Dissipative Optomechanics in a Michelson-Sagnac Interferometer

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Dissipative optomechanics studies the coupling of the motion of an optical element to the decay rate of a cavity. We propose and theoretically explore a realization of this system in the optical domain, using a combined Michelson-Sagnac interferometer, which enables a strong and tunable dissipative coupling. Quantum interference in such a setup results in the suppression of the lower motional sideband, leading to strongly enhanced cooling in the non-sideband-resolved regime. With state-of-the-art parameters, ground-state cooling and low-power quantum-limited position transduction are both possible. The possibility of a strong, tunable dissipative coupling opens up a new route towards observation of such fundamental optomechanical effects as nonlinear dynamics. Beyond optomechanics, the suggested method can be readily transferred to other setups involving nonlinear media, atomic ensembles, or single atoms.

Recent progress in the engineering of high-quality micromechanical oscillators coupled to high-finesse cavity modes has paved the way towards sensing and control of mechanical motion at the quantum limit [1–5]. The rapid developments in the field of optomechanics bear important implications for both applied and basic science, ranging from applications in high-sensitivity metrology [6–8] and quantum information processing [9–13] to fundamental tests of quantum mechanics at large mass and length scales [14–16].

In the conventional paradigm of optomechanics, the interaction of the mechanical oscillator with a cavity mode is dispersive in the sense that the cavity resonance frequency experiences a shift depending on the displacement of the mechanical oscillator arising from conservative radiation pressure or optical gradient forces. This coherent dispersive interaction has been employed for sideband-cooling to the quantum mechanical ground state [17,18], as well as for the observation of optomechanical normal-mode splitting [19–21] and optomechanically induced transparency [22,23]. The complementary paradigm of dissipative coupling—where the width κ_c of the cavity resonance, rather than its frequency ω_c, is dependent on the mechanical displacement x—was introduced very recently in a theoretical study [24] in the context of electromechanics. This situation, rather unusual for cavity quantum electrodynamics, was shown [24] to give rise to remarkable quantum noise interference effects which dramatically relax requirements for cooling to the ground state without sideband resolution; it also allows reaching the standard quantum limit (SQL) for the imprecision in position measurements. While it is already clear that a strong and tunable dissipative optomechanical coupling would

greatly enrich the toolbox of optomechanics, its full significance for the quantum control of optomechanical systems, e.g., for ponderomotive squeezing [25], nonlinear dynamics [26,27], or pulsed protocols [28], is yet to be explored.

Unfortunately, an optomechanical setup having a strong dissipative coupling in the absence of dispersive coupling has not yet been found. Consider, e.g., a Fabry-Pérot interferometer (FPI) of length L and resonance frequency $\omega_c = \pi n c / L$ ($n \in \mathbb{N}$), with one movable ideal end mirror and an input coupler of transmissivity τ, such that the cavity linewidth is $\kappa_c = c|\tau|^2/(4L)$. When the mirror moves, the cavity length changes, and with it both ω_c and κ_c. The corresponding shifts per zero-point fluctuation x_0 of the mirror oscillator, $g_\omega = (\partial \omega_c/\partial x)x_0$ and $g_\kappa = (\partial \kappa_c/\partial x)x_0$, quantify the strength of dispersive and dissipative couplings, respectively. Their ratio is thus given by the cavity's quality factor, $g_\omega/g_\kappa = \omega_c/\kappa_c \gg 1$, such that the dispersive coupling dominates by far. A similar conclusion applies for a movable membrane coupled to a single mode of a FPI. One may obtain $g_\kappa \simeq g_\omega$ by coupling the membrane to multiple [29,30] transverse modes, such that both types of coupling contribute to the dynamics; however, in such a setup, one cannot "switch off" the dispersive interaction to take advantage of the quantum noise interference effects present in a purely dissipative coupling. This situation persists in the case of evanescent coupling to microdisk resonators [31].

