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EDITORIAL

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1. Introduction

Cosmological observational data [1][1], when fitted using models based on the assumption of a spatially homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) model plus small perturbations, are usually interpreted as implying that the spatial geometry is flat, there is currently an accelerated expansion and the majority of the matter in the Universe is dark, non-baryonic and cold, giving rise to the so-called ΛCDM-concordance model (where the dark energy is interpreted as a positive cosmological constant Λ). The concordance model of cosmology is now operating on a well-established and tightly constrained empirical basis.

However, although the concordance ΛCDM model is remarkably successful, there does exist significant tensions between the data and the model [2]. Furthermore, if our Universe is not a perturbation of an exact flat FLRW solution, the conventional data analyses and their interpretation are not necessarily valid [3]. For example, the standard analysis of type Ia supernovae (SNIa) and CMB data in FLRW models cannot be applied directly when backreaction effects (i.e. significant effects of large scale inhomogeneities on observations) are present.

In addition, it is widely believed that dark energy is the biggest puzzle in standard cosmology today. One of the reasons for this is that the value of the cosmological constant Λ relevant for cosmology seems absurdly small in the context of quantum physics. These and other issues have motivated models which include a dynamical dark energy component. Furthermore, in contrast to the dark matter component of the concordance model, which can at least in principle be observed directly or indirectly in local experiments, it cannot be expected that dark energy will have locally observable effects. A more detailed discussion of dark energy is outside the scope of this overview, and we simply refer the reader to recent reviews (see, for example, [4] and the reviews cited in the articles in this focus section).

In fact, Supernovae data can be fitted using inhomogeneous cosmological models which do not incorporate dark energy, but where the full effects of general relativity (GR) come into play. Indeed, it has been shown that the Lemaître–Tolman–Bondi (LTB) models without dark energy can be used to fit the observed data, although it may be necessary to place the observer at the centre of a rather large-scale underdensity [5]. This motivates a careful study of exact inhomogeneous cosmological models and the effects of inhomogeneities on cosmological observations.

The fact that the Universe is presently inhomogeneous means that the assumptions implicit in the FLRW approximation must be empirically justified at the present epoch. In particular, spatial homogeneity only applies to the present day in an average sense. By assuming spatial homogeneity and isotropy on the largest scales in the cosmological models used to fit observational data, the inhomogeneities affect the observed dynamics through correction (backreaction) terms, and allow for behaviour in the Universe which is potentially qualitatively and quantitatively different from the FLRW models used in the data reduction process. This

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[1] A more complete set of references is given in the articles in this issue and the review articles cited.

leads to the possibility that there are large effects on the observed expansion rate (and hence on other observables) due to the backreaction of inhomogeneities in the Universe.

The effects of averaging[2] have been claimed by some authors to be as large as 10–30% [6, 7]. It has further been suggested that inhomogeneities related to structure formation could be responsible for what in the context of the concordance model is interpreted as accelerated expansion [8]. Based on the above remarks, it appears that a deeper understanding of the averaging problem in cosmology and the effects of inhomogeneities on cosmological observations, broadly referred to here as backreaction, is of considerable importance for the correct interpretation of cosmological data.

Inhomogeneities in the Universe can influence observations through backreaction effects from local inhomogeneities, and also through perturbative effects. There have been a number of sessions and plenary talks on these topics at recent conferences, eg. the Invisible Universe and Marcel Grossmann meetings which took place in Paris 2009 (see the review articles by Peebles [9], Kolb [10], Wiltshire [6], Schwarz [7] and van den Hoogen [11]). In addition, perturbative effects were treated in a recent special focus section of *Class. Quantum Grav.* [24], in which a collection of invited papers study nonlinear perturbations in cosmological models and whether such nonlinear effects can lead to deviations from Gaussianity which can provide an important probe of models for the origin of structure in the very early Universe (that may be revealed in future high redshift galaxy surveys).

