is then necessary to look for some approximate results. In this regard, one particular interesting case is the limit of small rotation, which should correspond to many real life situations in nature. To study the equations in the small rotation limit, we keep the mass m fixed and take $a \to 0$. We expand the unknown functions as

$$\begin{aligned}
\phi_{1b} &= \phi_{10} + \phi_{11}a + \phi_{12}a^2 + \cdots\,, \\
\phi_{2b} &= \phi_{20} + \phi_{21}a + \phi_{22}a^2 + \cdots\,, \\
\phi_v &= \phi_{v0} + \phi_{v1}a + \phi_{v2}a^2 + \cdots\,.
\end{aligned} \tag{17}$$

Up to the order $\mathcal{O}(a^2)$, we find that

$$\begin{aligned}
\phi_{v0} &= \frac{1}{2}\sqrt{c_\psi}f^{1/4}\,, \quad \phi_{20} = \phi_{10}\,, \quad \phi_{21} = \phi_{11}\,, \\
\phi_{v1} &= \sqrt{c_\psi}\cos\theta\frac{(33m^2 + 2mr + 2r^2)\sqrt{f} - 2r^2}{60m^2 r f^{1/4}}\,,
\end{aligned}$$

$$
\begin{aligned}
\phi_{v2} &= \sqrt{c_\psi}\Big[\frac{4m-r}{8r^3 f^{3/4}} + \frac{(1278m^5 - 375m^4 r + 140m^3r^2 - 120m^2r^3 - 8r^5)\cos^2\theta}{3600m^4r^3f^{3/4}} \\
&\quad + \frac{(33m^2+2mr+2r^2)\cos^2\theta}{900m^4 f^{1/4}}\Big]\,, \\
\phi_{22} &= \phi_{12} - \frac{r^2 u'_{t2}}{4mf}\sqrt{c_\psi\sqrt{f} - 4\phi_{10}^2} - 2\cos\theta\sin\theta\sqrt{c_\psi\sqrt{f}-4\phi_{10}^2} \\
&\quad \Big[\frac{2(1260m^4 - 180m^3r - 213m^2r^2 - 142mr^3 - 142r^4)}{23625m^4 r\sqrt{X}} \\
&\quad + \frac{r(9m^2+12mr+50r^2) + 24r^3\ln(1+\sqrt{f})}{675m^4\sqrt{fX}}\Big]\,, \qquad (18)
\end{aligned}
$$

where $f = 1-\frac{2m}{r}$, and $u'_{t2} = \frac{d}{d\theta}u_{t2}(\theta)$ is undetermined at the order $\mathcal{O}(a^2)$. Correspondingly,

$$
\begin{aligned}
\tilde{u} &= c_\psi(\tilde{u}_t dt + \tilde{u}_p d\phi)\,, \quad \tilde{u}_\varrho = \tilde{u}_{\varrho 0} + \tilde{u}_{\varrho 1}a + \tilde{u}_{\varrho 2}a^2 + \cdots\,, \quad \varrho = t, p\,, \\
\tilde{u}_{t0} &= \frac{4f^{3/2}}{15m}\Big(1+\frac{3m}{r}\Big)\,, \quad \tilde{u}_{p0} = \tilde{u}_{t1} = \tilde{u}_{p2} = 0\,, \\
\tilde{u}_{p1} &= \frac{4\sin^2\theta}{15m}\Big[1 - (1+\frac{3m}{r})f^{3/2}\Big]\,, \\
\tilde{u}_{t2} &= u_{t2} + \frac{8(18m^2 - 6mr - 25r^2)\cos^2\theta}{675m^3r^2} \\
&\quad + \frac{2\sqrt{f}(20m^4 + 5m^3r + 3m^2r^2 + 2mr^3 + 2r^4)}{45m^3r^4} \\
&\quad + \frac{4\cos^2\theta\sqrt{f}(6930m^4 - 1215m^3r - 1149m^2r^2 + 284mr^3 + 284r^4)}{23625m^3r^4} \\
&\quad - \frac{32\cos^2\theta}{225m^3}\ln(1+\sqrt{f})\,. \qquad (19)
\end{aligned}
$$

In the limit $a \to 0$, the black hole horizon is located at $r_0 \approx 2m - \frac{a^2}{2m}$. The result in (18) diverges at $f = 0$, i.e., $r = 2m$. We expect such a divergence to occur exactly on the horizon for the exact solution. As we will explain in the next section, such divergence is not a problem for us because we can only trust our result outside the black hole horizon.