Here, we show that, in a Michelson-Sagnac interferometer (MSI) with a movable membrane [32,33], cf. Fig. 1, a strong and tunable optomechanical coupling can be achieved for which $g_\omega = 0$ but where the effective dissipative coupling strength [20] can be of the order of the
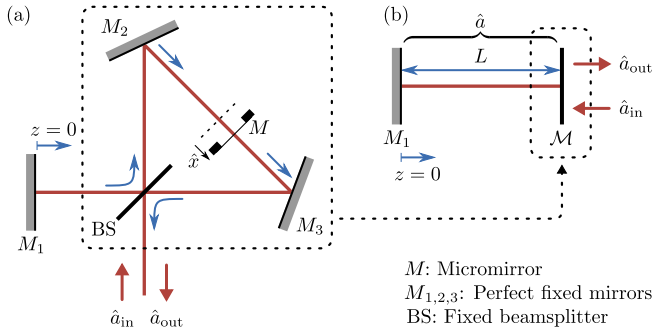
FIG. 1 (color online).   (a) Topology of the Michelson-Sagnac interferometer. (b) Effective cavity; the properties of $\mathcal{M}$ depend on M.

cavity linewidth. The idea is as follows: Consider the MSI operated at a point where the transmissivity τ of the effective mirror is close to zero; τ will then depend sensitively on the membrane displacement x. Combining this compound "MSI mirror" with a perfect mirror in its dark port will result in an effective FPI whose linewidth depends on x dominantly via τ and not via the change in the (effective) cavity length L; see Fig. 1. In contrast to a true FPI, therefore, g_κ and g_ω have a different functional dependence on x. This feature gives rise to a topology (i) where the optomechanical interaction can be tuned at will between being strongly dissipative or dispersive and (ii) which can realize dissipative optomechanics leading to ground-state cooling of the mechanical oscillator via quantum noise interference, as discussed in Ref. [24]. The distinctive signature of this interaction, an asymmetric Fano line shape, is observable in the spectrum of the cavity output field. We also report on the suitability of using this system for (iii) sensitive position transduction.

This Letter is structured as follows. We shall first describe the physical model and write down the resulting Hamiltonian and input-output relation. These are then used to derive the equations of motion for the cavity field and oscillator motion. The resulting dynamics is solved to obtain the steady-state mechanical occupation number analytically in the weak-coupling limit and also numerically in both weak- and strong-coupling regimes.

Model.—The Hamiltonian including both dispersive and dissipative effects in an opto- or electromechanical [24] system can be written as [34]

$$\hat{H}_{OM} = (-\Delta + g_\omega \hat{x})\hat{a}^\dagger \hat{a} + \omega_m b^\dagger b + \int d\omega\, \omega a_\omega^\dagger a_\omega$$

$$+ i\left(\sqrt{2\kappa_c} + \frac{g_\kappa}{\sqrt{2\kappa_c}}\hat{x}\right)\int \frac{d\omega}{\sqrt{2\pi}}(\hat{a}_\omega^\dagger \hat{a} - \text{H.c.}). \quad (1)$$

This Hamiltonian is written in a frame rotating at the optical frequency of some driving field ω_d; $\Delta = \omega_d - \omega_c$ is the detuning from cavity resonance (ω_c) whose annihilation operator is $\hat{a}$. $\hat{x} = (\hat{b} + \hat{b}^\dagger)/\sqrt{2}$ is the dimensionless displacement, and $\hat{b}$ is the annihilation operator

for the mechanical oscillator of frequency ω_m. The second line in Eq. (1) provides the coupling of the cavity mode $\hat{a}$ to the modes $\hat{a}_\omega$ of the external field coupling into and out of the effective FPI and will give rise to the finite width κ_c of the resonance. The two terms proportional to $\hat{x}$ describe the shifts of the cavity resonance and width with mechanical displacement, with strengths characterized by $g_{\omega,\kappa}$, respectively. This Hamiltonian generalizes the dispersive-only Hamiltonian that is considered in most works on optomechanical systems. Before we proceed to make the connection of Hamiltonian (1) and the physics of a MSI [32,33], we note that the standard cavity-input–output relation must be generalized to accommodate the effect of dissipative coupling [34]:

$$\hat{a}_{\text{out}} - \hat{a}_{\text{in}} = [\sqrt{2\kappa_c} + (g_\kappa/\sqrt{2\kappa_c})\hat{x}]\hat{a}. \quad (2)$$

This relation provides a boundary condition connecting light leaving the effective FPI with light entering it, and the intracavity dynamics. It can be identified with the familiar relation $\hat{a}_{\text{out}} - \hat{a}_{\text{in}} = \sqrt{2\kappa(\hat{x})}$, taking $\kappa(\hat{x}) = \kappa_c + g_\kappa \hat{x}$ and truncating the square root to first order in $\hat{x}$.