In this focus section, we will concentrate on the physical state of the present Universe and the problem of going beyond perturbation theory. The following topics will be covered: a general overview and a discussion of the relevant issues, inhomogeneous cosmological models (including non-Copernican models), the current observations and physics of the Universe and averaging and backreaction. We note that some of the backreaction discussion may be applicable to quantum gravity [12].

2. Outline of focus section

In the first contribution of this focus section, Ellis [13] discusses inhomogeneous spacetime models, both exact solutions and the averaging process with backreaction effects leading to effective contributions to the averaged energy–momentum tensor, each of which may affect large scale cosmological dynamics. This review serves, together with this editorial, as an overview of this focus section, and also puts the discussion into a historical context (and indicates the central role played by Ellis in formulating the problems under consideration). While we have placed the review of Ellis at the head of the section, in ordering the remaining articles we have attempted to group these according to subject. There are several review articles, and review material in many of the papers, and we will not make a strict distinction between these. Thus, the paper of Kolb, who in addition to Ellis has been instrumental in the recent surge of interest in backreaction effects in cosmology, is located towards the end of the focus section.

The paper of Ellis is followed by an article by Bolejko *et al* [14], where exact solutions of Einstein's equations that generalize the FLRW models are reviewed in detail and, in particular, the importance of an inhomogeneous framework in the analysis of cosmological observations and especially on distance measurements is discussed. The authors emphasize

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[2] Here, we will use this broadly to refer to the fact that, on the one hand, our local observations necessarily leads to observables which average contributions from the inhomogeneous Universe using models based on FLRW cosmological models. We will also refer to the various existing attempts in the literature to systematically study averaged or coarse-grained versions of the Einstein equations as a means to model the effect of inhomogeneities as the averaging problem (see below for further discussion).

that the inhomogeneous cosmological models are not an alternative to the FLRW models, but an exact perturbation of the latter, and also that the effect of inhomogeneities should be taken into account even in models incorporating a cosmological constant.

The next three papers concern some of the relevant cosmological observations. Sylos-Labini [15] reviews whether or not the observed galaxy structures are compatible with the standard model of galaxy formation. This depends crucially on the *a priori* assumptions encoded in the statistical methods employed to characterize the data and on the *a posteriori* assumptions made in the interpretation of the results. Evidence is presented that, in the available samples, the galaxy distribution is actually spatially inhomogeneous but statistically homogeneous and isotropic.

It has been proposed that the observed dark energy can be explained by the effect of large-scale nonlinear inhomogeneities such as voids, and that such models are compatible with all observations. In the next paper, Marra and Notari [16] discuss how observations constrain cosmological models featuring large voids, particularly non-Copernican models in which the observer is close to the centre of a very large void. This theme is continued in the article by Zibin and Moss [17], where constraints on inhomogeneous void models using the linear kinetic Sunyaev–Zel'dovich (kSZ) effect due to structure within the void are discussed.

Wiltshire [18] then reviews the problems of coarse-graining and averaging of inhomogeneous cosmologies, and their backreaction on average cosmic evolution, from a physical viewpoint. A brief overview of the current status of averaging is presented, focusing on comparing different notions of average homogeneity and on the interpretation of observational results. The emphasis is on the conceptual basis and a physically-motivated critique of the various techniques is discussed, rather than (an exhaustive discussion) of mathematical techniques in averaging, which have been reviewed recently by, for example, van den Hoogen [11].

The study of backreaction effects is aimed at the construction of physical cosmological models in relativistic cosmology and a thorough reinterpretation of observational data. In the article by Buchert [19] it is shown that backreaction terms can be encoded by spatially averaged geometrical invariants, which are absent and interpreted as missing dark fundamental sources in the standard model. A number of fundamental issues are discussed within a covariant framework, including a description of the backreaction term as an effective scalar field. Räsänen [20] provides a further discussion of backreaction, and conjectures that the observed slower expansion and shorter distances than those predicted in spatially homogeneous and isotropic cosmological models with ordinary matter and gravity might be due to the known breakdown of homogeneity and isotropy related to structure formation.