Another more fatal divergence may come in the form $\ln r$, as is hinted by the presence of $\ln(1+\sqrt{f})$ in (18) and (19). This will indicate that the result is not well defined at the spatial infinity. We have checked that such divergence can be removed at the order $\mathcal{O}(a^3)$, by appropriately adjusting the function u_{t2}. But it is not certain if the required cancellation exists to all orders in the expansion of a. It is then very important if one can solve the last two equations in (14) exactly. In the following, we will assume that such divergence can always be removed.

In any case, it is already quite remarkable that, as we will show in the next section, the leading order results from (1) does display features that look like a realistic physical quantity. This is why we think that we might be on the right track. We will discuss the result in more detail in the next section.

3 Features of the Fluid Flow

Our most important quantity is the fluid flow. From (1) and (19), we find that

$$\begin{aligned} \rho &= \frac{4c_\psi}{15m} f\left(1+\frac{3m}{r}\right) + \mathcal{O}(a^2)\,, \\ u^t &= \frac{1}{\sqrt{f}} + \mathcal{O}(a^2)\,, \quad u^r = u^\theta = 0\,, \\ u^\phi &= \frac{a}{r^2\sqrt{f}}\left[1 - f^{-1/2}(1+\frac{3m}{r})^{-1}\right] + \mathcal{O}(a^2)\,. \end{aligned} \tag{20}$$

Now let's look at the most important features of the result:

- Firstly, $u \to \partial_t$ as $r \to +\infty$. This means that the fluid is static at the spatial infinity. This is exactly what we expect for the fluid underlying our spacetime. What's more, one also has $u^\phi \sim a$, which means that there is no spatial flow underlying the Schwarzschild black hole. This also fits very well with our intuitive picture about the fluid. These two points are the most important reasons making us think that (20) might be describing something physical.

- Secondly, $\rho \to 0$ as $f \to 0$. For the exact solution, we expect the density to vanish exactly on the horizon. If we take the fluid picture seriously, then this result indicates that the geometrical picture is not valid on the horizon. The reason is the following:

  If the spacetime is emergent from some underlying condensed matter system, then the geometrical picture is only valid in the large distance limit, where the discrete nature of the underlying material is irrelevant. Now if $\rho \to 0$ at some point in the emergent spacetime, it does not mean that the density of the fluid really vanishes there. It only means that we have come to a location where a tiny region of the underlying material is projected to a huge region in the emergent spacetime. To be more explicit, Let's use the Schwarzschild black hole as an example. It is reasonable to assume that the underlying material is best described by a flat metric, which coincides with that of the Schwarzschild black hole at the spatial infinity. Let's also assume that the density of the underlying material is everywhere finite in the flat metric. Near the black hole horizon, a shell of depth dr of the underlying material is mapped to a shell of depth $\frac{dr}{\sqrt{f}}$ in the Schwarzschild spacetime. The density of the underlying material thus vanishes on the horizon when "observed" from the Schwarzschild spacetime. If this picture is true, then we are probing smaller and smaller distances in the underlying material as we come closer and closer to the black hole horizon. The discrete nature of the underlying material will become important and the geometric picture will break down before the horizon is reached. For this reason, we will only trust our result outside the black hole horizon.

- Thirdly, u^ϕ falls as r^{-2} when $r \to +\infty$, but it diverges at $f = 0$. Similarly, u^t also diverges at $f = 0$. For the reason explained above, the divergence on the horizon should not be a big concern for us, because we can only trust our result out side the black hole horizon.

- Finally, $\rho \to 4c_\psi/15m$ as $r \to +\infty$. At the spatial infinity, the density of the fluid is that of a flat spacetime, and should not depend on any physics near the center. As a result, we expect $c_\psi \propto m$. In a more general case, we expect c_ψ to be proportional to the total energy contained in the geometry.