The generalized optomechanical Hamiltonian (1) is realized in the MSI shown in Fig. 1 operating close to a dark-port condition, i.e., when most light is directly reflected at the first beam splitter (BS). The whole system can then be described as an effective Fabry-Pérot interferometer of length L, operating in the good-cavity limit, formed between the perfect end mirror M_1 and an effective end mirror $\mathcal{M}$; $2L$ is the length of the Sagnac mode M_1-BS-M_2-M_3-BS. The reflectivity ρ and transmissivity τ of $\mathcal{M}$ depend on the complex reflectivity R ($\mathfrak{r}$) and transmissivity T ($\mathfrak{t}$) of the BS (of the micromirror), as well as the displacement δ of M from the midpoint of M_2-M_3:

$$\rho = -(R^2 \mathfrak{r} e^{2i\delta} + T^2 \mathfrak{r} e^{-2i\delta} + 2RT\mathfrak{t})e^{-i\arg\mathfrak{t}}, \quad \text{and} \quad (3a)$$

$$\tau = [(RT^* e^{2i\delta} - \text{c.c.})\mathfrak{r} - (|R|^2 - |T|^2)\mathfrak{t}]e^{-i\arg\mathfrak{t}}. \quad (3b)$$

Assuming the close-to-dark-port condition $|\tau| \ll 1$, a quantization along the standard routes of cavity QED [34] yields a Hamiltonian of the form in Eq. (1) with the usual result $\kappa_c = -c/(2L)\ln|\rho| \approx c|\tau|^2/(4L)$, as well as dispersive and dissipative couplings

$$g_\omega = -2(\omega_c x_0/L)[(|R|^2 - |T|^2) + \tau\cos(\arg\mathfrak{t})], \quad (4a)$$

$$g_\kappa = -\sqrt{2}i|\tau|e^{i\phi}(\omega_c x_0/L)[2RT + \rho\cos(\arg\mathfrak{t})], \quad (4b)$$

where $\phi \approx 0$ close to resonance. These results guarantee that the values of g_ω and g_κ can be controlled independently by choosing δ (i.e., positioning the membrane) and the reflectivity of the central BS appropriately. The need for a sharp resonance demands $|R| \approx |T|$. We note, however, that $|R| \neq |T|$ is required to be able to set $g_\omega = 0$ with $|\tau| > 0$. In the following, we will specialize to this most interesting case of a purely dissipative coupling. We will provide experimental case studies below and

show that strong coupling, where $|\bar{a}||g_\kappa| \gg \kappa_c$ ($\bar{a}$ being the intracavity amplitude), can be achieved for a moderate driving power of a few hundred μW.

The next step in our investigation is to derive the Heisenberg equations of motion for $\hat{a}$, $\hat{x}$, and $\hat{p} = (\hat{b} - \hat{b}^\dagger)/(i\sqrt{2})$. The Hamiltonian (1) implies a full and rich nonlinear dynamics [35], including such effects as bistability and self-induced oscillations, but in this new context of dissipative optomechanics. Here, we will focus on the linear dynamics by assuming a strong classical driving field $\overline{a_{\text{in}}}$, which allows us to write the linear equations of motion for the fluctuations around the steady state as [34]

$$\dot{\hat{a}} = (i\Delta - \kappa_c)\hat{a} - \sqrt{2\kappa_c}\hat{a}_{\text{in}} - g_\kappa(\overline{a_{\text{in}}}/\sqrt{2\kappa_c} + \bar{a})\hat{x}, \quad (5)$$

$$\dot{\hat{p}} = -\omega_m\hat{x} - 2\kappa_m\hat{p} - \sqrt{2\kappa_m}\hat{\xi}$$
$$- ig_\kappa/\sqrt{2\kappa_c}[(\overline{a_{\text{in}}}^*\hat{a} + \bar{a}\hat{a}_{\text{in}}^\dagger) - \text{H.c.}], \quad (6)$$