We then turn to the perturbative approach to averaging, which is one of the important themes of this focus section. In contrast to studying nonlinear models, there is a large literature studying backreaction effects on the basis of linearly perturbed FLRW models. Some researchers claim that the weak field perturbative approximation is adequate to describe the nonlinear structures because the gravitational potential is very small even though the density contrasts are very large, and consequently the backreaction effect is negligible (see, for example, [30] and references therein). However, criticism of these (standard) arguments are presented in the articles by Ellis [13], Wiltshire [18] and Räsänen [20] in this section. In particular, the issue is discussed in detail in the contributions by Kolb [21] and Clarkson and Umeh [22], who argue that a weak-field approximation to a spatially homogeneous model is not sufficient and that the effects are non-Newtonian and non-perturbative.

In both of these articles it is argued that the linear models of perturbation theory are not adequate to study the affects of inhomogeneities; second-order perturbations are non-negligible and can be of the same size as first-order terms; fourth-order terms can be of the same size as second-order terms, leading to a breakdown in perturbation theory; indeed, perturbation theory leads to corrections to the background of order unity and higher order terms can diverge. In particular, Clarkson and Umeh [22] focus on calculations of observable quantities such as the all-sky average of the distance–redshift relation. Since the effect of backreactions can be significant, Kolb [10] discusses the proposal that the backreaction of inhomogeneities on scales much less than the Hubble radius mimics dark energy.

Finally, as an alternative to averaging, Clifton [23] presents a brief summary of attempts to build cosmological models containing discrete structures, rather than a fluid, highlighting that cosmological models can result in observables in such a spacetime that evolves differently from the smoothed out version.

3. Discussion and open problems

A number of fundamental issues in inhomogeneous cosmology have been addressed in this focus section. The motivations for this research are discussed in detail in the contributions to this issue (and the reviews cited) and include the facts that there is really no direct convincing observational evidence for spatial homogeneity at late epochs, data analysis often includes FLRW-model-dependent assumptions and observations do not prove that there is dark energy or that the Universe is accelerating in the usual sense (they do so only if one assumes that the Universe is spatially homogeneous and isotropic and the dynamics of the Universe are governed by GR). In addition, it is a fact that there are exact inhomogeneous models that are compatible with all current observational data [5]. Moreover, we often observe averaged quantities, but make our predictions for the local quantities. An additional complication is that we actually average over a light-cone volume rather than a spatial volume.

Although this focus section is devoted to discussing alternatives to the standard FLRW model, we have attempted to present opposing views where possible and appropriate. However, there are a number of things not explicitly discussed here. Let us briefly remark on two.

First, we should restate the fact that the ΛCDM model is essentially consistent with all the present tests. As emphasized by Peebles [9] (and in many other recent reviews), the consistency of independent constraints on the relevant parameters in ΛCDM makes a close to compelling case that we have not been seriously misled. That certainly does not mean that the ΛCDM cosmology is the final word on the structure and physics of the Universe, and it is certainly not impossible to find and demonstrate the viability of alternatives to the ΛCDM cosmology. In addition, the many open issues in this subject make it reasonable to expect that a more accurate cosmology will have more interesting physics in the invisible sector of the Universe, and maybe also in the visible part. The standard position, as championed by Peebles for example, is the belief that that this physics has a negligibly small effect on the general expansion, and that a more proactive strategy for discovering a possibly more accurate cosmology is to search for apparent anomalies in ΛCDM [9]. Of course, there is still the necessity to explain the theoretically unnatural value for Λ.

However, standard arguments like these are based on Newtonian physics and intuition, which are generally incorrect in GR (e.g, the LTB examples). As discussed by Kolb [10], this way of framing the argument makes manifest the fact that there are dynamical assumptions that enter, and it cannot simply be claimed that small peculiar velocities rule out the backreaction proposal. Statements that the expansion of the Universe is accelerating or that dark energy exists are made within the framework of a particular cosmological model; if the cosmological model is inappropriate or incomplete, the interpretation of the observations may be wrong. In particular, the most important point regarding dark energy (and, for that matter, the acceleration of the expansion of the Universe) is that all evidence for dark energy is indirect, because the only effect of dark energy is on the expansion history of the Universe. This situation has led many to consider alternative explanations of the observations. As an alternative to a cosmological constant, we might try to add something to the right-hand side of the field equations (FE) such as, for example, quintessence and scalar fields (braneworlds, extra dimensional models, etc), or the left-hand side of the FE, such as modified gravity (e.g., $f(R)$-models).