Now we see that (20) fulfills all our expectations about the speculated fluid flow, and we also have a consistent explanation to all the displayed features.

To this end, it is also interesting to note that if

$$\zeta^\mu = -\bar{\psi}\gamma^\mu\psi\,, \tag{21}$$

then (10) and (16) lead to

$$\zeta = c_\psi(f_a\partial_\phi + \partial_t)\,. \tag{22}$$

We see that $c_\psi^{-1}\zeta$ interpolates between the time-like Killing vector at the spatial infinity and the null Killing vector on the horizon. Most interestingly, if we let

$$\zeta^\mu = \rho u^\mu\,, \tag{23}$$

then

$$\rho \;=\; c_\psi\frac{f_y\sqrt{X}}{r^2+a^2}\,,\quad u = \frac{c_\psi}{\rho}(f_a\partial_\phi + \partial_t)\,. \tag{24}$$

Just like (20), this result also displays all the interesting features listed below (20). Thus, (24) is another possible candidate description of the speculated fluid flow.

One potential advantage of (21) and (23) over (1) is that they are easier to generalize to higher dimensions. For example, for the Myers-Perry black hole in five dimensions [16],

$$\begin{aligned} ds^2 = \eta_{ab}e^a{}_\mu e^b{}_\nu dx^\mu dx^\nu = \eta_{ab}e^a e^b\,,\quad \eta = diag\{++++-\}\,,\\ e^1 = \frac{\sqrt{r^2+y^2}}{\sqrt{X}}dr\,,\quad e^2 = \sqrt{r^2+y^2}d\theta\,,\\ e^3 = \frac{\cos\theta\sin\theta}{y\sqrt{r^2+y^2}}\Big[\frac{a^2(d\phi_1 - f_a dt)}{f_a} - \frac{b^2(d\phi_2 - f_b dt)}{f_b}\Big]\,, \end{aligned}$$

$$
\begin{aligned}
e^4 &= \frac{ab}{ry}\Big[\frac{\sin^2\theta(d\phi_1 - f_a dt)}{f_a} + \frac{\cos^2\theta(d\phi_2 - f_b dt)}{f_b}\Big]\,,\\
e^5 &= \frac{\sqrt{X}}{\sqrt{r^2+y^2}}\Big[dt - a\sin^2\theta d\phi_1 - b\cos^2\theta d\phi_2\Big]\,,\\
f_a = \frac{a}{r^2+a^2}\,,\quad f_b = \frac{b}{r^2+b^2}\,,\quad X &= \frac{(r^2+a^2)(r^2+b^2)}{r^2} - 2m\,,\\
y &= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta}\,,
\end{aligned}
\tag{25}
$$

one can check that (21) leads to (with some simple choice of the spinor degrees of freedom)

$$
\zeta = c_\psi(f_a\partial_{\phi_1} + f_b\partial_{\phi_2} + \partial_t)\,. \tag{26}
$$

Again, $c_\psi^{-1}\zeta$ interpolates between the time-like Killing vector at the spatial infinity and the null Killing vector on the horizon. Using (23), we can also interpret the result as a fluid flow underlying the five dimensional Myers-Perry black hole.

4 Summary

In this note, we consider an emergent picture of the spacetime: the material underlying our spacetime is assumed to be a condensed matter system (a fluid), all known matter fields are excitations in the fluid, and our observed spacetime is derived from the equations obeyed by the matter fields.

We are particularly interested in possible connections between the geometrical properties of the emergent spacetime and the properties of the underlying fluid. We have presented some partial results in support of such possible connections. In particular, we have constructed from the Riemann curvature tensor a vector field, which behaves just like the speculated fluid flow. Now if we take the fluid picture seriously, then our result indicates that the geometric picture breaks down on the horizon, and we can only trust our result outside the black hole horizon.

In the calculation, we have also noticed a second possible formulation of the fluid flow. It also displays all the desired quantitative features, and is more easily generalized to other dimensions of the spacetime. We shall consider this formulation in more detail in future works.

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