and $\dot{\hat{x}} = \omega_m\hat{p}$, with the mechanical motion having the damping rate κ_m. $\bar{a}$ is the coherent part of the cavity field, and $\hat{\xi} = \hat{\xi}^\dagger$, which models Brownian-motion–type noise acting on the mechanical oscillator, is assumed to obey $\langle\hat{\xi}(t)\hat{\xi}(t')\rangle = (2n_{\text{th}} + 1)\delta(t - t')$, where n_{th} is the thermal occupation number in the absence of driving and $\langle\hat{\xi}\rangle = 0$. The condition for the c-number component $\overline{a_{\text{in}}}$ of the input field to be "large enough" is obtained by evaluating the contributions of both components of the input field to $\langle\hat{a}^\dagger\hat{a}\rangle$. This linearization condition can be stated as $|\overline{a_{\text{in}}}|^2 \gg [(|\Delta| + \omega_m)^2 + \kappa_c^2]/(2\kappa_c)$ [34] and will be assumed to be satisfied in the following. In the weak-coupling limit, when $|g_\kappa|^2|\overline{a_{\text{in}}}|^2 \ll 4\kappa_c^3$, we assume that the cavity field follows the mechanical motion adiabatically and solve the linearized dynamics to obtain the steady-state mechanical occupation number:

$$\langle\hat{b}^\dagger\hat{b}\rangle = \frac{n_{\text{th}}\kappa_m}{\bar{\kappa}_m} + \frac{|g_\kappa|^2}{4\bar{\kappa}_m}\frac{|\overline{a_{\text{in}}}|^2}{\Delta^2 + \kappa_c^2}$$
$$\times \frac{\kappa_c(2\Delta - \bar{\omega}_m)^2 + \bar{\kappa}_m(\Delta^2 + \kappa_c^2 + \kappa_c\bar{\kappa}_m)}{\kappa_c[(\Delta - \bar{\omega}_m)^2 + (\kappa_c + \bar{\kappa}_m)^2]}. \quad (7)$$

In this equation, $\bar{\omega}_m$ and $\bar{\kappa}_m$ are the optically shifted mechanical oscillator frequency and damping rate, respectively, whose expressions are rather involved and will not be reproduced here [34]. The preceding relation is valid even for moderately strong input powers, as can be shown by comparing this analytical result with that obtained by solving the above equations of motion exactly using the method in Ref. [36]. In the "cryogenic optomechanics" limit, where $\kappa_c \gg \bar{\kappa}_m \gg n_{\text{th}}\kappa_m$ and $\bar{\omega}_m \approx \omega_m$, this result simplifies to

$$\langle\hat{b}^\dagger\hat{b}\rangle \approx \frac{|g_\kappa|^2}{4\bar{\kappa}_m}\frac{|\overline{a_{\text{in}}}|^2}{\Delta^2 + \kappa_c^2}\frac{(2\Delta - \omega_m)^2}{(\Delta - \omega_m)^2 + \kappa_c^2}. \quad (8)$$

This expression for $\langle\hat{b}^\dagger\hat{b}\rangle$ deserves some comments. We recall that, in dispersive optomechanics, cooling is

optimized when the upper motional sideband is strongly enhanced ($\Delta = -\omega_m$) and the lower sideband is strongly suppressed ($\omega_m \gg \kappa_c$). In the present case, Fano-like interference can be observed in the back action force noise spectrum $S_{\hat{F}\hat{F}}(\omega)$, cf. the inset of Fig. 2, where $\hat{F}$ [the second line in Eq. (6)] is the force operator acting on the micromirror motion. This resonance neutralizes the lower sideband when $\Delta = \omega_m/2$; the Fano line shape also means that optimal enhancement of the upper sideband requires not $\omega_m \gg \kappa_c$—indeed, that would be disingenuous—but $\omega_m = \omega_m^{\text{opt}} = \frac{2}{3}\sqrt{\frac{2\sqrt{13}-5}{3}}\kappa_c \approx 0.6\kappa_c$, which is much less demanding than the sideband-resolved condition. Figure 2 shows how the mechanical occupation number, at the optimal detuning $\Delta = \omega_m/2$ and with an input power $P_{\text{in}} = 10$ nW, changes as the ratio ω_m/κ_c is varied and is minimized when $\omega_m \approx \omega_m^{\text{opt}}$.