However, as has been emphasized in this focus section, perhaps a more conservative left-hand side is to include the *effects of cosmic structure* (i.e. the effects of cosmic averaging and nonlinear structure formation itself). In this scenario, the Friedmann equation has corrections due to inhomogeneities in the Universe. A critical discussion of some of the attempts to explain the phenomenon of dark energy as an effective description of complex gravitational physics and the proper interpretation of observations was presented in [7].

Related to this is the question of whether the perturbative approach in the standard model is adequate. At some level, the perturbative approach to backreaction effects can be at best a self-consistent approach (although self-consistency in itself is quite remarkable). Indeed, it might be expected on general grounds that in a matter-dominated Universe, nonlinear effects such as averaged curvature and kinematic backreaction will eventually lead to a ‘breakdown’ of perturbation theory. This is discussed in detail in this focus section, and especially in the articles by Kolb [21] and Clarkson and Umeh [22].

Second, there is no doubt that averaging produces correction (correlation) terms to the Einstein FE. The only dispute left is about the order of magnitude of the effect. However, the more theoretical (mathematical) issues of averaging are not the focus of the current issue. Some brief remarks are made in the review by Wiltshire [18] and a recent review is given by van den Hoogen [11]. For completeness, let us make some further brief remarks in the overview here.

The Universe is not isotropic or spatially homogeneous on local scales. If the effects of inhomogeneities on our smoothed out idealized model cannot be neglected, then they must be incorporated into the model. The gravitational FE on large scales are obtained by averaging the Einstein FE of GR, which leads to the question of whether GR is the correct theory of gravity on cosmological scales. It is necessary to use an exact covariant approach which gives a prescription for the correlation functions that emerge in an averaging of the full tensorial Einstein FE. Hence, a precise mathematical definition of averaging tensor fields on a manifold, and an analytical foundation which connects the inhomogeneities with the observations, is needed.

There are a number of approaches to the averaging problem. Ellis [25] was the first to give a detailed description of the issues and recognized the averaging problem as a fundamental problem in mathematical cosmology. Following the work of Ellis, a number of spacetime averaging procedures have been proposed. These include the covariant averaging procedure of Isaacson and Zalaletdinov [26] which lead to modified gravitational FE for cosmology (in [27] it was suggested that the gravitational correlation within macroscopic gravity behaves like a spatial curvature term). Other promising approaches to spacetime averaging (such as the use of a complete set of scalar curvature invariants [28]) and the widely used 1+3 framework space averaging approach of Buchert [29] (also discussed in this focus section) were reviewed in [11].

3.1. Open problems

A number of open problems have been discussed in the contributions in this focus section such as, for example, a discussion of the validity of the assumption that cosmological matter can be modelled by dust (by Wiltshire [18]), which we will not reproduce here. Let us finish by highlighting some of the more important open problems. These include:

- Determine the cosmological observations in exact inhomogeneous models.
- Provide a rigorous (fully covariant) definition of the spacetime average of a tensor on a differential manifold. Obtain a practical approach to determine the effective 'averaged' or 'coarse-grained' gravitational FE on cosmological scales obtained by averaging the Einstein FE on smaller scales for which it has been tested. Determine whether the correction (correlation) terms are significant.
- Understand the relation between statistical averages and actual observations, and understand physical/observational effects of averaging inhomogeneities; determine whether SNIa data can be explained without dark energy.
- Determine how to treat light-cone averages in realistic cosmological calculations.
- Determine how to *fit* the real-world observations to a smoothed out averaged model. Also determine whether we can do so in such a way as to pointwise determine the deviation between the background model and the real Universe [25].

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