Cooling.—Let us now turn our attention towards predicting the cooling performance of the model investigated above. We shall call the following "system I": mechanical oscillator effective mass $m = 100$ ng, frequency $\omega_m = 2\pi \times 103$ kHz, and quality factor $Q = \omega_m/(2\kappa_m) = 2 \times 10^6$, such that the zero-point fluctuation is $x_0 = \sqrt{\hbar/(2m\omega_m)} \simeq 1$ fm; $|R|^2/|T|^2 = 0.486/0.514$; $|r|^2/|t|^2 = 0.362/0.638$; driving wavelength $\lambda_c = 1064$ nm; and $L = 7.5$ cm, which are experimentally realizable [33] and yield $\kappa_c = 2\pi \times 196$ kHz and $g_\kappa \approx 2\pi \times 0.1$ Hz ($g_\kappa/x_0 \approx 2\pi \times 79$ kHz/nm). We limit the input power to the regime where heating from the power absorbed is not the dominant process. An input power of 10 mW corresponds to an effective coupling strength $G = |\bar{a}||g_\kappa| \approx 0.1\kappa_c$. By starting from an environment temperature $T_{\text{env}} = 300$ K, we can decrease the occupation number by over 3 orders of magnitude, as illustrated in Fig. 3(a).

Consider next a hypothetical, but still physically realizable, situation ("system II"), where M has a smaller mass ($m = 50$ pg [29]), higher mechanical quality ($Q = 1.1 \times 10^7$ [37]), and higher reflectivity ($|r|^2/|t|^2 = 0.818/0.182$,
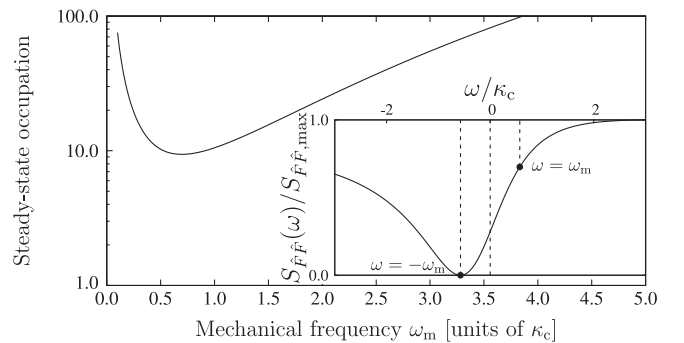

FIG. 2. Calculated steady-state occupation number for system II. Throughout this plot, $\Delta = \omega_m/2$ and $P_{\text{in}} = 10$ nW. Inset: Normalized back action force noise density, illustrating its Fano profile, when $\omega_m \approx 0.6\kappa_c$ and $\Delta = \omega_m/2$; the noise density at $\omega = -\omega_m$ is zero.
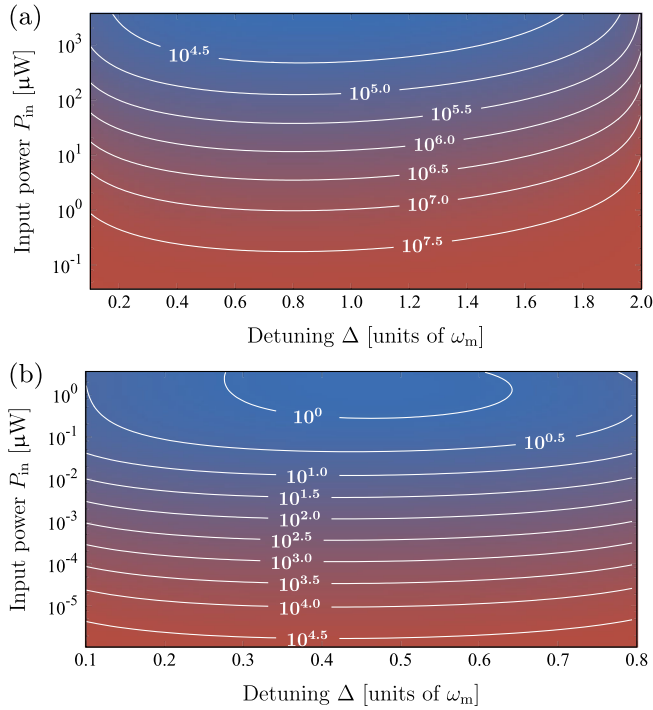
(a)

(b)

FIG. 3 (color online).   Full numerical solution for the occupation number $\langle \hat{b}^\dagger \hat{b} \rangle$ for (a) system I and (b) system II.

possible by patterning the membrane [38]), where BS is more balanced ($|R|^2/|T|^2 = 0.496/0.504$ and $\kappa_c = 2\pi \times 59$ kHz), and thus $g_\kappa \approx 2\pi \times 2.6$ Hz ($g_\kappa/x_0 \approx 2\pi \times 65$ kHz/nm). For this system, we can achieve the strong-coupling condition $G \gtrsim \kappa_c$. If we also assume cryogenic operation at $T_{\text{env}} = 0.3$ K, we can see from Fig. 3(b) that ground-state cooling is possible, despite the poor reflectivity of the mechanical oscillator, and that $\omega_m \sim \kappa_c$.

Position transduction.—Optomechanical systems are one promising approach towards extremely sensitive position transduction [1]. The coupling of a mechanical oscillator to multiple cavity modes was recently explored in Ref. [39]; a common feature of "multiple-light-mode–single-mechanical-mode" systems is a Fano-like profile in $S_{\hat{F}\hat{F}}(\omega)$ (Fig. 2, inset). The corresponding antiresonance allows one to reach the SQL for measurement imprecision at a significantly lower input power than a single-light-mode–single-mechanical-mode optomechanical system. Indeed, let us quantify the achievable resolution of a position measurement by the ratio $\mathcal{N}/\mathcal{S}$, where the "noise" $\mathcal{N}^2$ is the contribution to the symmetrized homodyne output spectrum evaluated at $\omega = \omega_m$, $\bar{S}_{\text{out}}(\omega_m)$, due to $\hat{a}_{\text{in}}$, and the "signal" $\mathcal{S}^2$ is that due to $\hat{\xi}$, normalized to the free mechanical motion noise spectrum $\bar{S}_{\text{free}}(\omega_m)$:

$$\bar{S}_{\text{out}}(\omega_m) = \mathcal{N}^2 + \mathcal{S}^2 \bar{S}_{\text{free}}(\omega_m). \quad (9)$$

At $\Delta = 0$ and under identical conditions, it can be shown that $\mathcal{N}/\mathcal{S}$ reaches the same lower bound (the SQL) in both dissipative (at a power P_κ) and dispersive (P_ω) cases, but

with $P_\kappa = (2\kappa/\omega_m)^2 P_\omega$. In the sideband-resolved regime, therefore, one can obtain significantly better position resolution at low powers in comparison with the dispersive case.

Comments.—We set $g_\omega = 0$ early on in this Letter, which can be realized by placing M at a point where the field intensity surrounding it is close to minimal, greatly reducing the power absorbed P_{abs} by the membrane. For the parameters in Fig. 3(a), using the "thermal link" from Ref. [40], the membrane temperature rises by ca. 60 K at $P_{\text{in}} = 1$ mW. The resulting temperature rise has a significant effect on the base occupation number of the micromirror but still allows strong cooling of the micromirror motion. It is a feature of our topology that the ideal position of M corresponds to both where P_{abs} is greatly reduced and where the competing dispersive optomechanics is switched off.

Significantly, we note that this situation does not persist in the case of the "membrane-in-the-middle" geometry. In a single-transverse-mode model for this latter situation, the dispersive and dissipative optomechanics cannot be independently turned off and are both zero at the nodes of the cavity field; this leads to a stronger restriction arising from the power absorbed. Moreover, dispersive (dissipative) optomechanical cooling requires $\Delta < 0$ ($\Delta > 0$); most treatments of this geometry do not include the dissipative optomechanics component of the dynamics [41] and (at least within a single-mode model) may therefore overestimate the cooling efficiency in the non-sideband-resolved regime.

Conclusions.—In this Letter, we have presented an experimentally feasible realization of an optomechanical system that can be fully tuned between strong dissipative and dispersive dynamics. The cooling mechanism we presented works best in the non-sideband-resolved regime, unlike the usual dispersive case. For an existing set of experimental parameters, we predict a strong cooling effect arising from a Fano-type resonance in the back action force acting on the mechanical motion. In the case of more optimistic parameters, we predict ground-state cooling of the mechanical motion. In the opposite, sideband-resolved regime, our system promises significantly improved position measurement resolution. The usage of the proposed implementation of a dissipative optomechanical coupling for nonlinear (quantum) dynamics [26,27] and for pulsed dynamics [28] remains to be explored.

Moreover, we believe that the topology of a MSI with a highly reflective mirror in its dark port—the signal-recycling configuration used in the context of gravitational wave detectors [42,43]—and the resulting dissipative intracavity dynamics provides attractive possibilities for other fields in cavity QED, especially in combination with single atoms or ensembles of cold atoms.

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